

ESSAYS ON REPURCHASE AGREEMENTS, BAILOUTS,  
AND THE MACROECONOMIC EFFECTS  
OF BANK FAILURES

by

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# ABSTRACT

In this dissertation, I study different aspects of the financial system within times of crisis with special attention paid to money market mutual funds (MMMF) and their use of repurchase their agreements ("repos").

Repos play a significant role in global credit market activity. Yet, research regarding the underlying components of repos remains nascent. In the first chapter, I examine the role of risk tolerance on lending and collateral within these agreements.

In the second chapter, I study a model in which a risk-pooling intermediary engaging in the repo market, such as a MMMF, is exposed to runs. Inspired by events during the financial crisis, I show that bailouts are part of an efficient social insurance scheme in the event that a run emerges. However, this result does not imply that optimal intervention completely isolates shadow-banking intermediaries from a crisis. In fact, optimal public sector intervention imposes costs on money market funds by requiring them to liquidate some collateral. On the other hand, a commitment to no bailouts contributes to financial instability as the repo market collapses in the wake of a run without a public safety net.

Chapter three expands my analysis of financial stress into the realm of traditional depository institutions. This chapter examines the effects of exogenous bank failures in the United States from 1973 - 2006. My results support the previous literature which suggests that real economic activity is lower following a shock. However, in addition to a linear model, I consider the possibility of a threshold value within banking shocks. Upon examination, I find that the effects of exogenous

failures become asymmetric around \$3 billion. Larger shocks are shown to impact the economy significantly. By comparison, smaller shocks have a completely ambiguous effect.

## DEDICATION

This dissertation is dedicated to my friends and family. To my friends, Nicholas John Pihakis and John Bernard Halbert III, I want to say thanks for always challenging me intellectually. You both continue to set the bar high and I do my best to keep up and challenge you back. To my parents, Larry Lee Otto and Dawn Charlotte Otto, thank you for your constant love and support. You are always there to lend me your ear and a helping hand – both of which I take for granted far too often. It is truly a privilege to be a part of your lives.

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# CONTENTS

ABSTRACT .....	ii
DEDICATION.....	iv
ACKNOWLEDGMENTS.....	v
LIST OF TABLES .....	ix
LIST OF FIGURES .....	x
CHAPTER 1: Repos, Risk Aversion, and Haircuts.....	1
1 Introduction .....	1
2 The Model.....	5
2.1 Endowments, Preferences, and Technologies.....	5
2.2 Timing of Actions.....	6
3 Optimal Choices.....	8
3.1 Repo Solutions.....	12
4 Numerical Analysis .....	14
4.1 Leverage Elasticity .....	19
5 Conclusion.....	20
References.....	21
CHAPTER II: Repos, Bailouts, and Instability in the Shadow Banking System....	23
1 Introduction .....	23
2 The Model.....	27
2.1 Preferences and Technologies.....	28
2.2 Unexpected Shocks and Financial Instability .....	30

3	The Planner's Allocation .....	31
3.1	Period 2: Settlement (Repurchase of Collateral) .....	34
3.2	Period 1: Collateral Liquidation and Bailout Funding .....	36
3.2.1	Optimizing Consumption in the Good Aggregate State .....	37
3.2.2	Optimizing Consumption in the Bad Aggregate State.....	39
3.3	Period 0: The Planner's Investment in the Repo Market, Short-Term Returns, and Taxes .....	42
3.4	Fragility.....	43
4	Bailouts and Collateral Liquidation.....	45
5	Equilibrium under Discretion .....	46
5.1	Intermediaries' Decisions .....	48
5.2	Fiscal Authority's Tax Decision.....	48
5.3	Bailouts vs. No Bailouts.....	49
6	Comparing Bailout Cases.....	51
7	Conclusion .....	52
	References.....	54
	Appendix .....	56
	CHAPTER III: The Macroeconomic Effects of Bank Failures .....	75
1	Introduction .....	75
2	The Basic Model .....	78
2.1	The Data.....	78
2.2	The Methodology.....	79
3	Aggregate Effects of Bank Failure Shocks.....	81
A.	Building the Framework.....	81



B. Durable and Non-Durable Goods Markets .....	83
C. Interest Rates and Credit Extension.....	84
D. Private Saving .....	85
4 Asymmetric Shocks.....	86
4.1 Threshold Testing.....	86
4.2 Small vs. Large Shocks: The Macroeconomic Effects .....	88
5 Conclusion .....	89
References .....	91
Appendix .....	93

## LIST OF TABLES

1.1 Benchmark Repo Market .....	14
1.2 Benchmark Repo Market as $\gamma_0$ Changes .....	15
1.3 Benchmark Repo Market as $\pi_0$ Changes .....	16
1.4 Repo Market under $\gamma_1$ as $\pi_0$ Changes .....	17
1.5 Benchmark Repo Market as $p$ Changes .....	18
1.6 Leverage Elasticity, Case 1 .....	19
1.7 Leverage Elasticity, Case 2 .....	19
3.1 Bank Failure Statistics (Normalized Failures in Levels) .....	78
3.2 F-Test on $\phi_1$ .....	88

## LIST OF FIGURES

3.1 Aggregate 8-Variable System Response to a FFR Shock .....	93
3.2 Aggregate 8-Variable System Response to a BF Shock.....	93
3.3 Aggregate 9-Variable System Response to a FFR Shock .....	94
3.4 Aggregate 9-Variable System Response to a BF Shock.....	94
3.5 System Response for the Nondurable Goods Market to a BF Shock.....	95
3.6 System Response for the Durable Goods Market to a BF Shock.....	95
3.7 Response of the Prime Interest Rate to a BF Shock .....	96
3.8 Response of the 30-yr Mortgage Rate to a BF Shock.....	96
3.9 Response of Consumer Loans to a BF Shock.....	97
3.10 Response of Business Loans to a BF Shock.....	97
3.11 Response of Private Savings to a BF Shock.....	98
3.12 Response of Small Time Deposits to a BF Shock .....	98
3.13 Response of Demand Deposits to a BF Shock .....	99
3.14 Response of Bank Cash Holdings to a BF Shock.....	99
3.15 Bank Failure Shock Series .....	100
3.16 Plot of the Residual Sum of Squares.....	100
3.17 Small/Large Shock Series .....	101
3.18 Aggregate System Response to a Large BF Shock.....	101

# CHAPTER I:

## Repos, Risk Aversion, and Haircuts

### 1 Introduction

Due to their significant role in credit market activity, there has been a lot of attention devoted towards understanding repo markets in recent years. Notably, Bernanke (2010) states: “Leading up to the crisis, the shadow banking system, as well as some of the largest global banks, had become dependent on various forms of short-term wholesale funding...In the years immediately before the crisis, some of these forms of funding grew especially rapidly; for example, repo liabilities of U.S. broker dealers increased by 2-1/2 times in the four years before the crisis.” In addition, Hördahl and King (2008, p. 37) estimate “gross amounts outstanding [of repos] at year-end 2007 of roughly \$10 trillion in each of the US and Euro markets, and another \$1 trillion in the UK repo market.” The figures for the United States are on par with the size of the traditional commercial banking market.

What is a repurchase agreement? Repo lending consists of two basic parts. In the first leg of a repo, cash-lenders provide loans to borrowers in exchange for collateral. The second leg completes the transaction with borrowers repurchasing the collateral from the cash-lender at a higher price.<sup>1</sup> For example, a money market fund (cash-lender) purchases collateral from a broker-dealer (borrower) with an agreement by the dealer to repurchase the collateral at a higher price on a later date. The higher price reflects interest on the loan or the “repo rate.”

In comparison to depositors at traditional banks, participants in the repo market are generally non-banking institutions which do not have access to many of the same guarantees

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<sup>1</sup>The lender in essence is purchasing the collateral. If the borrower defaults in the agreement, the lender can immediately sell the collateral. By doing so, the lender is able to avoid lengthy Chapter 11 bankruptcy proceedings. For more discussion of bankruptcy policy rules and repo markets, see Antinolfi et al. (2012).

and safeguards from failures as traditional banks such as deposit insurance. Instead, repo investors utilize collateral as a form of insurance against bad outcomes. That is, the collateral in a repo allows investors to recoup their losses in the event of some failure. Consequently, the amount of collateral that lenders require in a repo depends on their tolerance for risk.

In particular, the level of “haircuts” is an important component of security repurchase agreements. Haircuts are the extent to which repo transactions are *overcollateralized*. Imagine a case where a borrower sells \$100 worth of collateral to a lender for a loan of \$90. This transaction is said to be overcollateralized by 10% or, alternatively, there is a 10% haircut on the collateral sold by the borrower. If the borrower repurchases the collateral for \$95, then the repo rate would be roughly 5.5%.

A number of recent contributions have shown that subjective perceptions of risk affect behavior among financial market participants. For example, Malmendier and Nagel (2011) find that people who were alive during the Great Depression invest less in equities. In addition, Guiso, Sapienza, and Zingales (2011) observe that investors in Italy became more risk averse during the recent financial crisis. Koudijs and Voth (2014) contend that lenders most exposed to risk in the 18th century Amsterdam stock market imposed higher haircuts on collateralized loans. In the case of repo markets during the recent crisis, Gorton and Metrick (2010, p.5) catalogue the rise of haircuts from 2005 - 2008: “ABX data show that the deterioration of the subprime market began in early 2007.”<sup>2</sup> Moreover, haircuts ranged from nearly 0 in early 2007 to 5% in the fall of 2007. In the following year, concerns regarding the subprime mortgage market continued and haircuts rose to 25%.

Following the bankruptcy of Lehman Brothers, repo markets seized up. As Copeland, Martin, and Walker (2010) note, it is critically important for policymakers to understand the underlying factors which contribute to failures in repo markets. Adrian et al. (2013) stress that the relationships between (a) funding amount, (b) repo rates, (c) haircuts, and (d) counterparty risk need to be studied in order to convincingly address the structure of

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<sup>2</sup>The ABX index, or asset-backed securities index, measures the overall value of mortgages in the subprime mortgage market.

repo markets and the potential for instability in the financial system. We suggest that risk aversion also needs to be addressed when considering the underlying structure of repo market activity.

Though the structure of a security repurchase agreement may appear to be relatively straightforward, policymakers and regulators have little guidance for predicting how repo markets are likely to respond to different economic conditions. In particular, Bernanke (2012) emphasizes: “Unfortunately, data on the shadow banking sector, by its nature, can be more difficult to obtain. Thus, we have to be more creative to monitor risk in this important area.” The association between risk and haircuts is particularly elusive as Dang, Gorton and Holmström (2013b, p. 3) explain: “The existence of repo haircuts is a puzzle, as standard finance theory would suggest that risk simply be priced and the market price reflects risk and risk aversion of the market.” That is, Dang et al. (2013b) intimate that standard finance theory only accounts for uncertainty ex-ante rather than ex-post when default occurs. Consequently, standard finance theory cannot be used to study repo markets.

The objective of this paper is to study a repurchase agreement from a risk-sharing perspective. The environment we analyze is absent commitment and the decisions are determined by a social planner. In contrast to previous research based on risk neutral agents, we study a repo market in which all participants are risk averse. Lenders in our framework require some form of deposit insurance (overcollateralization) as well as compensation (interest) for the risk they bear during the lending process. As a result, we demonstrate that activity in repo markets critically depends on the degree of risk aversion among participants in the financial system.

The absence of commitment among repo market participants is an important friction in our framework. As emphasized by Mills and Reed (2012), repo markets are limited by two-sided moral hazard. Typically, models of collateralized lending are only concerned about strategic default by the borrower (Lacker 2001; Manove, Padilla, Pagano 2001; Rampini 2005; Kehoe and Levine 2008). In a repurchase agreement, however, lenders can sell collateral in

a secondary market. This option reflects the insurance role of collateral. In addition, absent a commitment mechanism, lenders can enter the secondary market at any time. Therefore, both borrowers and lenders must receive appropriate incentives in order to avoid strategic default.

Gorton and Metrick (2012) observe that increases in the risk of default led to higher haircuts imposed on collateral, indicating that there is greater reliance on the insurance role of collateral in environments with more risk. In addition, Adrian et al. (2013, p. 13) state that “in response to a rise in perceived risk... cash lenders might ask for higher interest rates, higher-quality collateral, increased haircuts, shorter maturities, or all of the above.” Our model rationalizes lender behavior with respect to increases in risk. Higher probabilities of default risk in our framework leads to less repo funding and higher haircuts – thus, an increase in required collateral per unit of funding. It is important to remember that the very reason repurchase agreements pay a return above the risk-free rate is due to the fact that the agreements are risky.

Previous work such as Dang et al. (2013b) motivate the need for haircuts through the concept of “information sensitivity” which relies on an additional layer of uncertainty – institutions holding collateral may be uncertain about its value.<sup>3</sup> Haircuts emerge to compensate lenders needing to sell the collateral to sophisticated secondary market buyers who have access to information regarding the collateral’s true value. Cash-lenders, unable to access the information, set haircuts in the first leg of the repo to protect them from potential downside risk.

In comparison to their work, lenders in our model have complete information about the value of the collateral they possess. Lower values of collateral significantly affect repo market activity in ways that are similar to higher degrees of risk aversion. Haircuts emerge as part of an efficient risk-sharing arrangement between repo market participants. Consequently, there are distinct policy implications for the repo market from our model in comparison to

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<sup>3</sup>For more on “information-sensitivity,” see Dang, Gorton, and Holmström (2013a).

Dang et al. (2013b).

The paper is organized as follows. Section 2 describes the model. Section 3 derives closed-form solutions for all variables. In section 4 we stress how default risk, risk aversion, and the market value of collateral affect the level of haircuts and overall funding in the repo market through numerical exercises. Section 5 concludes. The Appendix provides proofs of major results.

## 2 The Model

### 2.1 Endowments, Preferences, and Technologies

The economy lasts for two periods. There are two different goods available for consumption,  $\alpha$  and  $\beta$ . There are also two different types of agents,  $A$  and  $B$ , each with a population mass of 1. The type  $A$  individual receives one unit of endowment of good  $\alpha$  while type  $B$  agents are endowed with one unit of  $\beta$ . The preferences of each agent are:

$$U^A(c_\alpha^A, c_\beta^A) = \frac{(c_\alpha^A)^{1-\gamma}}{1-\gamma} + \frac{(c_\beta^A)^{1-\gamma}}{1-\gamma} \quad (1)$$

$$U^B(c_\beta^B) = \frac{(c_\beta^B)^{1-\gamma}}{1-\gamma} \quad (2)$$

with coefficient of risk aversion  $\gamma$ . That is, type  $A$  individuals derive utility from consumption of both goods while type  $B$  only wants to consume good  $\beta$ .

Type  $A$  individuals have access to a risky investment technology which generally produces  $R > 1$  units of good  $\beta$  at  $T = 2$  for every unit of good  $\beta$  invested at  $T = 1$ . However, with probability  $\pi$ , the risky investment technology does not yield any returns. Type  $A$  and  $B$  both have safe storage technologies for storing goods not allocated to  $A$ 's risky investment technology.

Differences in the endowments of each agent and differences in access to the investment technology lead to the potential for funding in  $A$ 's risky investment technology by a type  $B$



individual. However, there is not an enforcement mechanism in the economy. As a result, a type  $B$  will require a mutual transfer of good  $\alpha$  from  $A$  to be willing to lend. In this way, good  $\alpha$  acts as a form of collateral. Since a type  $A$  individual will seek to borrow funds from a type  $B$ , hereafter we also refer to  $A$ 's as borrowers and  $B$ 's as lenders. The amount of funding provided by the type  $B$  individual is denoted as  $d_\beta$  while the amount of collateral is  $d_\alpha$ .

Moreover, because collateral is transferred at the same time that funding is provided to borrowers, the lending arrangement represents a repurchase agreement between a borrower and a lender. The repo price for each unit of collateral is equal to  $r$ . One might think of a lender in this manner as a cash-rich intermediary such as a money market mutual fund and a borrower as a broker-dealer. In turn,  $\pi$  reflects the default or counterparty risk of a dealer-bank. Notably, as knowledge regarding failures of subprime mortgages began to surface in 2007-2008, default risk involving these securities grew significantly higher.

It is well known that cash-lenders like money market funds do not actively participate in the markets for various types of debt obligations which serve as collateral in security repurchase agreements. Consequently, the lender may face difficulties selling a collateral security in the event of default by a borrower.<sup>4</sup> In order to capture the discounted sale, lenders only possess a weak transformation technology for converting the collateral good into units of  $\beta$ . More specifically, the conversion rate is equal to  $p \leq 1$ . Note, however, the lender can choose to access the weak technology at any time. Such a lack of commitment, as articulated by Mills and Reed (2012), introduces frictions into repo arrangements. As a result, both borrowers and lenders must face appropriate incentives in order to avoid strategic default.

## 2.2 Timing of Actions

At the beginning of period 1, agents receive their endowments. Afterwards, agents  $A$

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<sup>4</sup>These difficulties may include transactions costs such as search costs and bargaining costs. Moreover, at times during the crisis, fire-sales of various types of assets took place which also caused their values to decline significantly.

and  $B$  meet. A social planner determines the quantities of goods  $\alpha$  and  $\beta$  to be exchanged between the agents. Assuming that the agents accept the proposal suggested by the planner, exchange takes place. Agent  $A$  promptly places the acquisition of  $\beta$  into his risky investment technology. All remaining endowments are placed into  $A$ 's and  $B$ 's respective storage technologies. After endowments are allocated to the various technologies, period 1 ends.

At the beginning of period 2, returns from the investment technology occur. If  $A$ 's technology does not yield any output,  $A$  defaults on the lender. Consequently,  $A$  transfers the collateral into units of good  $\beta$  using the weak transformation technology. Following a productive return, both agents meet and the planner suggests the amount of good  $\beta$  to be transferred by the  $A$  in order to re-acquire the amount of  $\alpha$  that was put forward as collateral.

It is important to note neither agent is required to accept any trade proposal. Therefore, the suggestions offered by the planner can be rejected at any stage.<sup>5</sup> If the trade is acceptable by both agents, then the repurchase occurs and both agents consume. If the trade is unacceptable or agent  $A$  was forced to default, then agent  $B$  utilizes the transformation technology and converts all of the collateral into units of good  $\beta$ . After consumption, period 2 concludes. The timeline of moves and events is described as follows:

$t=1.0$  Borrowers and lenders receive their endowments.

$t=1.1$  Borrowers exchange their collateral goods for investment goods from lenders.

$t=2.0$  Borrowers realize the rewards from their investment technology with probability  $(1 - \pi)$ .

$t=2.1$  Borrowers repurchase the collateral goods from the lenders with their investment proceeds.

$t=2.2$  All agents consume.

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<sup>5</sup>In comparison to the bilateral repos studied in our model, an additional third party known in a tri-party repo acts as a clearing bank by providing intermediation services for the cash-lender and borrower. In particular, the clearing bank (JPMorgan Chase or the Bank of New York Mellon in the United States) stewards the transaction by providing collateral management and repurchase settlement services.

### 3 Optimal Choices

The model is solved using a backwards-induction approach. We begin by considering the options available to agent  $B$  in the event that  $A$  did not receive any productive returns or simply defaulted on  $B$  earlier in period 2. It is also possible that agent  $B$  did not accept repayment of the loan from  $A$ . Upon converting the collateral good ( $\alpha$ ) to units of  $\beta$  at rate  $p$ , the lender's consumption would equal  $1 - d_\beta + p \cdot d_\alpha$ . Alternatively,  $B$ 's consumption of  $\beta$  without converting the collateral would equal  $1 - d_\beta$ . Thus, it is trivial to determine that an agent  $B$  in this scenario would convert the collateral good into units of  $\beta$  since lenders only derive utility from consumption of  $\beta$ .

We proceed to studying the decision of a borrower to repay the loan from a lender during the second leg of the repurchase agreement. A social planner will propose a level of repayment that maximizes aggregate utility across both parties:

$$\max_r \left[ \frac{1^{1-\gamma}}{1-\gamma} + \frac{(Rd_\beta - r \cdot d_\alpha)^{1-\gamma}}{1-\gamma} \right] + \frac{(1 - d_\beta + r \cdot d_\alpha)^{1-\gamma}}{1-\gamma} \quad (3)$$

Solving the optimization problem yields:

$$r(d_\alpha, d_\beta) = \frac{(R+1)d_\beta - 1}{2d_\alpha}. \quad (4)$$

From (4), the planner's suggestion for the repo price reflects the desire to maximize aggregate welfare through the distribution of income between both the borrower and lender. A higher price on the loan provides the lender with more consumption of good  $\beta$  but lowers the amount that the borrower would obtain. However, by participating in the second leg of the repo, the borrower is able to consume all of his endowment of good  $\alpha$ .

In the absence of commitment, either agent may walk away from the social planner's proposal. For the borrower, incentive compatibility requires:

$$1 + \frac{(Rd_\beta - r(d_\alpha, d_\beta) \cdot d_\alpha)^{1-\gamma}}{1-\gamma} \geq \frac{(1-d_\alpha)^{1-\gamma}}{1-\gamma} + \frac{(Rd_\beta)^{1-\gamma}}{1-\gamma} \quad (5)$$

By comparison, the lender's utility from accepting repayment of the loan is:

$$\frac{(1-d_\beta + r(d_\alpha, d_\beta) \cdot d_\alpha)^{1-\gamma}}{1-\gamma} \geq \frac{(1-d_\beta + p \cdot d_\alpha)^{1-\gamma}}{1-\gamma} \quad (6)$$

Obviously, this constraint is satisfied if the repo price exceeds the value of liquidating the collateral:

$$r(d_\alpha, d_\beta) \geq p$$

Or, using the solution for  $r^*$ , the lender will find the repayment to their benefit if the level of lending meets the following condition:

$$d_\beta \geq \frac{2pd_\alpha + 1}{(R+1)} \quad (7)$$

In order to determine the subgame perfect equilibrium for the model, we proceed by working back to studying actions in period 1. In the initial leg of the repo, the planner suggests an amount of funding ( $d_\beta$ ) to be provided by type  $B$  (the lenders) to type  $A$  (the borrowers). In order to help resolve the lack of commitment friction, the planner also suggests an amount of collateral ( $d_\alpha$ ) to be provided by the borrowers to the lenders. Consequently, the planner solves the following problem:

$$\begin{aligned} & \max_{d_\alpha, d_\beta} (1 - \pi) \left[ \frac{\left( \frac{1+(R-1)d_\beta}{2} \right)^{1-\gamma}}{1 - \gamma} \right] + \pi \left[ \frac{(1 - d_\beta + pd_\alpha)^{1-\gamma}}{1 - \gamma} \right] + \\ & (1 - \pi) \left[ 1 + \frac{\left( \frac{1+(R-1)d_\beta}{2} \right)^{1-\gamma}}{1 - \gamma} \right] + \pi \left[ \frac{(1 - d_\alpha)^{1-\gamma}}{1 - \gamma} \right]. \end{aligned} \quad (8)$$

From (8), the planner's suggestion for  $d_\beta$  is:

$$d_\beta(d_\alpha) = \frac{1 + pd_\alpha - \frac{(\chi)^{\frac{1}{\gamma}}}{2}}{\left[ 1 + (R - 1) \frac{(\chi)^{\frac{1}{\gamma}}}{2} \right]}, \quad (9)$$

where  $\chi = \frac{\pi}{(1-\pi)(R-1)}$ . Since  $\chi$  represents the ratio between the gains and losses from each additional unit of investment, it will henceforth be called the *loss ratio*. Given that no risk-averse agent would ever invest in a project that yields negative expected returns, we will assume  $\chi < 1$ .

From (8), the planner's suggestion for  $d_\alpha$  is:

$$d_\alpha(d_\beta) = \frac{1 - p^{\frac{\gamma-1}{\gamma}} (1 - d_\beta)}{\left( 1 + p^{\frac{\gamma-1}{\gamma}} \right)} \quad (10)$$

Although the planner's decisions in (9) and (10) contain endogenous variables, we find the *direct effects* of certain parameters provide important insight into the components of repurchase agreements:

**Lemma 1.** *On the basis of (9), for a given  $d_\alpha$ , default risk has a negative direct effect on the amount of lending. However, as observed from (10), for a given  $d_\beta$ , default risk does not have an effect on the reliance on collateral.*

Default risk enters directly into the planner’s decision for lending. In addition, the loss ratio captures the tradeoff between the benefits and costs of a repo arrangement in (9). As default risk raises the loss ratio, the gains from lending will be lower and less lending in the repo market would take place.

On the other hand, (10) does not reflect such a tradeoff. In particular, default risk is absent from any direct effect on collateral. The planner establishes a risk-sharing contract and in the event that the borrower’s technology does not yield any returns, both the borrower and lender are affected by the default. Consequently, an increase in default risk is equally shared by both parties and therefore the amount of collateral to be offered is independent of the default probability. As a result, default risk primarily plays a role in the size of the repo market.

We proceed to characterize the impact of risk aversion on repo market activity in Lemma 2:

**Lemma 2.** *Higher degrees of risk aversion have a negative direct effect on lending and a positive direct effect on the amount of collateral.*

As observed from (9), higher degrees of risk aversion negatively affect the extent of repo lending. The primary mechanism in which the extent of risk aversion directly affects repo volumes occurs through the loss ratio,  $\chi$ . An increase in the loss ratio reduces the relative gains from lending and therefore the planner proposes that the lender offer less funding. If the ratio of losses to gains were zero ( $\pi = 0$ ), then this direct effect would vanish.

With respect to  $d_\alpha$ , the effect in (10) is the opposite of (9) and provides support for the insurance role that collateral plays in our environment. Due to the transactions costs from the weak transformation technology,  $p \leq 1$ . In turn, higher degrees of risk aversion are associated with an increased reliance on collateral. However, if  $p = 1$ , there would be no cost to liquidating collateral. Essentially, collateral would provide full insurance against default. Therefore, the extent of risk aversion would not matter.

Next, we turn to the determination of repo rates. In particular, we are able to show how repo rates depend on the total amount of credit demand as follows:

$$RR(d_\beta) = \frac{r \cdot d_\alpha - d_\beta}{d_\beta} = \frac{(R-1)}{2} - \frac{1}{2d_\beta} \quad (11)$$

Notably, (11) shows how repo rates depend on demand for repo funding by expressing the repo rate in terms of  $d_\beta$ . As the demand for repos increases, repo rates rise. In order to understand the mechanics of the repo rate, it is useful to begin by looking at the mechanics behind the repo price for collateral from (4). The repo price, the price of repurchasing the collateral, balances the loss of utility by a borrower against the increase in utility of a lender once the returns from the borrower's investment technology are realized.

By comparison, the repo *rate* is the interest rate on the loan extended through the repurchase agreement. It is easily shown that the total costs of repurchasing all of the collateral are independent of the amount of collateral that was posted in the repo. Instead, the costs of repurchase depend on two factors: the returns from the borrower's technology and the amount of the loan in the repo agreement. In particular, the costs of reacquiring the collateral are increasing in the amount of funding obtained. The more funding received, then the higher the total investment returns generated by the borrower. As the planner seeks to maximize aggregate utility across both parties, the planner shares some of the income received by the borrower with the lender in the form of higher costs of reacquiring the collateral. In turn, the repo rate positively depends on the amount of the loan.

### 3.1 Repo Solutions

The (non-degenerate) solutions to the planner's proposals for repo activity are:

$$d_\beta^* = \frac{1 + \left[1 - (\chi)^{\frac{1}{\gamma}} \left(\frac{1}{2}\right)\right] \left(\frac{1}{p} + \frac{1}{p}^{\frac{1}{\gamma}}\right) - p^{\frac{\gamma-1}{\gamma}}}{\left[1 + (\chi)^{\frac{1}{\gamma}} \left(\frac{R-1}{2}\right)\right] \left(\frac{1}{p} + \frac{1}{p}^{\frac{1}{\gamma}}\right) - p^{\frac{\gamma-1}{\gamma}}} \quad (12)$$

$$d_{\alpha}^* = \frac{\frac{1}{p} \left[ 1 + (\chi)^{\frac{1}{\gamma}} \left( \frac{R-1}{2} \right) \right] - \frac{1}{p} (\chi)^{\frac{1}{\gamma}} \left( \frac{2-R}{2} \right)}{\left[ 1 + (\chi)^{\frac{1}{\gamma}} \left( \frac{R-1}{2} \right) \right] \left( \frac{1}{p} + \frac{1}{p} \right) - p^{\frac{\gamma-1}{\gamma}}} \quad (13)$$

Both agents will agree to the planner's proposal as long as the following conditions are satisfied:

*Agent A:*

$$(1 - \pi) \left[ 1 + \frac{\left( \frac{y + (R-1)d_{\beta}^*}{2} \right)^{1-\gamma}}{1 - \gamma} \right] + \pi \left[ \frac{(x - d_{\alpha}^*)^{1-\gamma}}{1 - \gamma} \right] \geq 1 \quad (14)$$

*Agent B:*

$$(1 - \pi) \left[ \frac{\left( \frac{1 + (R-1)d_{\beta}^*}{2} \right)^{1-\gamma}}{1 - \gamma} \right] + \pi \left[ \frac{(1 - d_{\beta} + p d_{\alpha}^*)^{1-\gamma}}{1 - \gamma} \right] \geq 1 \quad (15)$$

As noted in the introduction, there were substantial increases in haircuts on collateral during the crisis. Following Gorton and Metrick (2012) and Dang et al. (2013b), the solution for the haircut is equal to:

$$H^*(d_{\alpha}^*, d_{\beta}^*) = \frac{d_{\alpha}^* - d_{\beta}^*}{d_{\alpha}^*} = 1 - \frac{1 + \left[ 1 - (\chi)^{\frac{1}{\gamma}} \left( \frac{1}{2} \right) \right] \left( \frac{1}{p} + \frac{1}{p} \right) - p^{\frac{\gamma-1}{\gamma}}}{\left[ \frac{1}{p} + (\chi)^{\frac{1}{\gamma}} \left( \frac{R-1}{2p} \right) \right] - \frac{1}{p} (\chi)^{\frac{1}{\gamma}} \left( 1 - \frac{R}{2} \right)} \quad (16)$$

As is clear from (16), the level of haircuts depends on the degree of risk aversion, the probability of default risk, and the returns to the borrower's investment technology. To motivate our work further, we turn to numerical exercises which match data on repo market activity during the time of the recent financial crisis. In all of the examples below, we verify that the participation constraints in (14) and (15) are satisfied.



## 4 Numerical Analysis

In this section, we expand on our previous work to incorporate some important aspects of repo market activity in recent years. For example, as shown quite forcefully during the financial crisis, changes in investors’ risk aversion can have a substantial impact on the repo market.

To do so, we start with a benchmark parameter set so that the predictions of the model mimic repo market variables observed in the data. Gorton and Metrick (2012) present evidence that haircuts (averaged across asset classes) were around 8% in November 2007, immediately preceding the beginning of the recession in the United States. Observations for repo rates are available from the Securities Industry and Financial Markets Association (SIFMA). Their data shows overnight repo rates on repos involving agency and mortgage-backed collateral averaged around 5% in 2007.

We study variations on the following benchmark set of parameters:  $R = 3$ ,  $p = 0.9$ ,  $\pi_0 = 1.5\%$ , and  $\gamma_0 = 6.5$ .<sup>6</sup> Under this set of parameters, we obtain:

**Table 1.1**

Benchmark Repo Market			
Loan	Collateral	Haircut	Repo Rate
0.5317	0.5830	8.801%	5.955%

In comparison to the prediction of zero repo rates of Dang et al. (2013b), the level of haircuts and repo rates in our numerical analysis are consistent with the data from Gorton and Metrick (2012) and SIFMA. As emphasized by Mills and Reed (2012) and Ewerhart and Tapking (2008), the desire for risk-sharing is a strong motivating factor in the repo arrangements between the borrower and lender. For example, it will be apparent below that the lender will accept a lower repo rate in return for a higher haircut.<sup>7</sup>

<sup>6</sup>Mehra and Prescott (1985) cite that values of the coefficient of relative risk aversion up to 10 are plausible. Lansing and LeRoy (2014) also consider a range of values up to 10.

<sup>7</sup>The trade off between the marginal rates of insurance and investment returns is a common risk-sharing result – see also Ewerhart and Tapking (2008) for a related discussion on repo markets and counterparty risk.

As noted in the introduction, there have been several documented instances where increases in risk intolerance had a real affect on investment. The most recent case being that of the financial crisis as documented by Gorton and Metrick (2012). The empirical work of Coudert and Gex (2006, 2007) also shows that risk aversion indicators rise just before crises. In order to capture this feature of the repo market, we consider different measures of risk-aversion for repo market activity:

**Table 1.2**

Benchmark Repo Market as $\gamma_0$ Changes				
$\gamma$	Loan	Collateral	Haircut	Repo Rate
6.5	0.5317	0.5830	8.807%	5.955%
6.6	0.5280	0.5836	9.530%	5.307%
6.7	0.5245	0.5843	10.233%	4.668%
6.8	0.5210	0.5849	10.917%	4.036%
6.9	0.5177	0.5855	11.582%	3.413%

As *both* agents become more risk averse, haircuts on collateral increase. Moreover, risk aversion dramatically affects repo market activity by causing the amount of lending to shrink. To this extent, increases in risk aversion can ultimately cause repo markets to collapse. The collapse occurs due to the steady decline in repo funding.<sup>8</sup> In fact, loan offers decrease in the face of higher collateral postings – leading to a larger haircut. Consequently, the predictions of our model line-up well with the change in investor sentiment prior to the financial crisis.

In this manner, our work makes a clear contribution to the literature on repo markets. Standard models are confined to studying repo activity under risk-neutrality and therefore are not suited for capturing changes in the degree of risk aversion among market participants. Rather, they study investor sentiment through changes in the risk of collateral or the riskiness

<sup>8</sup>The repurchase agreement is no longer incentive compatible for both the borrower and lender when risk aversion increases above 7.

of the borrower’s investment opportunities. Yet, as we discuss below, it is clearly important to study default risk when investors are risk averse.

The model also allows us to consider the effects of an increase in the default rate among borrowers in repo markets. Consequently, the model provides insights about repo market behavior following the two pivotal moments during the financial crisis – the collapse of Bear Stearns and the failure of Lehman Brothers – which clearly raised concerns about default. Naturally, the results are qualitatively similar to the effects of higher risk aversion:

**Table 1.3**

Benchmark Repo Market as $\pi_0$ Changes				
$\pi$	Loan	Collateral	Haircut	Repo Rate
1.5	0.5317	0.5830	8.807%	5.955%
1.6	0.5284	0.5836	9.453%	5.376%
1.7	0.5253	0.5841	10.061%	4.822%
1.8	0.5224	0.5846	10.636%	4.292%
1.9	0.5197	0.5851	11.181%	3.784%

That is, at higher default rates, there is less lending activity along with an increase in haircuts. The repurchase agreement is no longer incentive compatible for both the borrower and lender when default risk increases above 2.1%.

However, the twin distortions from higher default rates along with higher risk aversion have a dramatic impact on the repo market. At a slightly higher degree of risk aversion ( $\gamma_1 = 6.8 > 6.5 = \gamma_0$ ), the market is much more fragile as default rates rise:

**Table 1.4**

Repo Market under $\gamma_1$ as $\pi_0$ Changes				
$\pi$	Loan	Collateral	Haircut	Repo Rate
1.5	0.5210	0.5849	10.917%	4.036%
1.6	0.5179	0.5854	11.537%	3.453%
1.7	0.5149	0.5859	12.122%	2.896%

Notably, the repo market completely breaks down at lower thresholds of default as lenders no longer participate. The expected return from lending in the repo market is not incentive compatible when there is an increase in default rates at a higher degree of risk aversion. Rather, the utility lenders would receive by simply consuming their endowments is higher than the expected utility associated with lending to borrowers. As risk aversion in the market was easily higher after the default of Lehman Brothers than the collapse of Bear Stearns, the model shows us that repo markets would break down more quickly at higher degrees of risk aversion.

Available data on repo activity is in-line with the predictions of our analysis. The decline in repo activity was much larger after the bankruptcy of Lehman Brothers than the collapse of Bear Stearns when risk aversion in the market was not as high. For example, available survey evidence on the amount of total repo activity declined marginally after the Bear Stearns event. To be specific, data from the International Capital Markets Association (ICMA) sample of firms in Europe was around 3.24 trillion euros in December 2007. Later, in June 2008, the number stood at 3.17 trillion euros. Following Lehman, the decline was substantial. In December 2008, repo activity fell to 2.31 trillion euros – more than a 25% drop.

Data on haircuts during the crisis period is also highly suggestive of a breakdown in repo lending. In June 2007, Fitch Ratings data show that haircuts on AA and A- rated securities posted as collateral were equal to 25%. By comparison, two years later, haircuts increased

to 100%, implying that lenders refused to lend against such securities.<sup>9</sup>

It is also well known that there were “fire-sales” of assets originating from the shadow banking system during the crisis. In this exercise, we turn to the effect of changes in the liquidation value of collateral on repo activity:

**Table 1.5**

Benchmark Repo Market as $p$ Changes				
$p$	Loan	Collateral	Haircut	Repo Rate
0.9	0.5317	0.5830	8.807%	5.955%
0.85	0.5372	0.5858	8.286%	6.933%
0.80	0.5423	0.5895	8.011%	7.803%
0.75	0.5469	0.5944	7.990%	8.570%
0.70	0.5509	0.6003	8.232%	9.237%
0.65	0.5544	0.6075	8.746%	9.808%
0.60	0.5573	0.6161	9.542%	10.282%
0.55	0.5597	0.6262	10.630%	10.659%

Notably, our results are in stark comparison to Dang et al. (2013b). They argue that haircuts should only increase as *uncertainty* about the value of the collateralized asset increases. Therefore, because the lender does not initially know the true value of the collateral they, in turn, require a haircut. That is, in their model with risk-neutral agents, haircuts are used to pass-through the potential costs of information acquisition from a buyer in a secondary market to the borrower in a repo.

By comparison, in our framework, collateral and haircuts are part of an efficient risk-sharing arrangement between a risk-averse borrower and a risk-averse lender. In the event of default, lenders would like to have a certain amount of income. Haircuts are a way of over-collateralizing a secured credit arrangement in order to achieve optimal risk-sharing.

<sup>9</sup>Haircut data from Fitch as reported by the ICMA (Comotto, 2012).

## 4.1 Leverage Elasticity

Using the benchmark case, this section calculates point estimates of haircut elasticity with respect to the degree of risk aversion, default risk, and the liquidation value of collateral. We will call these point estimates *leverage elasticity*. Essentially, leverage (or haircuts) allows the quantification of risk lenders are willing to bear in the agreement. Leverage elasticity allows the measurement of sensitivity to the chosen environmental factors. Table 1.6 shows the effect a one percentage increase in risk aversion and default risk has on haircuts at the benchmark parameter levels. Liquidation value is a one percentage decrease.

**Table 1.6**

Leverage Elasticity, Case 1			
	$\gamma_0$	$\pi_0$	$p$
$\% \Delta H^*$	5.36%	1.13%	-1.27%

In Table 1.7, we change the parameter values for default risk and risk aversion as a robustness check, where  $\pi_r = 6\%$  and  $\gamma_r = 4$ .

**Table 1.7**

Leverage Elasticity, Case 2			
	$\gamma_r$	$\pi_r$	$p$
$\% \Delta H^*$	26.63%	8.34%	-4.56%

The results in Tables 1.6 and 1.7 clearly show that risk aversion has the largest impact on the leverage elasticity in repo markets. Yet, standard analyses of repo activity ignores risk aversion by focusing on risk neutral market participants. Therefore, we argue that existing work fails to incorporate a potentially important element of repo markets.

## 5 Conclusion

This paper studies security repurchase agreements in the presence of risk aversion. In so doing, we show that repo activity between borrowers and lenders reflects optimal risk sharing. Out of this risk sharing behavior, haircuts arise naturally in order to provide lenders with insurance against default risk. Interestingly, we find that the twin distortions from higher risk aversion along with counterparty risk or lower market values of collateral render repo markets highly susceptible to a collapse of activity. Therefore, policymakers must pay close attention to conditions in financial markets which can affect the repo market. In particular, when central banks notice spikes in risk aversion indicators they could anticipate liquidity in repurchase markets to wane quickly.

There are a number of interesting avenues for future research. First, the model could be extended to consider repos of different maturities when lenders are subject to liquidity risk as in Diamond and Dybvig (1983). Second, multiple types of collateral could be incorporated with varying degrees of liquidity in secondary markets. Consequently, as shown by Boulware, Ma, and Reed (2014), the level of activity, the type of collateral posted, and the maturity of repo arrangements would respond to changes in financial conditions.

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# CHAPTER II:

## Repos, Bailouts, and Instability in the Shadow Banking System

### 1 Introduction

The shadow banking system consists of many institutions which operate in a similar manner as traditional banks. However, in contrast to traditional banks, these non-bank financial institutions are subject to much less regulation. In turn, they are able to offer higher returns but do not have access to deposit insurance or the discount window. In particular, money market mutual funds (MMMFs) are an important group of intermediaries in the shadow banking system. As they mimic the demand deposit aspects of traditional banks, they are an attractive investment. Yet, without government backstops, they may also be susceptible to excessive withdrawals. To address this source of instability in the shadow banking system, the objective of this paper is to analyze the behavior of MMMFs facing runs in a financial crisis. We also investigate whether government intervention can prevent extreme economic losses.

MMMFs play a critical role in U.S. capital markets. Over the past 30 years, total net assets held by MMMFs have risen from roughly \$233 billion to over \$2.7 trillion.<sup>10</sup> Such an increase represents growth in asset holdings from 5% of GDP to over 16% of GDP. Moreover, Kacperczyk and Schnabl (2013) point out: “Money market funds are the largest provider of short-term financing to financial institutions,..., and they are also the largest provider of liquidity to corporations, issuing about the same amount of demand deposits as the entire U.S. commercial banking sector.” (p. 1074).

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<sup>10</sup>2014 Investment Company Fact Book, published by the Investment Company Institute.

The principal way that MMMFs extend funding is through security repurchase agreements (or ‘repos’). For example, at the end of the third quarter of 2014, over \$575 billion of funding were extended through repos. In particular, the amount of repo funding by MMMFs peaked at \$600 billion in the third quarter of 2008 – the most significant point in the recent financial crisis. In fact, Gorton and Metrick (2012) characterize the financial crisis as a large-scale “run on repo.”

A typical repurchase agreement starts with a lender purchasing illiquid assets (collateral) from a borrower. The purchase is considered a “true sale” in which the lender obtains full property rights to the collateral throughout the duration of the repo. Consequently, the lender may sell or trade the collateral at any time.<sup>11</sup> The second leg of the repo occurs when the borrower repurchases the collateral from the lender. The repurchase price (repo price) will be higher than the lender’s initial purchase price. The percentage difference between the two prices reflects the interest rate on the loan or the “repo rate.”

Although repos are short-term, the maturities can range anywhere from one day to three months.<sup>12</sup> However, any open-ended fund such as a MMMF is still characterized by a maturity mismatch. This mismatch allows for the possibility of a “bank run” synonymous with the seminal work of Diamond and Dybvig (1983). The excessive withdrawals during a run cause the fund to be under-capitalized. Notably, the Financial Stability Oversight Council (FSOC) states in a November 2012 report:

“In the event of shareholder redemptions in excess of a MMF’s available liquidity, a fund may be forced to sell less-liquid assets to meet redemptions. In times of stress, such sales may cause funds to suffer losses that must be absorbed by the fund’s remaining investors, further reinforcing the first-mover advantage.”

The first-mover advantage problem is of particular importance in the current structure

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<sup>11</sup>For additional discussion, see Garbade (2006), International Capital Market Association (2011) and Morrison, Roe, and Sontchi (2014).

<sup>12</sup>The weighted average maturity of a money market fund portfolio cannot exceed 60 days as stated by the SEC in 2010.

of MMMFs. Shares of a MMMF are calculated based on the net asset value (NAV) of the fund's portfolio. Therefore, as the value of the fund's portfolio diminishes so do the value of its shares. Subsequently, investors who withdraw their funds before the portfolio value falls will suffer no losses. This creates an environment where investors will withdraw their funds en masse when losses appear possible. MMMFs will then quickly seek out liquidity to meet the unexpectedly high demand for redemptions. In fact, Begalle, Martin, McAndrews, and McLaughlin (2013) argue that MMMFs face strong incentives to sell assets immediately, even at greatly reduced prices, during an impending crisis. As we explain below, events during the recent financial crisis have made this dramatically clear.

In only four days following the bankruptcy of Lehman Brothers, more than \$300 billion 'ran' from money market funds. In order to limit the damage and promote the stability of MMMFs, the U.S. Treasury instituted a temporary guarantee program on money market fund investor shares. The guarantee program essentially provided unlimited deposit insurance for the 38,000 shareholder accounts worth around \$3.8 trillion.<sup>13</sup> Although the Treasury paid out no claims during the twelve months it was in place, lawmakers have since installed several barriers against the renewal of such a program. Specifically, Title I of the Dodd-Frank Act established the Financial Stability Oversight Council which requires the council to eliminate expectations on the part of shareholders, creditors, and counterparties of non-bank financial companies that the government will shield them from losses in the event of failure.

Ideally, policymakers would like for MMMFs to self-insure against runs – as evident by the recent increases in capital requirements. However, holding more capital cannot insure against large scale financial crises like that of 2007-2008. As Diamond and Dybvig (1983) demonstrate, bank runs are possible whenever an institution holds long-maturity assets and issues short-maturity liabilities.<sup>14</sup> Therefore, it is important that policymakers understand how intermediation breaks down in the shadow banking system during times of crisis. To

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<sup>13</sup>Information provided by the Investment Company Institute 2014 Factbook.

<sup>14</sup>Money market funds are required to hold a minimum of 10 percent of their assets in cash daily.

understand this behavior, it is important to first construct a process that resembles the type of borrowing and lending occurring in the shadow banking sector. Consequently, our framework is designed to mimic that of a MMMF engaging in repo markets in the presence of self-fulfilling investor runs. However, the structure can be generalized to any “open-end” fund that participates in repo lending.<sup>15</sup>

Our framework builds on the work of Diamond and Dybvig (1983) and Keister (2014). Notably, the framework studies an environment in which investors are exposed to liquidity risk and there is limited commitment on the part of financial market participants. Fiscal policy is modeled as a decision to fund public goods with lump-sum taxes. In contrast to Diamond and Dybvig and Keister, investment in our model is not reversible as the intermediary must decide how much to lend to borrowers through security repurchase agreements. As previously noted, MMMFs represent one of the largest sources of funding in repo markets.<sup>16</sup> Thus, the risk-pooling intermediary should be viewed as a participant in the shadow banking system in contrast to standard Diamond-Dybvig intermediaries who directly have access to investment returns. Consequently, the investment decision of the intermediaries in our framework is non-trivial compared to standard models of risk-pooling institutions.

Moreover, intermediaries cannot easily extract funds from the repo market when additional liquidity is needed. Instead, collateral will be utilized. In particular, Gorton and Metrick (2012) note that collateral is the most important part of a repo. To begin, the repo price on collateral is analogous to the interest rate on loans, and collateral itself serves as deposit insurance. The extent of over-collateralization (also interpreted as the ‘haircut’) is similar to a reserve ratio in the traditional banking system – a feature of the repurchase market that, like a reserve requirement, can be used to increase market stability. In addition, collateral also serves as an incentive device for borrowers to engage in the repurchase. With all this, in the event of large-scale runs in the shadow banking system, collateral clearly plays

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<sup>15</sup>A open-ended fund will buy back shares that its investors wish to sell at any time.

<sup>16</sup>MMMFs rank second in total assets held in the repurchase market. The largest category of lenders in repos are brokers and dealers.

a critical role. Yet, in such an environment, would public sector intervention be appropriate to promote stability among MMMFs and the repo market?

Inspired by events during the financial crisis, we show that bailouts of such risk-pooling intermediaries are part of an efficient social insurance scheme when a run emerges. In a decentralized setting, income taxes which fund public goods are inefficiently high. As a result, deposit funding in the financial system will be low, causing funding offered to borrowers to be inefficient and repo rates to be too high. However, in the unlikely event that a run emerges, the high level of public resources can be used to minimize the costs of “runs on repos” in which there would be large-scale liquidation of collateral as a result of the liquidity crisis. Nevertheless, this observation does not imply that optimal intervention completely isolates shadow-banking intermediaries from a crisis. In fact, optimal public sector intervention imposes costs on money market funds by requiring them to liquidate some collateral. On the other hand, a commitment to no bailouts contributes to financial instability as the repo market collapses in the wake of a run without a public safety net.

The remainder of the paper is as follows. Section 2 outlines the environment for the model. Section 3 studies investment decisions and repo activity in a Planner’s allocation. Next, section 4 considers a setting where fiscal authorities have discretion to bailout intermediaries in the event that a run by investors occurs. Section 5 compares outcomes under discretion to an environment where fiscal authorities are committed against public-sector bailouts of private institutions. Section 6 provides concluding remarks. Proofs of major results are provided in the Appendix.

## 2 The Model

The model extends Keister (2014) to include a money market where risk-pooling intermediaries engage in security repurchase transactions with borrowers as in standard repo agreements between a MMMF and another intermediary such as an investment bank or a broker-dealer.<sup>17</sup> Similar to Diamond and Dybvig (1983) and Keister (2014), investors are

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<sup>17</sup>A broker-dealer is in the business of buying and selling securities. For example, a broker-dealer may be

exposed to idiosyncratic liquidity risk and seek access to the risk-pooling services provided by MMMFs. In contrast, borrowers seek access to funding that can be obtained from such intermediaries. As in Cipriani et al. (2014) and Allen and Gale (2000), we study the effects of unanticipated shocks whereby investors run on MMMFs. This reflects behavior experienced by MMMFs following the bankruptcy of Lehman Brothers which led to a collapse of repo markets.

## 2.1 Preferences and Technologies

There are two different types of agents in our model. The first is a typical depositor (henceforth called an “investor”) who invests their endowment into a standard *risk-pooling intermediary* (or intermediary for short). This intermediary insures investors against liquidity risk as in Diamond and Dybvig (1983). The second agent is a “borrower”. *Borrowers* have access to an investment technology but do not have any resources to operate the technology. However, they do possess a collateral good that can be sold to the risk-pooling intermediary with the intention of buying it back at a later date. In this manner, the risk-pooling intermediary and borrowers take part in a repurchase transaction which is a common financing arrangement among participants in the shadow banking sector.<sup>18</sup>

The economy lasts for three periods,  $t = 0, 1, 2$ . In the economy, there is a population mass 1 of investors indexed by  $i \in [0, 1]$ . Investors value consumption of a private good and access to the economy’s public good as follows:

$$U^I(c_1, c_2, g; \omega_i) = u(c_1 + \omega_i c_2) + u(g), \quad (17)$$

where  $u$  is a constant relative risk aversion utility function with coefficient  $\gamma$ .<sup>19</sup>

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a subsidiary of a large commercial bank. Alternatively, a broker-dealer may be a subsidiary of an investment bank which is a modern, market-based intermediary in the shadow banking system rather than a traditional deposit-based intermediary.

<sup>18</sup>Boulware, Ma, and Reed (2014) study the impact of monetary policy shocks on repo activity by the primary dealers of the Federal Reserve System which are largely composed of investment banks.

<sup>19</sup>Keister (2014) allows the utility function from consumption of the private good to differ from the utility from access to the economy’s public good. However, as we explicitly model the repo market, we have additional variables to determine including the repo rate in each aggregate state and the investment by

The indicator variable  $\omega_i$  indicates whether the investor is *impatient* or *patient*. That is, its value determines whether the investor desires consumption in period 1 or period 2. The value of  $\omega_i$  is random with support  $\Omega_i \equiv \{0, 1\}$  and  $\pi$  is the probability of being impatient. By the law of large numbers,  $\pi$  also represents the total fraction of investors who are impatient but the realization of the individual's preferences is private information. If  $\omega_i = 0$ , the investor is impatient, and they are patient if otherwise.

In contrast to investors, borrowers do not face any idiosyncratic risk and they do not derive utility from access to the economy's public good. In particular, a borrower only values consumption of the private good in period 2. The population mass of the borrowers is equal to 1 and their preferences are given by:

$$U^B(b_2) = u(b_2) = \frac{(b_2)^{1-\gamma}}{1-\gamma} \quad (18)$$

Each investor is endowed with 1 unit of the private good in period 0. Each borrower is endowed with some amount  $z$  of the collateral good in period 0. Private production in the economy primarily occurs through the borrowers' investment technology which can transform units of the private good in period 0 into  $R > 1$  units of the private good in period 2. The risk-pooling intermediaries only have access to a storage technology in which one unit of the private good in period 0 yields one unit of the private good in the following period.

Two other technologies also exist. One technology, held by the fiscal authority, transforms units of the private good one-for-one into units of the public good in any period. The other technology, held by both the financial intermediary and the borrower, can transform units of the collateral good into units of the private good in periods 1 and 2. However, the transformation rate varies between the market participants. Notably, the rate at which the intermediary can transform collateral is equal to  $p_F$  while the borrowers' transformation rate

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the intermediary before the realization of the aggregate state. We also endogenously determine the amount of collateral liquidation which can occur under each aggregate state. With the presence of five additional endogenous variables in our model, it is important that investors have the same utility function for both types of goods.



is  $p_B \geq p_F$ . As described by Gorton and Metrick (2012), broker-dealers often make markets in securities that they use as collateral. On the other hand, money market mutual funds are not well-positioned to sell securities that they hold as collateral. Therefore, the assumption that  $p_B \geq p_F$  reflects the different valuations of collateral across market participants.

Withdrawals are executed following a sequential-service constraint. This concept allows the investor to observe their position in the withdrawal order and make their consumption decision accordingly. Investors, based upon their “position in line”  $i$ , choose when to withdraw funds and will receive payments by the financial intermediary contingent on the number of withdrawals that have already taken place. One might interpret that the earliest investors are perhaps the most savvy among all the investors. For example, investors with a small value of  $i$  could be interpreted as “institutional” investors and others as “retail” investors.

## 2.2 Unexpected Shocks and Financial Instability

Crises in the model are based on an unexpected shock regarding the “state of the world”.<sup>20</sup> There are two possible states. Let  $S = (\alpha, \beta)$  be the set of extrinsic signals representing the good and bad states, respectively. The good state relates to normal investor sentiment regarding the degree of early liquidation to be experienced by MMMFs. However, the bad state signals an impending financial crisis where all individuals will seek to withdraw funds in period 1 regardless of their individual realizations of their discount rates. That is, investors construct their withdraw profile:

$$y_i : \Omega \times S \rightarrow \{0, 1\}, \quad (19)$$

where  $y_i$  corresponds to withdrawing early or waiting until period 2. The realization of a financial crisis causes fragility in the system when some proportion of patient investors decide

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<sup>20</sup>According to the International Capital Market Association, about half of all repurchase agreements have terms of one month or less. Given the short length of these agreements and the low probability of a financial crisis from historical data, we consider crises to be unexpected shocks in our environment. For example, Hsiang and Jina (2014) cite data from Reinhart and Rogoff (2009) which indicates that the historical probability of a financial crisis is less than 0.1%. This shock is modeled in a manner similar to Cipriani et. al. (2014) and Allen and Gale (2000).

to withdraw their funds early:

$$y_i(1, \beta) = 0 \tag{20}$$

As we will show, a crisis across MMMFs can have negative consequences for the repo market:<sup>21</sup>

**Definition 1:** *Instability in the repo market occurs when participation in the money market (at any point in time) is no longer incentive compatible for either market participant.*

In contrast to Keister (2014), the actions of the investors who deposit funds at the risk-pooling intermediary have repercussions that extend beyond the intermediary. Since our model contains a money market, we are able to make distinct statements regarding the financial instability of money markets created by investor runs. Notably, we will demonstrate that the repo market breaks down as MMMFs experiencing unanticipated demand for liquidity will sell the collateral acquired in the initial stage of the repo. For example, there were widespread “fire sales” of assets shed by MMMFs after the bankruptcy of Lehman Brothers in Fall 2008. Consequently, in the absence of a policy response, repo rates can increase significantly and credit intermediation breaks down.

### 3 The Planner’s Allocation

In order to draw direct comparisons to Keister (2014), we focus on the actions of a narrow-minded Planner who seeks to maximize aggregate expected utility across investors. This reflects recent discussions since the crisis which have focused on providing recommendations to improve the stability of MMMFs. Towards that end, we want to directly show how efficient allocations (from the perspective of the funds) would be determined. Moreover, modeling

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<sup>21</sup>Rosengren (2014) recognizes the potential for poor behavior to spill across to numerous financial markets: “Financial instability occurs when problems (or concerns about potential problems) within institutions, markets, payments systems, or the financial system in general significantly impair the supply of credit intermediation services – so as to substantially impact the expected path of real economic activity.”

the repo market provides important insights regarding instability in the shadow banking system. The ex-ante aggregate utility of investors is:

$$\int_0^1 [U^I (c_1 (i), c_2 (i), \tau; \omega_i)] di \quad (21)$$

The timeline of moves and events is described as follows:

- t=0.1* Investors' endowments are taxed and the remainder is deposited with the risk-pooling intermediaries.
- t=0.2* The Planner chooses portfolios to maximize expected utility across investors.
- t=0.3* The initial stage of repurchase agreements occurs where intermediaries and borrowers exchange private goods and collateral goods in a competitive market.
- t=1.0* Investors with  $i \leq \theta$  decide to withdraw funds.

*The aggregate state is realized by the financial intermediary and fiscal authority.*

- t=1.1* Funding decisions regarding the remaining impatient and patient investors are made.
- t=2.0* Borrowers choose how much collateral to repurchase.
- t=2.1* All remaining investors withdraw their funds.

Within the timing of events listed, there are three unique stages. Importantly, at *t=0.3*, the initial leg of repurchase transactions occurs where intermediaries transfer units of the private good to borrowers in exchange for the collateral good. In order to avoid the potential for strategic default by the borrower, the intermediary requires that the funds lent must be

secured by collateral. Moreover, repo agreements generally involve some degree of overcollateralization or a “haircut”.<sup>22,23</sup> For each unit of funding provided, the borrower must transfer  $\delta$  units of the collateral good to the intermediary. Notably,  $\delta > 1$  represents the level of overcollateralization. Consequently, the total amount of collateral received is  $x\delta$  where  $x$  is the amount of investment in the repo market by the risk-pooling intermediaries.

Second, after the aggregate state has been revealed, the intermediary will make a decision regarding collateral liquidation. Specifically, at  $t=1.1$ , intermediaries will liquidate some portion ( $\lambda_s$ ) of their collateral if they find it optimal to distribute a greater amount of funding than they currently have on hand to impatient investors. Collateral is liquidated at rate  $p_F$  using the transformation technology discussed earlier. In this manner, investor runs can trigger instability in the repo market as borrowers would not be able to repurchase all of the collateral that they transferred to the intermediary.

The third important stage further highlights the potential for liquidity crises to spillover to the repo market. At  $t=2.0$ , borrowers meet up with financial intermediaries and determine the price necessary to repurchase the collateral – thus completing the final leg of the repurchase agreement. After a liquidity disruption, the repo price will increase in the absence of a policy response. In fact, the price might be so high that it is no longer incentive compatible for borrowers to repurchase their collateral. In turn, strategic default by the borrower may take place.

We will now solve for the Planner’s allocation in the model. It is important to note that we only want to study subgame perfect equilibrium outcomes. Therefore, we proceed using a backwards-induction approach. Since there is a strategic game between the investors and the Planner, we will consider the following ‘partial run’ strategy profile of investors:

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<sup>22</sup>For general information about haircuts, see Gorton and Metrick (2012). Additional work regarding their use can be found in Dang, Gorton, and Holmström (2013) and Otto and Reed (2014).

<sup>23</sup>To determine the “haircut” simply use the formula  $1 - \frac{1}{\delta}$ . The definition of a haircut in this way is the same used in Dang, Gorton, and Holmström (2013).

$$\begin{aligned}
y_i(\omega_i, \alpha) &= \omega_i \quad \forall i \\
y_i(\omega_i, \beta) &= \begin{cases} 0 \\ \omega_i \end{cases} \text{ for } \begin{cases} i \leq \theta \\ i > \theta \end{cases}
\end{aligned} \tag{22}$$

Under this withdrawal strategy, patient investors in state  $\alpha$  will always wait to withdraw. On the other hand, *some* patient investors in state  $\beta$  who have the opportunity to withdraw before the planner realizes the state of the world will choose to withdraw early. Hence, only a segment of the population will run in state  $\beta$ .

### 3.1 Period 2: Settlement (Repurchase of Collateral)

Before the intermediary distributes the receipts from the repo market to the remaining patient agents, it first meets up with the borrower to complete the final leg of the repo transaction. Note that depending on the state of the world, the intermediary liquidated a fraction,  $\lambda_s$ , of the collateral amount  $x\delta$ . Therefore, at this stage, the intermediary possesses  $(1 - \lambda_s)x\delta$  (which is constrained to be less than  $z$ ) units of the collateral good in state  $s$ . As a result, this is also the amount of collateral available for repurchase. It remains to be determined how much the borrower will pay to re-acquire the remaining amount of collateral.

The repo price,  $r_s$ , is the state-contingent price of repurchasing collateral that was initially used to secure funding. As mentioned, the collateral available for repurchase depends on the state of the world. Thus, income raised by the intermediary in the repo market would equal  $r_s(1 - \lambda_s)x\delta$ . The funds raised by selling the collateral available in state  $s$  back to the dealer will be distributed to the remaining investors who seek to redeem their funds in state  $s$ .

The number of remaining patient investors,  $(1 - \theta)(1 - \hat{\pi}_s)$ , would then obtain

$$c_{2s} = \frac{r_s(1 - \lambda_s)x\delta}{(1 - \theta)(1 - \hat{\pi}_s)} \tag{23}$$

in income from the intermediary.<sup>24</sup> In return, income for the borrowers in the repurchase

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<sup>24</sup>The values of  $\hat{\pi}_s$  generated from the strategy profiles in (22) are represented as  $\hat{\pi}_\alpha = \frac{\pi - \theta}{1 - \theta}$  and  $\hat{\pi}_\beta = \pi$ . The value of  $\hat{\pi}_s$  therefore denotes the state-dependent fraction of investors who withdraw early.

would be:

$$b_{2s} = R \cdot x - r_s(1 - \lambda_s)x\delta + p_B(z - \lambda_s\delta x) \quad (24)$$

As a tractable alternative to Nash bargaining, we consider that the repo price paid in state  $s$  is chosen to maximize aggregate welfare across patient investors and borrowers. Optimal pricing in this manner insures that our results concerning fragility will not be driven by inefficiencies from the price determination mechanism.

The determination of the repo price is as follows. The total measure of individuals who wait until period 2 in state  $s$  to redeem their funds is equal to  $(1 - \theta)(1 - \hat{\pi}_s)$ . The population of borrowers seeking to repurchase their collateral is equal to 1. The repo price depends on the total number of investors redeeming funds in period 2 and borrowers' utility:

$$\max_{r_s} (1 - \theta)(1 - \hat{\pi}_s)u(c_{2s}) + u(b_{2s}), \quad (25)$$

**Proposition 1.** *The repo price is:*

$$r_s(\lambda_s, x, \delta) = \left( \frac{(1 - \theta)(1 - \hat{\pi}_s)}{1 + (1 - \theta)(1 - \hat{\pi}_s)} \right) \frac{[R \cdot x + p_B(z - \lambda_s x \delta)]}{(1 - \lambda_s)x\delta} \quad (26)$$

In turn, the repo rate is  $r_s(\lambda_s, x, \delta)\delta - 1$ .

The social returns from completing the second leg of the repo are equal to  $\frac{R \cdot x + p_B(z - \lambda_s x \delta)}{x}$ . However, given the total units of collateral available for repurchase, the repo price depends on the proportion of remaining investors who are patient as a fraction of the total size of the repo market in period 2. That is, the repo price is higher if there are more patient investors to be funded by the investment returns from the repo market.

**Corollary 1:** *Based upon the interest rates in the repo market, the state-dependent amount of consumption among the borrowers and patient investors is:*

$$c_{2s}(\lambda_s, x, \delta) = \frac{[R \cdot x + p_B(z - \lambda_s \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_s)} = b_{2s}(\lambda_s, x, \delta) \quad (27)$$

From the perspective of the patient investor, consumption is just the amount of income earned by borrowers dispersed among the size of the repo market in period 2. As is easily observed, however, the extent of collateral liquidation adversely affects consumption among patient investors. Therefore, Corollary 1 implies that the planner must carefully consider their decision whether to liquidate collateral in period 1.

Consumption for the borrowers also decreases with collateral liquidation. More importantly, borrowers in this environment with limited commitment are not forced to accept the repo rate that is proposed. Notably, the incentive compatibility constraint of borrowers in the settlement stage of a repurchase transaction is:

$$(1 - \lambda_s) p_B \delta x \geq \left( \frac{(1 - \theta)(1 - \hat{\pi}_s)}{1 + (1 - \theta)(1 - \hat{\pi}_s)} \right) [R \cdot x + p_B (z - \delta x)] \quad (28)$$

In other words, the amount of consumption gained through repurchasing the collateral must be greater than the proportion of private goods forfeited by walking away and strategically defaulting. Alternatively, borrowers' incentives can be expressed in terms of the amount of funding obtained from the intermediary:

$$x \geq \frac{(1 - \theta)(1 - \hat{\pi}_s) p_B z}{p_B \delta [(1 + (1 - \theta)(1 - \hat{\pi}_s)) (1 - \lambda_s) + (1 - \theta)(1 - \hat{\pi}_s)] - R(1 - \theta)(1 - \hat{\pi}_s)} \quad (29)$$

Higher investment translates into more collateral which in turn increases the incentive for borrowers to repurchase. From equation (29), it is also clear that liquidation of collateral by the risk-pooling intermediary tightens the participation constraint.

## 3.2 Period 1: Collateral Liquidation and Bailout Funding

Prior to the realization of the aggregate state, each intermediary would have committed to providing returns which offer  $c_1$  to all investors who show up in period 1 to withdraw their funds. After  $\theta \in [0, \pi]$  investors have withdrawn funds, both the intermediary and the fiscal authority realize the aggregate state of the economy. If  $\theta = \pi$ , the intermediary would

only learn that the aggregate economy is in crisis after all of the anticipated demand for redemptions (which should only have occurred by those truly impatient) were realized. This is the standard approach in Diamond-Dybvig models of fragility.

However, if  $\theta < \pi$ , the intermediary has a chance to revise payments prior to realizing the total anticipated demand for redemptions by depositors. That is, the Planner would have the ability to adjust payments to impatient investors (along with those who are privately patient) prior to the anticipated demand for short-term redemptions that should have taken place.

After the state of the world has been revealed, the Planner solves the following problem of maximizing aggregate utility among all of the remaining  $(1 - \theta)$  investors (by construction,  $c_1$  has already been allocated to each of the  $\theta$  individuals who showed up at the intermediary):

$$V(\psi_s; \hat{\pi}_s) \equiv \max_{(c_{1s}, c_{2s})} (1 - \theta) (\hat{\pi}_s u(c_{1s}) + (1 - \hat{\pi}_s) u(c_{2s})) \quad (30)$$

To determine the state-contingent resource constraint facing the Planner, first note that there are a total of  $(1 - \theta)$  individuals seeking redemptions once the aggregate state has been revealed. Of that group,  $(1 - \hat{\pi}_s)$  will show up in period 2. Therefore, the total consumption among the patient individuals awarded ex-post will be:  $(1 - \theta)(1 - \hat{\pi}_s)c_{2s}$ . By comparison,  $(1 - \theta)\hat{\pi}_s c_{1s}$  would be provided to the impatient investors in state  $s$ .

Each intermediary initially had total funding in the amount  $1 - \tau$  where  $\tau$  represents the tax imposed in period 0. The resources available to the intermediaries in period 1 are also contingent upon  $x$  (funds were lent to borrowers in the repo market in period 0) and  $\theta c_1$  (total redemptions provided before the realization of the aggregate state).

We begin our analysis of period 1 choices with those occurring in state  $\alpha$ .

### 3.2.1 Optimizing Consumption in the Good Aggregate State

In the good state, each intermediary solves the following problem:



$$V(\psi_\alpha; \hat{\pi}_\alpha) \equiv \max_{c_{1\alpha}, \lambda_\alpha} (1 - \theta) \left( \hat{\pi}_\alpha u(c_{1\alpha}) + (1 - \hat{\pi}_\alpha) u \left( \frac{[R \cdot x + p_B(z - \lambda_\alpha \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \right) + \mu_\alpha [p_F \lambda_\alpha \delta x + 1 - \tau - x - \theta c_1 - (1 - \theta) \hat{\pi}_\alpha c_{1\alpha}] \quad (31)$$

In addition to showing the implications of a widespread liquidity crisis for risk-pooling intermediation, our model is also designed to understand the determination of repo funding and credit intermediation in the shadow banking sector. In the absence of a financial crisis, lenders enter the repurchase stage with all of the collateral purchased in the initial leg of the repo agreement:

**Proposition 2:** *Intermediaries will not liquidate any of the collateral in the good state. That is,  $\lambda_\alpha = 0$  iff*

$$\frac{u'(c_{1\alpha})}{u'(c_{2\alpha})} < \frac{p_B}{p_F} \left( \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \quad (32)$$

The condition on liquidation of collateral can be written as a condition on the marginal rate of substitution between early and late consumers. As long as the marginal rate of substitution is not too high – that is, the desire of the planner to allocate additional income to the impatient consumers is not too high – then the Planner would not choose to liquidate any of the collateral. This is more likely to hold if the relative collateral value between the borrower and the intermediary is higher. In other words, as the economic inefficiency of collateral liquidation becomes greater. In addition, the condition is more difficult to achieve as the information asymmetry between investors and the Planner grows (ie.  $\theta$  increases).

In state  $\alpha$ , bailouts are also unnecessary. Therefore, the amount of consumption across investors in the good state is:

$$c_{1\alpha}(c_1, x, \tau) = \frac{1 - \tau - x - \theta c_1}{(1 - \theta) \hat{\pi}_\alpha} \quad (33)$$

$$c_{2\alpha}(c_1, x, \tau) = \frac{[R \cdot x + p_B z]}{1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)} \quad (34)$$

Substituting the period 1 solutions for  $c_{1\alpha}(c_1, x, \tau)$  and  $c_{2\alpha}(c_1, x, \tau)$  shows that the condition in Proposition 2 can be expressed in terms of the extent of investment in the repo market:

**Corollary 2:** *Intermediaries do not liquidate collateral in the good state iff:*

$$x < \frac{1 - \tau - \theta c_1 - p_B z \left( \frac{(1-\theta)\widehat{\pi}_\alpha}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right) \left[ \left( \frac{1+(1-\theta)(1-\widehat{\pi}_\alpha)}{(1-\theta)(1-\widehat{\pi}_\alpha)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}}}{\left[ 1 + R \left( \frac{(1-\theta)\widehat{\pi}_\alpha}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right) \left[ \left( \frac{1+(1-\theta)(1-\widehat{\pi}_\alpha)}{(1-\theta)(1-\widehat{\pi}_\alpha)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \right]} \quad (35)$$

Corollary 2 reevaluates intermediaries' incentives' to liquidate collateral as an upper bound on investment. This upper bound highlights an important feature of the model. When intermediaries invest at lower levels, they have minimal access to collateral to liquidate. With fewer resources to liquidate, the marginal value of each unit of collateral increases further strengthening the condition in (32).

### 3.2.2 Optimizing Consumption in the Bad Aggregate State

In the event that a financial crisis is underway, given all investors are following (22), the additional withdrawals from the run causes consumption for remaining investors with  $i > \theta$  to decline. To maximize aggregate utility across the remaining investors, the Planner can inject capital back into the intermediaries by liquidating some of its collateral holdings. It may also choose to transfer an amount of resources to intermediaries,  $k$ , by diverting funding away from the public good so that the intermediary would not have to liquidate as much collateral to meet the demand for redemptions.

When considering the decisions of financial intermediaries in the bad state, it is helpful to first look at the resource constraint facing the Planner:

$$\frac{Rx + p_B z - [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] c_{2\beta}}{p_B \delta x} = \lambda_\beta \quad (36)$$

If the Planner intends to give more consumption to patient investors, then he cannot liquidate as much collateral. However, for a given amount of  $c_{2\beta}$ , the planner can allocate more of the collateral for liquidation if the returns from the borrowers' investment projects are higher (and thus the repo price is higher).

By comparison, looking at the constraint on impatient investors:

$$\frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta \quad (37)$$

The proportion of collateral that must be liquidated is lower if there is more income available after funding of the public good, investment, and payments to the earliest withdrawers. In contrast to the resource constraint for state  $\alpha$ , we now take into account the possibility of a bailout – denoted by  $k \geq 0$ . As we can see, choosing  $c_{1\beta}$  and  $c_{2\beta}$  pins down  $\lambda_\beta$ .

Constructing the resource constraint through the collateral liquidation variable ( $\lambda_\beta$ ) yields:

$$\psi_\beta = \frac{p_F}{p_B} R x + p_F z + 1 - \tau + k - x - \theta c_1, \quad (38)$$

Thus, the feasibility constraint for our intermediary can be described by:

$$(1 - \theta)\widehat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B} (1 + (1 - \theta)(1 - \widehat{\pi}_\beta)) c_{2\beta} = \psi_\beta \quad (39)$$

Therefore, the Planner's objective once the aggregate state has been revealed is equal to:

$$\begin{aligned} V(\psi_\beta; \widehat{\pi}_\beta) &\equiv \max_{c_{1\beta}, c_{2\beta}} (1 - \theta) (\widehat{\pi}_\beta u(c_{1\beta}) + (1 - \widehat{\pi}_\beta) u(c_{2\beta})) + \\ &\mu_\beta \left[ \psi_\beta - \left[ (1 - \theta)\widehat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] c_{2\beta} \right] \right] \end{aligned} \quad (40)$$

**Proposition 3:** *In the event of a crisis the financial intermediary liquidates some proportion of collateral to satisfy:*

$$\frac{u'(c_{1\beta})}{u'(c_{2\beta})} = \frac{p_B}{p_F} \frac{(1-\theta)(1-\hat{\pi}_\beta)}{[1+(1-\theta)(1-\hat{\pi}_\beta)]} \quad (41)$$

For intermediaries to liquidate collateral in an effort to satisfy (41) then it must be the case that intermediaries realize ex-post an over-investment of investor resources into the borrower's investment technology.

Because we construct the resource constraint through  $\lambda_\beta$ , we place constraints on the intermediary's liquidation value in order to produce reasonable solutions regarding  $\lambda_\beta$ . The following corollary ensures that intermediaries cannot acquire more or liquidate more collateral than they hold:

**Corollary 3:** *Suppose that  $p_F$  satisfies  $\underline{p}_F < p_F < \overline{p}_F$ . Under this condition,  $\lambda_\beta \in (0, 1)$ .*

The lower bound in Corollary 3 implies that the collateral must have sufficient liquidation value in order to sell it prematurely instead of waiting for the second leg of the repo to be completed. The upper bound implies that the collateral must not be too valuable – otherwise, the lender would have an incentive to sell all of it instead of raising funds in the repo market.<sup>25</sup>

Knowing the values of  $\hat{\pi}_s$  generated by 22, we are able to state:

**Corollary 4:** *The MRS in the bad state is lower than the upper bound in the good state.*

Hence, the lower marginal rate of substitution in the bad aggregate state – caused by the moral hazard problem among investors – leads intermediaries to liquidate collateral in the bad aggregate state. To see the role of the asymmetric information, note that the marginal rate of substitution in the bad aggregate state is the same in the good aggregate state if  $\theta = 0$ . In comparison to Keister (2014), the total size of the financial system has a population mass of 2 since there are two groups of agents that consume: borrowers and investors.

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<sup>25</sup>Corollary 3 is consistent with the incentive issues of a repurchase agreement that other work, such as Roe et al. (2014), has addressed.

In the event of a financial crisis, the Planner may find it in the best interest of the investors to hold back on provision of the public good and instead redistribute some of its tax revenues to promote social welfare. Thus, the Planner solves:

$$\underset{(\tau-k)}{Max} [V(p_F \lambda_\beta(x, \tau, k, c_1) \delta x + 1 - \tau + k - x - \theta c_1; \hat{\pi}_\beta) + u(\tau - k)] \quad (42)$$

so that the size of the bailout satisfies:

$$u'(\tau - k) = \mu_\beta. \quad (43)$$

### 3.3 Period 0: The Planner's Investment in the Repo Market, Short-Term Returns, and Taxes

As in Allen and Gale (2000) and Cipriani, Martin, McCabe, and Parigi (2014), we consider the bad state to be an event which is highly unlikely to occur. Therefore, any realization of the bad aggregate state is unexpected causing the Planner to solve the following ex-ante problem:

$$\underset{x, c_1, \tau}{Max} \theta u(c_1) + [V(\psi_\alpha; \hat{\pi}_\alpha) + u(\tau)] \quad (44)$$

*The Planner's Early Consumption Decision*

$$u'(c_1) = \mu_\alpha \quad (45)$$

*The Planner's Investment in the Repo Market*

$$u'(c_{1\alpha}) = u'(c_{2\alpha}) R \left[ \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right] \quad (46)$$

The term on the right-hand side represents the increase in utility among the patient investors in the good state from a higher amount of lending by intermediaries. The term on the left-hand side represents the decrease in utility among impatient agents in the good state.

As we can see above, only the consumption levels in the good aggregate state are considered when making the investment decision. This holds true because any shock regarding the aggregate state is unexpected at date 0 and, therefore, does not affect the optimal risk-sharing contract offered by the MMMF:

$$\frac{u'(c_{1\alpha})}{u'(c_{2\alpha})} = R \left[ \frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right] \quad (47)$$

The MRS above comes from the maximization of investment in period 0. This MRS looks very similar to that provided by the state  $\alpha$  FOC from consumption choice in period 1. However, here we can see that the right-hand side is pre-multiplied by the strength of the investment technology,  $R$ , instead of  $\frac{p_B}{p_F}$ . Corollary 5 (below) provides conditions where any investment solution which satisfies (47) remains consistent with the corner solution found from the period 1 optimization.

**Corollary 5.** *Let  $R$  be such that  $\frac{p_E}{p_B}R < 1$ . Then, any investment solution which satisfies (47) remains consistent with complete settlement of repos in the good state.*

Although the Social Planner invested amount  $x$  of the private good in period 0, a partial value of the investment may still be redeemed in period 1 through collateral liquidation.

*The Planner's Tax Decision*

$$u'(\tau) = \mu_\alpha \quad (48)$$

The Planner chooses a tax that evens consumption out amongst all impatient consumers. It also takes into account the optimal trade-offs between allocating income for consumption and the public good:

$$u'(\tau) = u'(c_{1\alpha}) = u'(c_1). \quad (49)$$

We conclude with the following statement:

**Proposition 4.** *Suppose that  $\delta > \max(\underline{\delta}_\alpha^*, \underline{\delta}_\beta^*)$ . Further,  $p_F$  satisfies the conditions in Corollary 3 and Corollary 5. Then, an allocation in the planner-run financial system exists in which complete settlement of repo financing occurs in the good state. In the bad state, only partial settlement occurs.*

### 3.4 Fragility

The financial system is fragile if there exists an equilibrium strategy profile with  $y_i(1, \beta) = 0$  for a positive measure of investors. As Keister (2014) explains: “Fragility thus captures the idea that the financial system is potentially susceptible to a run based on shifting investor sentiment” (p. 7). In particular, we will consider the partial-run strategy profile outlined by (22).

The strategy profile of (22) is a partial run because only some individuals would run in the bad aggregate state. Since the intermediary follows a sequential service strategy, as soon as it realizes the aggregate state is  $\beta$ , it will adjust interest payments to the remaining

investors.<sup>26</sup> This adjustment to interest payments is performed by the intermediary through a decision on collateral – more specifically,  $\lambda_\beta$ . Investors will not run if they are after the  $\theta$ th individual since the information asymmetry between the investors and the fund would no longer be present. According to (22), investors with  $i \leq \theta$ , will choose to withdraw early in aggregate state  $\beta$  regardless of their patience or impatience.

Let  $y^*$  represent the partial run strategy profile in (22). Furthermore,  $y^*$  is part of an equilibrium if  $c_1^* \geq c_{2\beta}^*$ . It is straightforward to show that there exist parameter values such that this condition is satisfied. However, we need to verify whether it's also a necessary condition. That is, suppose a run is occurring in state  $\beta$ . Does a run lead to the conclusion that:

$$c_1^* \geq c_{2\beta}^* \quad (50)$$

Note the fraction of the first  $\theta$  withdraws in state  $s$  is such that  $\varepsilon_s \in [0, 1 - \pi]$ . In other words, in state  $\beta$ , the number of investors who ‘run’ is bounded by the number who do not experience a shock to preferences:

$$\varepsilon_\alpha = 0; \quad \varepsilon_\beta = 1 - \pi \quad (51)$$

The notation representing the remaining impatient investors where  $\widehat{\varepsilon}_s$  is defined by the bounds in (51) is  $\widehat{\pi}_s$ .

We intend to show that if  $\varepsilon_\beta > 0$ , it must be the case that  $c_1^* \geq c_{2\beta}^*$  holds. Otherwise, the partial run strategy outlined above does not necessarily exist in equilibrium. If this is the case then some other strategy may dominate in equilibrium. The notation

$$\widetilde{\pi}_s = \widetilde{\pi}_s(\widetilde{\varepsilon}_s) \quad (52)$$

will be used to represent the remaining impatient investors after the state has been revealed where  $\widetilde{\varepsilon}_s$  is not defined by (51).

Let  $\widetilde{y}$  be any strategy profile where the number of withdraws from investors whose  $i \leq \theta$  leads to  $\widetilde{\pi}_s$ . In response to the strategy profile  $\widetilde{y}$ , the Planner allocates  $\{\widetilde{c}_1, \widetilde{c}_{1s}, \widetilde{c}_{2s}\}$  for  $s = \alpha, \beta$ . If there is an equilibrium in which investors follow  $\widetilde{y}$ , it must be the case that

$$\widetilde{c}_1 \geq \widetilde{c}_{2\beta}. \quad (53)$$

Due to the strategy adopted by the  $\varepsilon_s$  individuals,

$$\widehat{\pi}_s(\widehat{\varepsilon}_s) = \frac{\pi - (1 - \widehat{\varepsilon}_s)\theta}{1 - \theta} \quad (54)$$

represents the fraction of remaining impatient investors after  $\theta$  withdraws have been made. The conditional probability that someone shows up early after the first  $\theta$  withdraws depends on the number of those who are truly impatient relative to the fraction of the  $\theta$  withdraws made by those who were actually patient. For example, in state  $\beta$  under the  $y^*$  strategy,

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<sup>26</sup>Keister (2014) discusses numerous examples where financial intermediaries have adjusted their liabilities during banking crises. In a similar manner, BNP Paribas suspended redemptions of three of its funds in August 2007. Upon retrospection, this event foreshadowed the coming global financial crisis.

some proportion of the first  $\theta$  individuals served were actually patient investors ‘running’ on the intermediary. The possible fraction of patient investors who choose to run is  $(1 - \pi)$ . Consequently,

$$\widehat{\pi}_\beta(\widehat{\varepsilon}_\beta) \equiv \frac{\pi - (1 - (1 - \pi))\theta}{1 - \theta} = \frac{\pi - \pi\theta}{1 - \theta} = \pi. \quad (55)$$

After  $\theta$  investors have been served the Planner offers an incentive compatible arrangement such that  $c_{1\beta}$  will be below  $c_{2\beta}$ . Thus, the conditional probability of an investor redeeming their funds early at this point just depends on the number of remaining impatient relative to those that were already served.

**Proposition 5:** *A Planner-run financial system is fragile if and only if  $c_1^* \geq c_{2\beta}^*$  holds.*

Here we show that given state  $\beta$  the best decision for a patient investor is to run if their order in line is  $i \leq \theta$ . Investors in our environment do not benefit from choosing any strategy other than  $y^*$ . Therefore, moving forward in the analysis the investors will always follow the choices outlined by (22).

## 4 Bailouts and Collateral Liquidation

After the Planner realizes the state of the world, he is able to make decisions regarding the amount of collateral to liquidate and the amount of public goods to put in place. In state  $\alpha$ , a complete settlement of repo transactions occurs as shown by Proposition 4. In addition, all of the taxes collected will be allocated towards funding for public goods.

In the event of a crisis and investor runs, absent any decision made by the Planner, the intermediary will run out of liquidity before all of the truly impatient agents have consumed in period 1. Because the Planner has a desire to provide risk-sharing among all investors, he will utilize the collateral and tax resources at his disposal to inject additional liquidity into the intermediaries’ balance sheets. Both actions come with social costs.

Collateral liquidation in our environment, and unlike Keister (2014), is not modeled as completely reversible investment funding. Liquidation comes with heavy social costs, particularly for those consuming in period 2. Transferring tax resources to the intermediaries, on the other hand, is a more efficient process. Although, providing a bailout decreases the level of public goods available for all investors. In this way, the Planner also has a desire to mitigate the losses among taxpayers who value public goods. The following Proposition shows how the Planner attempts to maximize investor welfare across both tools:

**Proposition 6.** *The planner chooses a combination of bailouts and collateral liquidation when the bad state occurs,  $k^* > 0$  and  $\lambda_\beta^* > 0$ .*

Providing a bailout decreases the level of public goods available for both impatient and patient investors. In contrast, liquidating collateral leads to lower consumption for patient investors. Thus, the planner utilizes two tools to improve social welfare in the event of a crisis. Obviously, Proposition 6 demonstrates that the planner will implement a fiscal bailout by diverting funding from public goods. Yet, the Planner also imposes costs on intermediaries by liquidating some of the collateral held by institutions. In conjunction with Corollary 3,



Proposition 6 provides conditions under which the Planner will seek to avoid huge economic losses in the repo market.

## 5 Equilibrium Under Discretion

We now deviate from the Planner's allocation to studying equilibrium behavior in a decentralized world. In this case, the policymaker or fiscal authority has *discretion* over the amount of funds to “bailout” the intermediary in the bad aggregate state. The total size of the bailout across all intermediaries is defined as:

$$k \equiv \int k^j d\sigma(j). \quad (56)$$

Let  $k^j \geq 0$  denote the bailout payment by the fiscal authority to intermediary  $j$  for each investor. In addition, let  $\sigma(j)$  denote the distribution of investors who have allocated funds to intermediary  $j$ . Then,

$$\max_{\{k^j\}} \equiv \int V(\psi_\beta^j; \hat{\pi}_\beta) d\sigma(j) + u(\tau - k^j) \quad (57)$$

subject to the feasibility constraint:

$$\psi_\beta^j = \frac{p_F}{p_B} Rx^j(j) + p_F z + 1 - \tau + k^j - x^j - \theta c_1^j \quad (58)$$

For a given size of the bailout package  $k$  per investor, the amount of the bailout entails:

$$k^j = k + \theta(c_1^j - \bar{c}_1) \quad \forall j \quad (59)$$

where  $\bar{c}_1 \equiv \int c_1^j d\sigma(j)$  is the average level of  $c_1^j$  in the economy.

The logic behind this specification of the bailout is that the institution paid  $c_1^j$  to  $\theta$  individuals before it became aware of the aggregate state and did not know that it should restructure its interest payments to depositors. Thus, the bailout should be designed to correct the error from the information asymmetry between the investors (who know the aggregate state) and the intermediary (unaware of the aggregate state at the beginning of the period). In turn, we have:

$$\psi_\beta^j = \frac{p_F}{p_B} Rx^j + p_F z + 1 - \tau - x^j - \theta \bar{c}_1 + k \quad (60)$$

in which the intermediary takes  $\bar{c}_1$  as given.

In choosing  $k^j$ , the fiscal authority takes into account how the size of the bailout affects the decision making of the intermediaries. Recall from the Planner's problem that bailouts directly reduce the need for intermediaries to liquidate collateral. Consequently, the Lagrangian function of the fiscal authority is:

$$\begin{aligned} \max_k \mathcal{L} &= \int V(\psi_\beta^j; \hat{\pi}_\beta) d\sigma(j) + u(\tau - k) + \\ &\mu_\beta^j \left[ \frac{p_F}{p_B} R x^j + p_F z + 1 - \tau - x^j - \theta \bar{c}_1 + k - \right. \\ &\left. \left( (1 - \theta) \hat{\pi}_\beta c_{1\beta}^j + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \hat{\pi}_\beta)] c_{2\beta}^j \right) \right] \end{aligned} \quad (61)$$

When the aggregate state is revealed the fiscal authority chooses the size of the bailout and each intermediary solves:

$$\begin{aligned} V(\psi_\beta^j; \hat{\pi}_\beta) &\equiv \max_{c_{1\beta}^j, c_{2\beta}^j} (1 - \theta) (\hat{\pi}_\beta u(c_{1\beta}^j) + (1 - \hat{\pi}_\beta) u(c_{2\beta}^j)) \\ &+ \mu_\beta^j \left[ \frac{p_F}{p_B} R x^j + p_F z + 1 - \tau - x^j - \theta \bar{c}_1 + k - \right. \\ &\left. \left( (1 - \theta) \hat{\pi}_\beta c_{1\beta}^j + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \hat{\pi}_\beta)] c_{2\beta}^j \right) \right] \end{aligned} \quad (62)$$

This yields the FOCs:

$$u'(c_{1\beta}^j) = \mu_\beta^j \quad (63)$$

$$u'(c_{2\beta}^j) = \frac{p_F}{p_B} \frac{[1 + (1 - \theta)(1 - \hat{\pi}_\beta)]}{(1 - \theta)(1 - \hat{\pi}_\beta)} \mu_\beta^j \quad (64)$$

The optimal risk-sharing condition is

$$u'(c_{1\beta}^j) = u'(c_{2\beta}^j) \frac{p_B}{p_F} \frac{(1 - \theta)(1 - \hat{\pi}_\beta)}{[1 + (1 - \theta)(1 - \hat{\pi}_\beta)]} = \mu_\beta^j \quad (65)$$

The results are analogous to the Planner's allocation. Based upon the behavior of each intermediary, the envelope theorem applies and therefore the bailout is pinned down by:

$$u'(\tau - k) = \mu_\beta^j \quad (66)$$

By the envelope theorem, there are not any indirect effects of the bailout package on the interest payments to each investor. Simply put, the fiscal authority recognizes that the social cost of the bailout derives from lower provisions of the public good to everyone. The marginal benefit ( $\mu_\beta$ ) of lower provisions is the additional resources that can be transferred to the remaining  $(1 - \theta)$  depositors after the system has been subjected to a run.

Given that the market for investor funds and the money market are both perfectly competitive, the optimal decisions across intermediaries and borrowers will be the same. Therefore, in order to simplify the notation moving forward, we will denote the decentralized funding decisions by all intermediaries as  $\{c_1^D, c_{1s}^D, c_{2s}^D\}$  for  $s = \alpha, \beta$ .

## 5.1 Intermediaries' Decisions

The financial intermediary chooses the level of income to be provided to the earliest consumers and the level of investment, taking the level of taxes imposed by the fiscal authority ( $\tau^D$ ) as given:

$$\underset{x, c_1}{Max} \theta u(c_1^D) + [V(\psi_\alpha; \hat{\pi}_\alpha) + u(\tau^D)] \quad (67)$$

*Payments to the early investors:*

$$u'(c_1^D) = \mu_\alpha \quad (68)$$

*Investment in the Repo Market:*

$$u'(c_{1\alpha}^D) = u'(c_{2\alpha}^D) R \left[ \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right] \quad (69)$$

The decision rules adopted by the financial intermediary are the same as the Planner. However, the levels of activity will not necessarily be the same across cases as the fiscal authority's tax decision plays a major role.

## 5.2 Fiscal Authority's Tax Decision

The fiscal authority only has direct control over the level of taxes and provision of the public good. This decision is chosen to maximize aggregate investor expected utility as follows:

$$\underset{\tau^D}{Max} \theta u(c_1) + [V(\psi_\alpha; \hat{\pi}_\alpha) + u(\tau^D)] \quad (70)$$

The tax decision is:

$$u'(g_\alpha^D) = \mu_\alpha^D \left[ 1 + \frac{dx^D}{d\tau^D} \right]. \quad (71)$$

In turn, early consumption and investment are both functions of taxes,  $c_1(\tau^D)$  and  $x(\tau^D)$ .

As observed in the Planner's decision, lump-sum taxes are chosen to reflect an optimal trade-off between access to public goods and the loss of consumption across investors. However, as the fiscal authority attempts to maximize aggregate expected utility across investors, it also has a desire to influence the level of risk-sharing in the economy through investment in the repo market,  $dx/d\tau^D$ . We show in the Appendix that  $dx/d\tau^D$  is negative. Therefore, the level of taxes will be higher than the level chosen by the Planner:

$$\tau^D > \tau^* \quad (72)$$

The excessive level of taxation emerges here because the fiscal authority only has one tool to influence the level of risk-sharing whereas the planner can directly affect investment in the repo market. This mechanism is unique to our framework – as investment in Keister is

completely reversible, the investment decision is trivial. In our model, investment in the repo market is motivated by the desire to fund patient consumers. Consequently, repo funding by the intermediary affects the level of risk-sharing across all of the fund's investors.

In addition to affecting the degree to which investors obtain risk-sharing, the level of taxes affects the ability of the fiscal authority to deal with a liquidity crisis. To show this to be the case, we inspect the proportion of tax resources to intermediaries' liquid assets after the state of the world is revealed. The proportion is defined as:

$$\psi \equiv \frac{\tau}{1 - x - \theta c_1} \quad (73)$$

Hereafter, we refer to  $\psi$  as *public liquidity*. When an economy has higher public liquidity, the fiscal authority holds a larger proportion of the economy's liquid assets making them more agile in the event of a crisis. Using this measurement, our results suggest that the fiscal authority in a decentralized economy is more capable of dealing with a financial crisis as opposed to the Planner's case:

$$\psi_\tau^D > \psi_\tau^* \quad (74)$$

In essence, taxes reduce investment which has a direct effect on each intermediary's resource constraint. The higher level of taxes encourages intermediaries to provide more funding to impatient investors at the expense of patient investors. To compensate for inequality of consumption of the private good, the fiscal authority increases taxes in order to boost the level of public good because that consumption is shared across both investor cohorts. However, if a policymaker were to commit to no bailouts, then higher taxes would adversely affect the intermediaries' ability to respond to a crisis.

As in the Planner's allocation, we can show that a fiscal authority under discretion chooses a level of bailouts but also imposes costs on intermediaries in the system by inducing them to liquidate some of the collateral that they hold:

**Proposition 7.** *Suppose that  $\delta > \max(\delta_\alpha^D, \delta_\beta^D)$ . Further,  $p_F$  satisfies the conditions in Corollary 3 and Corollary 5. Then, an allocation under fiscal discretion exists in which complete settlement of repo financing occurs in the good state. In the bad state, only partial settlement occurs.*

We also show that the system can be fragile under discretion:

**Proposition 8.** *The shadow-banking system under fiscal discretion is fragile if and only if  $c_1^D \geq c_{2\beta}^D$  holds.*

### 5.3 Bailouts vs. No Bailouts

In contrast to a setting where the fiscal authority has discretion to provide a bailout in the event that the bad state emerges, we can also consider that the fiscal authority may be committed to not bailout institutions as recent legislation such as the Dodd-Frank Act would imply. Since the information set is the same in period 0 given a commitment to bailouts or not, the ex-ante decisions of the financial intermediaries and the fiscal authority are identical.

The only way to respond to a crisis under fiscal restrictions is to liquidate collateral. In this section, we will show that between the two choices, offering bailout funding in the event of a crisis is objectively the better decision. Consumption decisions in the no bailouts case are denoted as  $\{c_1^{NB}, c_{1s}^{NB}, c_{2s}^{NB}\}$  for  $s = \alpha, \beta$ .

**Proposition 9:** *Under the expectation that bailouts will not occur, the set of possible economies that lead to bank runs is strictly larger than that of a discretionary regime.*

Let  $\Phi$  be the set of possible parameter values that characterize an economy that leads to bank runs when the bad state occurs. Using the same set theoretic approach applied in Keister (2014), we are able to show that  $\Phi^D \subset \Phi^{NB}$ . Therefore, by committing to no bailouts policymakers increase the possibility of bank runs when investors realize the bad state of the economy.

**Proposition 10.** *Suppose that  $\underline{\delta}_\beta^D < \delta < \underline{\delta}_\beta^{NB}$ . This means there is a subset of parameter values in  $\Phi^{NB}$  where a run on intermediaries by investors leads to a collapse of the repo market in a no bailouts regime yet survives the run with bailout funding.*

Overcollateralization ( $\delta$ ) is an exogenous parameter directly linked to the repo price. In accordance with the risk sharing motive of the repurchase agreement, it holds true that the higher the haircut the lower the repo price. The Proposition shows that there is a range of  $\delta$  where instability in the repo market occurs if the fiscal authority refuses to rescue intermediaries but remains stable if the fiscal authority has discretion over bailout funding. We refer to a collapse of the repo market as a setting where the second leg of repos in the system is not settled. That is, strategic default by borrowers occurs. This emerges if  $r_\beta > p_B$ . Proposition 10 also shows that as the level of overcollateralization approaches  $\underline{\delta}_\beta^{NB}$  the possibility of a market collapse decreases. This result provides support for regulation on overcollateralization as a policy tool toward stability on repo market financing.

In a no bailouts regime, intermediaries only have one way to respond to runs by investors – they liquidate large amounts of collateral as observed during the fire sales of securities which occurred after the failure of Lehman Brothers.<sup>27</sup> In a statement made by the Treasury in 2009:

“An initial fundamental shock associated with the bursting of the housing bubble and deteriorating economic conditions generated losses for leveraged investors and banks... The resulting need by investors and banks to reduce risk triggered a wide-scale deleveraging in these markets and led to fire sales. As prices declined, many traditional investors exited these markets, causing declines in market liquidity.”

Our model examines such an occurrence. When investors become aware of state  $\beta$ , the idea of massive liquidation encourages strategic default by borrowers further reinforcing the first-mover advantage among investors with  $i \leq \theta$ . Consequently, credit intermediation in

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<sup>27</sup>A narrative of fire-sales during a financial crisis can be found in Schleifer and Vishny (2011).

the repo market is much more susceptible to breaking down if there is a commitment against bailouts in place.

## 6 Comparing Bailout Cases

We have shown that commitments against bailouts can have destabilizing effects on MMMFs and repo market activity. The possibility of such instability is precisely why the Treasury instituted their temporary guarantee program. Focusing our attention away from the most destabilizing choice, we begin comparing choices under discretion to those provided by a social planner. In this manner, we seek to understand how a Planner’s willingness to tolerate instability in the shadow banking system contrasts with fiscal authorities’ preferences. We also provide support for the use of taxation in order to bring greater ex-ante stability to the shadow banking system.

In the discretionary case, we showed that taxes are inefficiently high as the fiscal authority attempts to influence the level of investment indirectly through taxes:

$$c_1^D < c_1^*; x^D < x^* \quad (75)$$

As one would expect, the reduction in repo market investment within a decentralized economy leads to less consumption in the good aggregate state for all repo market participants (both investors and borrowers):

$$c_{1\alpha}^D < c_{1\alpha}^*; c_{2\alpha}^D < c_{2\alpha}^*; b_{2\alpha}^D < b_{2\alpha}^* \quad (76)$$

The optimal repo rate is also affected through the reduction in investment:

$$r_\alpha^D > r_\alpha^* \quad (77)$$

With investment resources becoming more scarce, interest rates increase to clear the market.

The investment technology under discretion is under-utilized causing economy-wide consumption of the private good to be at an inefficiently low level compared to the Planner’s case. However, in the bad aggregate state, higher taxes provide the fiscal authority with more resources for bailout funding – a mechanism that is less inefficient than prematurely selling collateral.

By Propositions 5 and 8, in the event that a fiscal crisis occurs, patient investors may have an incentive to withdraw early if  $i \leq \theta$ . To handle the excessive withdraws, financial intermediaries will begin liquidating collateral at rate  $p_F$ . In both the decentralized and social planner’s cases, intermediaries’ liquidity needs are accommodated with a bailout. The following comparison between cases highlights the benefits of higher taxation and the role of the government after a financial crisis is underway.

Liquidation of collateral is an inefficient means of raising resources. First, the liquidation price of collateral is less than both the repo price and the borrowers’ value of collateral. In this way, use of the intermediaries’ weak transformation technology can be thought of as a fire-sale. Second, premature sales of collateral adversely affect incentives to settle repos. For example, Proposition 1 shows that repos are re-priced in the bad aggregate state. An increase in repo rates during a crisis can distort incentives for borrowers to participate in the

second leg of the repo. That is, fewer collateral goods upon repurchase encourages strategic default.

Bailout funding is based on the lost marginal utility from access to the economy's public good. On the other hand, during a fire-sale, the liquidation value of the collateral ( $p_F$ ) may be low. Therefore, redistributing fiscal resources from the public good to the shadow banking system can prevent large economic losses:

$$c_{1\beta}^D > c_{1\beta}^*; c_{2\beta}^D > c_{2\beta}^*; b_{2\beta}^D > b_{2\beta}^* \quad (78)$$

As public liquidity is higher under discretion,  $\psi_\tau^D > \psi_\tau^*$ , the liquidity needs of intermediaries in a time of crisis are impacted by a greater degree under discretion than in a Planner-run financial system:

$$k^D > k^* \quad (79)$$

With more liquidity coming from bailouts, collateral liquidation is decreased:

$$\lambda_\beta^D < \lambda_\beta^* \quad (80)$$

Consequently, intermediaries are able to meet their withdraw obligations after a run without substantial liquidation of assets. Along with the increased stability of the repo market, the higher taxes in the decentralized economy also lessen the likelihood for runs to occur:

**Proposition 11:**  $\Phi^* \supset \Phi^D$ . *Furthermore, the decentralized financial system has a more stable repo market when crises occur.*

If a run by investors on MMMFs were to occur in either case, the decentralized system would be less likely to see a collapse of the repo market. Two characteristics of the decentralized economy create this featured stability. First, smaller initial investments by intermediaries in the repo market lead to lower returns. In this manner, policymakers face a trade-off between the efficiency of the shadow banking system versus its stability. Second, as  $k^D > k^*$ , the fiscal authority shoulders more of the burden than the Planner. Since the fiscal authority transfers more resources to intermediaries in the system, less collateral would be sold indicating the extent of "fire sales" would be lower. Consequently, repo rates will not be as volatile under discretion as in a Planner-run financial system. Therefore, the probability of a run occurring in the Planner's economy is strictly greater than in the decentralized case. That is, the Planner may be more willing to tolerate a crisis in the shadow banking system than fiscal authorities under discretion.

## 7 Conclusion

MMMFs are one of the primary ways both retail and institutional investors seek returns on their capital – a fact that has not escaped policymakers as the opening chapter of Dodd-Frank addresses the issue of increasing stability within non-bank financial institutions. Yet, failures of MMMFs are highly unlikely events. However, aggregate financial crises like that of

2007-2008 have shown to create runs across the mass of MMMFs which can lead to the failure of many funds at once. Without access to deposit insurance or the discount window, MMMFs must liquidate some of their illiquid assets in order to meet excessive early redemptions from runs.

In this paper we provide support for policies which can increase the stability of the repo market. It is important to note that these policy implications contrast with Title I of the Dodd-Frank Act which intends on eliminating the *expectation* of taxpayer funded bailouts. In particular, we find eliminating the expectation of bailouts increases the likelihood of runs and, in turn, can lead to a collapse of the repo market. Subsequently, extreme economic losses are borne by investors due to the extensive devaluation of the collateral assets held by MMMFs. Providing the expectation of bailouts, on the other hand, bolsters the repo market.

A benevolent planner choosing the efficient allocation of resources sets taxes lower than in a decentralized economy. This occurs because the fiscal authority only has one instrument to indirectly affect the amount of risk-sharing in the economy – the level of taxes. Lower taxes in the Planner’s allocation generally lead to higher returns for all repo market participants. By comparison, the higher level of taxes imposed by a fiscal authority with discretion leads to inefficiently low levels of investment and lower investor returns. Yet, in the event of a crisis, the fiscal authority has resources to capitalize institutions under distress.

In comparison to Keister (2014), public sector intervention in our model can play an important role in stabilizing the repo market by preventing massive liquidation of collateral which causes repo markets to collapse. Thus, policymakers aiming to increase stability of the repo market in times of crisis should carefully consider the use of public funds to stabilize shadow banking institutions. Nevertheless, as some liquidation of collateral is optimal, we also show that optimal policy does impose some losses among participants in the shadow banking system.

Understanding that bailouts for large financial institutions can be extremely unpopular, Proposition 10 gives policymakers an alternative. In particular, imposing minimum standards on haircuts (or haircut floors) have recently been proposed by the SEC Commissioner, Financial Stability Board, and European Parliament.<sup>28</sup> Proposition 10 offers support to these proposals. Higher minimum standards on haircuts lead to a lower probability of repo market collapse. Furthermore, there is a level of haircuts in which a bailout would not be necessary in order to prevent a market collapse. Obviously, however, minimum standards would come at a cost. Higher haircuts place greater restrictions on the amount of funding borrowers can access through the repo market since borrowers can only pledge as much collateral as they have hold. Therefore, the extent of overcollateralization is limited by the amount of collateral resources available to borrowers.

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<sup>28</sup>Policy recommendations regarding haircut floors can be found in Financial Stability Board (2013) and Comotto (2013). Commissioner of the SEC, Kara M. Stein, recently encouraged the agency to require some meaningful haircuts on repos as well.



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# Appendix

**Proof of Proposition 1.** The planner's objective:

$$\max_{r_s} (1 - \theta)(1 - \hat{\pi}_s)u(c_{2s}) + \eta(b_{2s}),$$

$$r_s = \left( \frac{(1 - \theta)(1 - \hat{\pi}_s)}{1 + (1 - \theta)(1 - \hat{\pi}_s)} \right) \frac{R \cdot x + p_B(z - \lambda_s \delta x)}{(1 - \lambda_s)x\delta}$$

**Proof of Corollary 1:**

*The state-dependent levels of consumption for investors are:*

$$c_{2s}(\lambda_s, x, \delta) = r_s \left( \frac{(1 - \lambda_s)x\delta}{(1 - \theta)(1 - \hat{\pi}_s)} \right)$$

$$c_{2s}(\lambda_s, x, \delta) = \frac{[R \cdot x + p_B(z - \lambda_s \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_s)}$$

*By comparison, for borrowers:*

$$b_{2s} = R \cdot x - r_s(1 - \lambda_s)x\delta + p_B(z - \lambda_s \delta x)$$

$$b_{2s} = R \cdot x - \left( \frac{(1 - \theta)(1 - \hat{\pi}_s)}{1 + (1 - \theta)(1 - \hat{\pi}_s)} \right) [R \cdot x + p_B(z - \lambda_s \delta x)] + p_B(z - \lambda_s \delta x)$$

*In the good state:*

$$b_{2\alpha} = R \cdot x - \left( \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) [R \cdot x + p_B z] + p_B z$$

$$b_{2\alpha} = R \cdot x - (1 - \theta)(1 - \hat{\pi}_\alpha)c_{2\alpha} + p_B z$$

$$b_{2\alpha} = c_{2\alpha}$$

*In the bad state:*

$$b_{2\beta} = R \cdot x - (1 - \theta)(1 - \hat{\pi}_\beta)c_{2\beta} + p_B(z - \lambda_\beta \delta x)$$

$$\frac{R \cdot x + p_B z - [1 + (1 - \theta)(1 - \hat{\pi}_\beta)]c_{2\beta}}{p_B \delta x} = \lambda_\beta$$

$$b_{2\beta} = c_{2\beta}$$

Therefore, regardless of the state:

$$b_{2s} = c_{2s}$$

**Proof of Proposition 2.**

In the good aggregate state, the Planner's objective is:

$$V(\psi_\alpha; \hat{\pi}_\alpha) \equiv \max_{c_{1\alpha}, \lambda_\alpha} (1 - \theta) \left( \hat{\pi}_\alpha u(c_{1\alpha}) + (1 - \hat{\pi}_\alpha) u \left( \frac{[R \cdot x + p_B(z - \lambda_\alpha \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \right) + \mu_\alpha [p_F \lambda_\alpha \delta x + 1 - \tau - x - \theta c_1 - (1 - \theta) \hat{\pi}_\alpha c_{1\alpha}]$$

It is optimal not to liquidate in the good state as long as:

$$\frac{dV(\psi_\alpha; \hat{\pi}_\alpha)}{d\lambda_\alpha} = (1 - \theta)(1 - \hat{\pi}_\alpha) u'(c_{2\alpha}) \left( \frac{-p_B \delta x}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) + \mu_\alpha p_F \delta x < 0$$

That is, if the additional utility received by the impatient investors upon liquidation of the collateral is not sufficient to cover the loss among the patient investors, then none of the collateral will be liquidated.

By comparison, the optimal choice of  $c_{1\alpha}$  is pinned down by the shadow value of resources in state  $\alpha$ :

$$\frac{dV(\psi_\alpha; \hat{\pi}_\alpha)}{dc_{1\alpha}} = u'(c_{1\alpha}) = \mu_\alpha$$

Liquidating the marginal unit of collateral would lower income that would be allocated to the late consumers, depending on the liquidation value that the borrower would be able to generate. The loss of consumption among the late consumers is given by  $\frac{p_B \delta x}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)}$ . The total loss of utility among the population of late consumers depends on the number remaining after the aggregate state has been revealed,  $(1 - \theta)(1 - \hat{\pi}_\alpha)$ .

$$\frac{dV(\psi_\alpha; \hat{\pi}_\alpha)}{d\lambda_\alpha} = (1 - \theta)(1 - \hat{\pi}_\alpha) u'(c_{2\alpha}) \left( \frac{-p_B}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) + u'(c_{1\alpha}) p_F < 0$$

$$\frac{u'(c_{1\alpha})}{u'(c_{2\alpha})} < \left( \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \left( \frac{p_B}{p_F} \right)$$

**Proof of Corollary 2:**

Defining the condition in terms of explicit consumption decisions:

$$\left( \frac{1 - \tau - x - \theta c_1}{(1 - \theta) \hat{\pi}_\alpha} \right)^{-\gamma} < \left( \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \left( \frac{p_B}{p_F} \right) \left( \frac{[R \cdot x + p_B z]}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right)^{-\gamma}$$

Solving for  $x$ :

$$x < \frac{1 - \tau - \theta c_1 - p_B z \left( \frac{(1 - \theta) \hat{\pi}_\alpha}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \left[ \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)}{(1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}}}{\left[ 1 + R \left( \frac{(1 - \theta) \hat{\pi}_\alpha}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \left[ \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)}{(1 - \theta)(1 - \hat{\pi}_\alpha)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \right]}$$

**Proof of Proposition 3:**

The Planner's objective once the aggregate state has been revealed is equal to:

$$V(\psi_\beta; \hat{\pi}_\beta) \equiv \max_{c_{1\beta}, c_{2\beta}} (1 - \theta) (\hat{\pi}_\beta u(c_{1\beta}) + (1 - \hat{\pi}_\beta) u(c_{2\beta})) + \mu_\beta \left[ \begin{aligned} & \frac{p_F}{p_B} R x + p_F z + 1 - \tau + k - x - \theta c_1 - \\ & \left( (1 - \theta) \hat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \hat{\pi}_\beta)] c_{2\beta} \right) \end{aligned} \right]$$

$$\frac{dV(\psi_\beta; \hat{\pi}_\beta)}{dc_{1\beta}} = u'(c_{1\beta}) = \mu_\beta$$

$$\frac{dV(\psi_\beta; \hat{\pi}_\beta)}{dc_{2\beta}} = u'(c_{2\beta}) = \frac{p_F [1 + (1 - \theta)(1 - \hat{\pi}_\beta)]}{p_B (1 - \theta)(1 - \hat{\pi}_\beta)} \mu_\beta$$

Therefore,

$$\frac{u'(c_{1\beta})}{u'(c_{2\beta})} = \frac{p_B (1 - \theta)(1 - \hat{\pi}_\beta)}{p_F [1 + (1 - \theta)(1 - \hat{\pi}_\beta)]}$$

**Proof of Corollary 3**

First, we solve for the condition that  $\lambda_\beta < 1$ . We do so by taking the solutions for  $c_{1\beta}$  and  $c_{2\beta}$  and maximizing over  $\lambda_\beta$ :

$$V(\psi_\beta; \hat{\pi}_\beta) \equiv \max_{\lambda_\beta} (1 - \theta) \left[ \begin{aligned} & \hat{\pi}_\beta u \left( \frac{p_F \lambda_\beta \delta x + 1 - \tau + k - x - \theta c_1}{(1 - \theta) \hat{\pi}_\beta} \right) + \\ & (1 - \hat{\pi}_\beta) u \left( \frac{[R \cdot x + p_B (z - \lambda_\beta \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_\beta)} \right) \end{aligned} \right]$$

Evaluating at  $\lambda_\beta = 1$  and establishing conditions in which 100% liquidation is not optimal:

$$\begin{aligned} & u' \left( \frac{p_F \delta x + 1 - \tau + k - x - \theta c_1}{(1 - \theta) \hat{\pi}_\beta} \right) p_F \delta x \\ < (1 - \theta) (1 - \hat{\pi}_\beta) u' \left( \frac{[R \cdot x + p_B (z - \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_\beta)} \right) \left( \frac{p_B \delta x}{1 + (1 - \theta)(1 - \hat{\pi}_\beta)} \right) \end{aligned}$$

$$\left( \frac{p_F}{p_B} \right) \left( \frac{[R \cdot x + p_B (z - \delta x)]}{p_F \delta x + 1 - \tau + k - x - \theta c_1} \right)^\gamma < \frac{(1 - \hat{\pi}_\beta)}{(\hat{\pi}_\beta)^\gamma} \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)} \right)^{\gamma-1}$$

Second, we solve for the condition that  $\lambda_\beta > 0$  by evaluating at  $\lambda_\beta = 0$ :

$$\begin{aligned} & (1 - \theta) \hat{\pi}_\beta u' \left( \frac{1 - \tau + k - x - \theta c_1}{(1 - \theta) \hat{\pi}_\beta} \right) \left( \frac{p_F \delta x}{(1 - \theta) \hat{\pi}_\beta} \right) \\ > (1 - \theta) (1 - \hat{\pi}_\beta) u' \left( \frac{[R \cdot x + p_B z]}{1 + (1 - \theta)(1 - \hat{\pi}_\beta)} \right) \left( \frac{p_B \delta x}{1 + (1 - \theta)(1 - \hat{\pi}_\beta)} \right) \end{aligned}$$

$$\left( \frac{p_F}{p_B} \right) \left( \frac{[R \cdot x + p_B z]}{1 - \tau + k - x - \theta c_1} \right)^\gamma > \frac{(1 - \hat{\pi}_\beta)}{(\hat{\pi}_\beta)^\gamma} \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)} \right)^{\gamma-1}$$

With the two conditions above, we can reduce the inequalities down to thresholds on the exogenous parameter  $p_F$ .

$$p_F < \bar{p}_F = p_B \frac{(1 - \hat{\pi}_\beta)}{(\hat{\pi}_\beta)^\gamma} \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)} \right)^{\gamma-1} \left( \frac{p_F \delta x + 1 - \tau + k - x - \theta c_1}{[R \cdot x + p_B (z - \delta x)]} \right)^\gamma$$

$$p_F > \underline{p}_F = p_B \frac{(1 - \hat{\pi}_\beta)}{(\hat{\pi}_\beta)^\gamma} \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)} \right)^{\gamma-1} \left( \frac{1 - \tau + k - x - \theta c_1}{[R \cdot x + p_B z]} \right)^\gamma$$

#### Proof of Corollary 4:

The lower MRS can essentially be described by the fraction of patient investors in the economy after the bad state occurs as opposed to the good state.

$$\frac{(1 - \theta) (1 - \hat{\pi}_\beta)}{[1 + (1 - \theta)(1 - \hat{\pi}_\beta)]} < \frac{(1 - \theta) (1 - \hat{\pi}_\alpha)}{[1 + (1 - \theta)(1 - \hat{\pi}_\alpha)]}$$

After substituting in the values for  $\hat{\pi}_\alpha$  and  $\hat{\pi}_\beta$ , we get

$$(1 - \theta) + (1 - \theta) (1 - \pi) < 1 + (1 - \theta)(1 - \pi)$$

### Derivation of the Condition in Corollary 5.

We know that  $u'(c_{1\alpha}) = \mu_\alpha$  and  $u'(c_{2\alpha}) \left[ \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right] > \frac{p_F}{p_B} \mu_\alpha$

$$u'(c_{1\alpha}) = u'(c_{2\alpha}) R \left[ \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right]$$

$$\mu_\alpha > R \frac{p_F}{p_B} \mu_\alpha$$

$$\frac{p_F}{p_B} R < 1$$

### Proof of Planner's Tax:

$$\text{Max}_\tau \theta u(c_1) + [V(1-\tau-x-\theta c_1; \hat{\pi}_\alpha) + v(g_\alpha)]$$

The optimal tax in the good aggregate state follows:

$$\text{Max}_\tau [V(1-\tau-x-\theta c_1; \hat{\pi}_\alpha) + v(g_\alpha)]$$

$$\frac{dV(1-\tau-x-\theta c_1; \hat{\pi}_\alpha)}{d\tau} + v'(g_\alpha) = 0$$

Recall

$$V(\psi_\alpha; \hat{\pi}_\alpha) \equiv (1-\theta)(\hat{\pi}_\alpha u(c_{1\alpha}) + (1-\hat{\pi}_\alpha)u(c_{2\alpha})) + \mu_\alpha \left[ \begin{array}{l} \frac{p_F}{p_B} R x + p_F z + 1 - \tau - x - \theta c_1 - \\ \left[ (1-\theta)\hat{\pi}_\alpha c_{1\alpha} + \frac{p_F}{p_B} [1+(1-\theta)(1-\hat{\pi}_\alpha)] c_{2\alpha} \right] \end{array} \right]$$

$$\frac{dV(\psi_\alpha; \hat{\pi}_\alpha)}{d\tau} = -\mu_\alpha$$

Plugging these values in:

$$\mu_\alpha = v'(\tau)$$

### Proof of the Planner's Solutions

To move further through our analysis, we need to find solutions for our variables of interest. We begin with the consumption allocations:

$$c_{1\beta}(\lambda_\beta, x, \delta, g_\beta) = \frac{p_F \lambda_\beta \delta x + 1 - \tau + k - x - \theta c_1}{(1-\theta)\hat{\pi}_\beta}$$

$$c_{2\beta}(\lambda_\beta, x, \delta) = \frac{[R \cdot x + p_B(z - \lambda_\beta \delta x)]}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}$$

Now, solving the the collateral liquidation condition:

$$u'(c_{1\beta}(\lambda_\beta, x, \delta)) = \left( \frac{(1 - \theta)(1 - \widehat{\pi}_\beta)}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_B}{p_F} \right) u'(c_{2\beta}(\lambda_\beta, x, \delta))$$

$$c_{1\beta} = \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} c_{2\beta}$$

$$\frac{p_F \lambda_\beta \delta x + 1 - \tau + k - x - \theta c_1}{(1 - \theta) \widehat{\pi}_\beta} = \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \frac{[R \cdot x + p_B(z - \lambda_\beta \delta x)]}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}$$

$$\lambda_\beta = \frac{\left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \left[ \frac{[R \cdot x + p_B z]}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)} \right] - \frac{1 - \tau + k - x - \theta c_1}{(1 - \theta) \widehat{\pi}_\beta}}{\left[ \frac{p_F \delta x}{(1 - \theta) \widehat{\pi}_\beta} + \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \frac{p_B \delta x}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)} \right]}$$

Using the feasibility constraint constructed for the bad state, we can solve for consumption across investors:

$$\frac{p_F}{p_B} R x + p_F z + 1 - g_\beta - x - \theta c_1 = \left[ (1 - \theta) \widehat{\pi}_\beta c_{1\beta} + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] c_{2\beta} \right]$$

From the first-order condition,

$$c_{1\beta} = \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} c_{2\beta},$$

we obtain

$$\begin{aligned} & \frac{p_F}{p_B} R x + p_F z + 1 - g_\beta - x - \theta c_1 \\ &= \left[ (1 - \theta) \widehat{\pi}_\beta + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{-\frac{1}{\gamma}} \right] c_{1\beta} \end{aligned}$$

Applying the relationship for the public good,

$$u'(c_{1\beta}) = \mu_\beta = u'(g_\beta)$$



$$c_{1\beta} = g_\beta$$

and solving for  $c_{1\beta}$ , we obtain:

$$c_{1\beta} = \frac{\frac{p_F}{p_B} Rx + p_F z + 1 - x - \theta c_1}{\left[ 1 + (1 - \theta)\widehat{\pi}_\beta + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{-\frac{1}{\gamma}} \right]}$$

Then, using the first-order condition once again we are able to find  $c_{2\beta}$ .

$$c_{2\beta} = c_{1\beta} \left[ \left( \frac{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}{(1 - \theta)(1 - \widehat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{-\frac{1}{\gamma}}$$

Consumption solutions in the good state are given by:

$$c_{1\alpha}(c_1, x, g_\alpha) = \frac{1 - g_\alpha - x - \theta c_1}{(1 - \theta)\widehat{\pi}_\alpha}$$

$$c_{2\alpha} = \frac{[R \cdot x + p_B z]}{1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)}$$

Since we know that

$$u'(c_{1\alpha}) = \mu_\alpha$$

$$v'(g_\alpha) = \mu_\alpha$$

$$u'(c_1) = \mu_\alpha$$

Therefore,

$$c_{1\alpha}(x, c_1) = \frac{1 - x}{(1 + \theta + (1 - \theta)\widehat{\pi}_\alpha)}$$

and

$$c_1 = \frac{1 - x}{(1 + \theta + (1 - \theta)\widehat{\pi}_\alpha)}$$

The investment condition yields

$$u'(c_{2a}) R \left[ \frac{(1 - \theta)(1 - \widehat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)} \right] = u'(c_{1\alpha})$$

$$c_{2a} \left[ R \frac{(1 - \theta)(1 - \widehat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}} = c_{1\alpha}$$

Recall

$$c_{1\alpha} = c_1 = \frac{1 - x}{1 + \theta + (1 - \theta) \widehat{\pi}_\alpha},$$

By simply substituting into the investment condition the solutions for  $c_1$  and  $c_{2\alpha}$ , the solution for  $x$  is shown to be:

$$x = \frac{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} - \frac{p_B z}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \left[ R \frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}}}{\left( \frac{R}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \left[ R \frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}} + \frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} \right)}$$

#### Proof of Proposition 4.

In order for settlement to occur in the second leg of the repo, the borrower's incentive constraint must be satisfied:

$$r_s < p_B.$$

After substituting the solution for the repo rate in state  $s$ :

$$r_s = \left( \frac{(1 - \theta)(1 - \widehat{\pi}_s)}{1 + (1 - \theta)(1 - \widehat{\pi}_s)} \right) \frac{R \cdot x + p_B z - p_B \lambda_s x \delta}{(1 - \lambda_s) x \delta}$$

and solving for  $\lambda_s$  we obtain

$$\lambda_s < 1 - (1 - \theta)(1 - \widehat{\pi}_s) \left[ \frac{R \cdot x + p_B z}{x \delta p_B} - 1 \right].$$

Consequently, the condition for settlement can be written in terms of an upper-bound on the liquidation of collateral.

We begin by verifying the condition is satisfied in the good state where  $\lambda_\alpha = 0$ :

$$0 < [1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)] x \delta p_B - (1 - \theta)(1 - \widehat{\pi}_\alpha) (R \cdot x + p_B z)$$

$$\left[ \delta p_B - \left( \frac{(1 - \theta)(1 - \widehat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)} \right) R \right] x^* > \left( \frac{(1 - \theta)(1 - \widehat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \widehat{\pi}_\alpha)} \right) p_B z$$

Given that

$$x^* = \frac{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} - \frac{p_B z}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \left[ R \frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}}}{\frac{1}{1+\theta+(1-\theta)\widehat{\pi}_\alpha} + \frac{R}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \left[ R \frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}}}$$

we find

$$\delta > \frac{\delta_R^*}{p_B} \equiv \frac{R}{p_B} \left( \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right) + z \cdot \frac{\left[ \frac{1}{1+\theta+(1-\theta)\hat{\pi}_\alpha} + \frac{R}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \left[ R \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}} \right]}{\left[ \frac{1}{1+\theta+(1-\theta)\hat{\pi}_\alpha} - \frac{p_B z}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \left[ R \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right]^{-\frac{1}{\gamma}} \right]} \left( \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right)$$

Since  $x^*$  is not a function of  $\delta$  and is entirely in terms of exogenous parameters, the threshold can be simplified:

$$\delta > \left( \frac{R}{p_B} + \frac{z}{x^*} \right) \left( \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right)$$

**Next, for the bad state.** Recall that the solution for  $\lambda_\beta$  is:

$$\lambda_\beta = \frac{\left[ \left( \frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \left[ \frac{[R \cdot x + p_B z]}{1+(1-\theta)(1-\hat{\pi}_\beta)} \right] - \frac{1-\tau+k-x-\theta c_1}{(1-\theta)\hat{\pi}_\beta}}{\left[ \frac{p_F \delta x}{(1-\theta)\hat{\pi}_\beta} + \left[ \left( \frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \frac{p_B \delta x}{1+(1-\theta)(1-\hat{\pi}_\beta)} \right]}$$

Plugging in the solution for  $\lambda_\beta$  into

$$\lambda_s < 1 - (1-\theta)(1-\hat{\pi}_s) \left[ \frac{R \cdot x + p_B z}{x \delta p_B} - 1 \right],$$

we find

$$\tau < (1-x-\theta c_1) + k + \left( \frac{p_F}{p_B} \right) x \delta p_B - \left[ \frac{\left( \frac{p_F}{p_B} \right) (1-\theta)(1-\hat{\pi}_\beta) + (1-\theta)\hat{\pi}_\beta \left[ \left( \frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}}}{(1-\theta)\hat{\pi}_\beta \left[ \left( \frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}}} \right] [R \cdot x + p_B (z - x \delta)]$$

From the first-order conditions in the planner's case, we know that tax imposed in period 0 is

$$\tau = c_1.$$

In addition, from the previous solutions:

$$c_1 = \frac{1-x}{(1+\theta+(1-\theta)\hat{\pi}_\alpha)}.$$

Plugging these equations into the following inequality

$$(1 + \theta) c_1 < (1 - x) + k + \left( \frac{p_F}{p_B} \right) x \delta p_B - [R \cdot x + p_B (z - x \delta)] \left[ \begin{array}{l} \left( \frac{p_F}{p_B} \right) (1 - \theta)(1 - \hat{\pi}_\beta) + \\ (1 - \theta) \hat{\pi}_\beta \left[ \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)(1 - \hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \end{array} \right]$$

yields:

$$\delta > \underline{\delta}_\beta^* = \frac{(R + \frac{p_B z}{x^*}) \left[ \left( \frac{p_F}{p_B} \right) (1 - \theta)(1 - \hat{\pi}_\beta) + (1 - \theta) \hat{\pi}_\beta \left[ \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)(1 - \hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \right]}{\left[ p_F [1 + (1 - \theta)(1 - \hat{\pi}_\beta)] + (1 - \theta) \hat{\pi}_\beta \left[ \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)(1 - \hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \right]} + \frac{\left( 1 - \frac{1}{x^*} \right) \frac{(1 - \theta) \hat{\pi}_\alpha}{(1 + \theta + (1 - \theta) \hat{\pi}_\alpha)} - \frac{k^*}{x^*}}{\left[ p_F [1 + (1 - \theta)(1 - \hat{\pi}_\beta)] + (1 - \theta) \hat{\pi}_\beta \left[ \left( \frac{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}{(1 - \theta)(1 - \hat{\pi}_\beta)} \right) \left( \frac{p_F}{p_B} \right) \right]^{\frac{1}{\gamma}} \right]}$$

Therefore, in order for settlement to take place in either aggregate state,  $\delta > \max(\underline{\delta}_\alpha^*, \underline{\delta}_\beta^*)$ .

### Proof of Proposition 5:

The proof of the Proposition relates to showing the following relationship hold true

$$\frac{\tilde{c}_1}{\tilde{c}_{2\beta}} \leq \frac{c_1^*}{c_{2\beta}^*}$$

and is broken down into three steps.

**Step 1.** The Planner always chooses  $x = x^*$  and  $c_1 = c_1^*$ .

From the first-order conditions in period 0 and 1, the relationship between impatient consumption is found to be

$$c_{1\alpha} = c_1.$$

Therefore, utilizing this relationship along with the investment condition:

$$u'(c_{1\alpha}) = u'(c_{2\alpha}) R \left[ \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right],$$

By CRRA preferences it must be true that

$$c_1 < c_{2\alpha}$$

so long as  $R \left[ \frac{(1 - \theta)(1 - \hat{\pi}_\alpha)}{1 + (1 - \theta)(1 - \hat{\pi}_\alpha)} \right] > 1$ .

Thus, a patient investor with  $i \leq \theta$  will strictly prefer to wait in state  $\alpha$ . Since the realization of the bad state is unexpected, decisions ex-ante are made only with regards to the strategy that investors choose in the good state. As no patient investor has an incentive to run in the good state then it directly follows that the social planner will always construct  $x = x^*$  and  $c_1 = c_1^*$ .

**Step 2.** Show that  $\widetilde{\mu}_\beta < \mu_\beta^*$ .

Due to CRRA preferences, we may write:

$$\phi c_{1\beta} = c_{2\beta}$$

for some scalar  $\phi$ .

From the feasibility constraint, the proportion of the population that is left to be funded after the aggregate state is revealed is given by:

$$\left[ ((1 - \theta)\widehat{\pi}_\beta)c_{1\beta} + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] c_{2\beta} \right]$$

Using the first-order condition

$$c_{1\beta} = \left[ \frac{p_F [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)]}{p_B (1 - \theta)(1 - \widehat{\pi}_\beta)} \right]^{-\frac{1}{\gamma}} c_{2\beta}$$

Let  $\left[ \frac{p_F [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)]}{p_B (1 - \theta)(1 - \widehat{\pi}_\beta)} \right]^{-\frac{1}{\gamma}} = \phi$ . Then  $\left[ (1 - \theta)\widehat{\pi}_\beta + \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] \phi \right] c_{1\beta}$  is increasing increasing in  $\widehat{\pi}_\beta$  for any level of risk aversion.

The feasibility constraint for the bad state is

$$p_F z + 1 - \left( 1 - \frac{p_F}{p_B} R \right) x - g_\beta - \theta c_1 - \left[ (1 - \theta)\widehat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] \right] c_{1\beta} = 0,$$

Since the Planner chooses  $c_1^*$  and  $x^*$  and, in turn the investors always follow the strategy outlined by  $y^*$ , the feasibility constraint across strategy profiles in the bad state can be reduced to:

$$\begin{aligned} & \widetilde{g}_\beta + \left[ (1 - \theta)\widetilde{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widetilde{\pi}_\beta)] \right] \widetilde{c}_{1\beta} \\ &= g_\beta^* + \left[ (1 - \theta)\widehat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] \right] c_{1\beta}^*. \end{aligned}$$

As  $\left[ (1 - \theta)\widetilde{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widetilde{\pi}_\beta)] \right] < \left[ (1 - \theta)\widehat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)] \right]$ , then it follows that one of the following must hold:

$$\widetilde{c}_{1\beta} > c_{1\beta}^*; \widetilde{g}_\beta > g_\beta^*$$

Both of which imply that  $\widetilde{\mu}_\beta < \mu_\beta^*$ .

### Step 3.

By Steps 1 and 2,

$$\frac{u'(c_1^*)}{\widetilde{\mu}_\beta} > \frac{u'(c_1^*)}{\mu_\beta^*}.$$

Substituting in the optimal risk-sharing condition for the bad state:

$$\begin{aligned} \mu_\beta^* &= \left( \frac{(1-\theta)(1-\widehat{\pi}_\beta)}{1+(1-\theta)(1-\widehat{\pi}_\beta)} \right) \left( \frac{p_B}{p_F} \right) u'(c_{2\beta}^*) \\ \widetilde{\mu}_\beta &= \left( \frac{(1-\theta)(1-\widetilde{\pi}_\beta)}{1+(1-\theta)(1-\widetilde{\pi}_\beta)} \right) \left( \frac{p_B}{p_F} \right) u'(c_{2\beta}) \end{aligned}$$

We find

$$\left( \frac{u'(c_1^*)}{u'(c_{2\beta})} \right) \geq \left( \frac{u'(c_1^*)}{u'(c_{2\beta}^*)} \right)$$

Therefore,  $\frac{\widetilde{c}_1}{c_{2\beta}} \leq \frac{c_1^*}{c_{2\beta}^*}$ . This completes the proof of fragility.

### Proof of Proposition 6:

Using the relationship between consumption among the impatient investors once again,

$$c_1 = c_{1\alpha}$$

Then, combine the result from the run condition:

$$c_{1\alpha} > c_{2\beta}.$$

Imposing CRRA preferences and the period 1 first-order conditions, the shadow values for each state are:

$$\mu_\alpha^* < \mu_\beta^*.$$

Thus, public good provision across states must be defined by

$$\tau^* - k < \tau^*.$$

The condition for collateral liquidation is simply the lower bound found in Corollary 3.

Proofs for the Decentralized Economy

### The Impact of Taxes on Investment:

From the Investment Condition:

$$u''(c_{2\alpha}^D) R \left[ \frac{(1-\theta)(1-\hat{\pi}_\alpha)}{1+(1-\theta)(1-\hat{\pi}_\alpha)} \right] \frac{dc_{2\alpha}^D}{d\tau^D} = u''(c_{1\alpha}) \frac{dc_{1\alpha}^D}{d\tau^D}$$

From the Period 2 Budget Constraint:

$$[1+(1-\theta)(1-\hat{\pi}_\alpha)] \frac{dc_{2\alpha}^D}{d\tau^D} = R \frac{dx^D}{d\tau^D}$$

From the Period 1 Budget Constraint:

$$(1-\theta) \hat{\pi}_\alpha \frac{dc_{1\alpha}^D}{d\tau^D} = -1 - \frac{dx^D}{d\tau^D} - \theta \frac{dc_1^D}{d\tau^D}$$

From the Period 0 First-Order Condition:

$$u''(c_{1\alpha}^D) \frac{dc_{1\alpha}^D}{d\tau^D} = u''(c_1^D) \frac{dc_1^D}{d\tau^D}$$

*In order to isolate the impact of taxes on investment, we combine all of the aforementioned conditons*

$$(1-\theta) \hat{\pi}_\alpha \frac{dc_{1\alpha}^D}{d\tau^D} = -1 - \frac{dx^D}{d\tau^D} - \theta \frac{u''(c_{1\alpha}^D)}{u''(c_1^D)} \frac{dc_{1\alpha}^D}{d\tau^D}$$

*and solve for  $dc_{1\alpha}^D/d\tau^D$ .*

$$\frac{dx^D}{d\tau^D} = \frac{-1}{\left[ 1+(1-\theta) \hat{\pi}_\alpha \frac{u''(c_{2\alpha}^D)}{u''(c_{1\alpha}^D)} \frac{R^2(1-\theta)(1-\hat{\pi}_\alpha)}{[1+(1-\theta)(1-\hat{\pi}_\alpha)]^2} + \theta \frac{u''(c_{2\alpha}^D)}{u''(c_{1\alpha}^D)} \frac{R^2(1-\theta)(1-\hat{\pi}_\alpha)}{[1+(1-\theta)(1-\hat{\pi}_\alpha)]^2} \right]}$$

*Let  $\chi = \frac{u''(c_{2\alpha}^D)}{u''(c_{1\alpha}^D)} \frac{R^2(1-\theta)(1-\hat{\pi}_\alpha)}{[1+(1-\theta)(1-\hat{\pi}_\alpha)]^2}$  so that*

$$\frac{dx^D}{d\tau^D} = \frac{-1}{[1+((1-\theta) \hat{\pi}_\alpha + \theta) \chi]}$$

$$-1 < \frac{dx^D}{d\tau^D} < 0.$$

## **Proof of Portfolio Choice between the Planner and Decentralized Cases**

*Suppose that  $c_1^D \geq c_1^*$ . From the feasibility constraint:*

$$(1-\theta) \hat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

$$(1-\theta) \hat{\pi}_\alpha c_{1\alpha} + x = 1 - \tau - \theta c_1$$

If  $c_1^D > c_1^*$  and we know  $\tau^D > \tau^*$  then either  $x^D < x^*$  or  $c_{1\alpha}^D < c_{1\alpha}^*$ . Both of which lead to  $\mu_\alpha^D > \mu_\alpha^*$  which contradicts  $c_1^D \geq c_1^*$ . Therefore, it must be true that  $c_1^D < c_1^*$ .

In turn from the investment condition:

$$\mu_\alpha = u'(c_{2\alpha}) R \left[ \frac{(1-\theta)(1-\widehat{\pi}_\alpha)}{1+(1-\theta)(1-\widehat{\pi}_\alpha)} \right]$$

which implies that since  $c_1^D < c_1^*$  then  $x^D < x^*$ .

### Proof of a Higher Tax Rate

Recall:

$$u'(\tau^*) = \mu_\alpha^*$$

$$u'(\tau^D) = \mu_\alpha^D \left[ 1 + \frac{dx^D}{d\tau^D} \right]$$

Suppose that  $\tau^D < \tau^*$ . Using the equations above, this means that

$$\mu_\alpha^D \left[ 1 + \frac{dx^D}{d\tau^D} \right] > \mu_\alpha^*$$

or alternatively, since we know  $-1 < \frac{dx^D}{d\tau} < 0$

$$\mu_\alpha^D > \mu_\alpha^*$$

This inequality also implies that  $c_1^D < c_1^*$  and  $x^D < x^*$ . From the feasibility constraint

$$(1-\theta)\widehat{\pi}_\alpha c_{1\alpha}^D + \tau^D = 1 - x^D - \theta c_1^D$$

Therefore, we must have  $c_{1\alpha}^D > c_{1\alpha}^*$  or  $\tau^D > \tau^*$ . Both of which are a contradiction. Consequently:  $\tau^D > \tau^*$ .

### Proof of Public Liquidity

Since  $-1 < \frac{dx^D}{d\tau^D} < 0$ , then

$$\frac{\mu_\alpha^*}{\mu_\alpha^D} > \frac{\mu_\alpha^D}{\mu_\alpha^D} + \frac{dx}{d\tau}$$

From the fiscal authority's tax decision,

$$u'(\tau^D) = \mu_\alpha^D \left[ 1 + \frac{dx}{d\tau} \right]$$

Therefore

$$\frac{u'(\tau^*)}{\mu_\alpha^*} > \frac{u'(\tau^D)}{\mu_\alpha^D}.$$

or



$$\frac{u'(\tau^*)}{u'(c_{1\alpha}^*)} > \frac{u'(\tau^D)}{u'(c_{1\alpha}^D)}.$$

Due to CRRA preferences

$$\frac{\tau^*}{c_{1\alpha}^*} < \frac{\tau^D}{c_{1\alpha}^D}$$

along with the budget constraint for period 1 in the good state:

$$(1 - \theta) \widehat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

$$(1 - \theta) \widehat{\pi}_\alpha \frac{c_{1\alpha}}{\tau} + 1 = \frac{1 - x - \theta c_1}{\tau} = \psi_\tau^{-1}$$

Using the inequality  $\frac{\tau^*}{c_{1\alpha}^*} < \frac{\tau^D}{c_{1\alpha}^D}$  implies that  $\psi_\tau^* < \psi_\tau^D$ .

### Proof of Proposition 7

The proof of this Proposition follows directly from that of Proposition 4.

### Proof of Proposition 8

The proof of this Proposition follows directly from that of Proposition 5.

Bailouts vs. No Bailouts

**Proof that  $c_{2\beta}^{NB} < c_{2\beta}^*$ .**

The conditions that  $c_1^{NB} < c_1^*$  and  $x^{NB} < x^*$  follow (except for notation) from the decentralized case. Using the feasibility constraints for the bad state:

$$p_F z + 1 - \left[1 - \frac{p_F}{p_B} R\right] x^* - \theta c_1^* = \left[(1 - \theta) \widehat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)]\right] c_{1\beta}^* + \tau^* - k^*$$

$$p_F z + 1 - \left[1 - \frac{p_F}{p_B} R\right] x^{NB} - \theta c_1^{NB} = \left[(1 - \theta) \widehat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1 - \theta)(1 - \widehat{\pi}_\beta)]\right] c_{1\beta}^{NB} + \tau^{NB}$$

We know that  $\tau^{NB} = \tau^D > \tau^* > (\tau^* - k^*)$ . Since  $c_1^{NB} < c_1^*$  and  $x^{NB} < x^*$ , the relationship between  $c_{1\beta}^{NB}$  and  $c_{1\beta}^*$  is not explicitly defined here.

Suppose that  $c_{1\beta}^{NB} > c_{1\beta}^*$ . This also directly implies that  $c_{2\beta}^{NB} > c_{2\beta}^*$ .

From the solution for  $c_{2\beta}$ :

$$c_{2\beta} = \frac{[R \cdot x + p_B (z - \lambda_\beta \delta x)]}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}$$

Therefore, the only way that  $c_{2\beta}^{NB} > c_{2\beta}^*$  when  $x^{NB} < x^*$  is if  $\lambda_\beta^{NB} < \lambda_\beta^*$ .

Recall the collateral liquidation variable is defined by

$$\frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

The provision of public goods in the bad state is redefined as  $\tau - k$ ; where  $k$  once again is the size of the bailout package.

$$\frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

$$\frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta} - (1 - \tau - x - \theta c_1) - k}{p_F \delta x} = \lambda_\beta$$

The inequality above would then imply that

$$\frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta}^{NB} - (1 - \tau^{NB} - x^{NB} - \theta c_1^{NB}) - k^{NB}}{p_F \delta x^{NB}} < \frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta}^* - (1 - \tau^* - x^* - \theta c_1^*) - k^*}{p_F \delta x^*}$$

Since  $k^{NB} = 0$ ,

$$(1 - \theta)\widehat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

Plugging this in

$$\frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta}^{NB} - (1 - \theta)\widehat{\pi}_\alpha c_{1\alpha}^{NB}}{p_F \delta x^{NB}} < \frac{(1 - \theta)\widehat{\pi}_\beta c_{1\beta}^* - (1 - \theta)\widehat{\pi}_\alpha c_{1\alpha}^* - k^*}{p_F \delta x^*}$$

From the inequalities in the the no bailouts economy we have:  $c_{1\beta}^{NB} > c_{1\beta}^*$ ,  $c_{1\alpha}^{NB} < c_{1\alpha}^*$ , and  $x^{NB} < x^*$ .

Therefore, the inequality cannot hold and thus it must be true that  $c_{1\beta}^{NB} < c_{1\beta}^*$ ,  $c_{2\beta}^{NB} < c_{2\beta}^*$ ,  $\lambda_\beta^{NB} > \lambda_\beta^*$ , and  $\lambda_\beta^{NB} > \lambda_\beta^D$ .

### Proof of Proposition 9

Patient investor consumption is defined by

$$c_{2\beta} = \frac{[R \cdot x + p_B (z - \lambda_\beta \delta x)]}{1 + (1 - \theta)(1 - \widehat{\pi}_\beta)}$$

Since  $\lambda_\beta^{NB} > \lambda_\beta^D$ ,  $c_{2\beta}^{NB} < c_{2\beta}^D$ . With the level of  $c_1$  the same regardless of bailouts, then

$$c_1^{NB} = c_1^D$$

$$c_1^D \geq c_{2\beta}^D$$

implies that

$$c_1^{NB} > c_{2\beta}^{NB}$$

Since the inequality is strict, this implies there exists economies in  $\Phi^{NB}$  that would not be a part of  $\Phi^D$ . Therefore,  $\Phi^{NB}$  is a superset of  $\Phi^D$ .

The no bailouts regime contains all of the downfalls of the bailouts regime as well as added fragility in the event of the bad state. In the no bailouts regime, all of the consumption variables for private good consumption are chosen the same as those in the bailouts regime; however, when the bad state occurs consumption for patient investors is lower – leading to a broader set of run parameters.

### Proof of Proposition 10

This is the stability condition for the decentralized case. The threshold for stability follows nearly exactly to that shown in Proposition 4.

$$\delta > \underline{\delta}_\beta^D = \frac{\left(R + \frac{p_B z}{x^D}\right) \left[ \left(\frac{p_F}{p_B}\right) (1-\theta)(1-\hat{\pi}_\beta) + (1-\theta)\hat{\pi}_\beta \left[ \left(\frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)}\right) \left(\frac{p_F}{p_B}\right) \right]^{\frac{1}{\gamma}} \right]}{\left[ p_F [1 + (1-\theta)(1-\hat{\pi}_\beta)] + (1-\theta)\hat{\pi}_\beta \left[ \left(\frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)}\right) \left(\frac{p_F}{p_B}\right) \right]^{\frac{1}{\gamma}} \right]} \\ + \frac{\left(1 - \frac{1}{x^D}\right) \frac{(1-\theta)\hat{\pi}_\alpha}{\left[ \left(1 + \frac{dx^D}{d\tau^D}\right)^{-\frac{1}{\gamma}} + \theta + (1-\theta)\hat{\pi}_\alpha \right]} - \frac{k^D}{x^D}}{\left[ p_F [1 + (1-\theta)(1-\hat{\pi}_\beta)] + (1-\theta)\hat{\pi}_\beta \left[ \left(\frac{1+(1-\theta)(1-\hat{\pi}_\beta)}{(1-\theta)(1-\hat{\pi}_\beta)}\right) \left(\frac{p_F}{p_B}\right) \right]^{\frac{1}{\gamma}} \right]}$$

Since we know the only difference between bailouts and no bailouts is  $k^{NB} = 0$ , then it must be true that for all  $k^D$  non-negligible from zero there exists a  $\delta$  where  $\underline{\delta}_\beta^D < \delta < \underline{\delta}_\beta^{NB}$ .

### Comparing between the Decentralized and Planner's Allocations:

Using the feasibility constraints for the bad state

$$p_F z + 1 - \left[1 - \frac{p_F}{p_B} R\right] x^* - \theta c_1^* = \left[ (1-\theta)\hat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1-\theta)(1-\hat{\pi}_\beta)] \right] c_{1\beta}^* + \tau^* - k^*$$

$$p_F z + 1 - \left[1 - \frac{p_F}{p_B} R\right] x^D - \theta c_1^D = \left[ (1-\theta)\hat{\pi}_\beta + \phi \frac{p_F}{p_B} [1 + (1-\theta)(1-\hat{\pi}_\beta)] \right] c_{1\beta}^D + \tau^D - k^D$$

Since  $c_1^D < c_1^*$  and  $x^D < x^*$ , then in order for the above conditions to hold with equality  $c_{1\beta}^D > c_{1\beta}^*$  or  $(\tau^D - k^D) > (\tau^* - k^*)$ . Both of which imply that  $\mu_\beta^D < \mu_\beta^*$ . This also directly implies that  $c_{2\beta}^D > c_{2\beta}^*$ .

From the equation for  $c_{2\beta}$  we know

$$c_{2\beta} = \frac{[R \cdot x + p_B(z - \lambda_\beta \delta x)]}{1 + (1 - \theta)(1 - \hat{\pi}_\beta)}$$

Therefore, the only way  $c_{2\beta}^D > c_{2\beta}^*$  when  $x^D < x^*$  is if  $\lambda_\beta^D < \lambda_\beta^*$ .

The economy in the decentralized state can be defined by:  $\mu_\alpha^D > \mu_\alpha^*$ ,  $\mu_\beta^D < \mu_\beta^*$ ,  $c_1^D < c_1^*$ ,  $x^D < x^*$ ,  $\tau^D > \tau^*$ ,  $(\tau^D - k^D) > (\tau^* - k^*)$ ,  $c_{1\alpha}^D < c_{1\alpha}^*$ ,  $c_{2\alpha}^D < c_{2\alpha}^*$ ,  $c_{1\beta}^D > c_{1\beta}^*$ ,  $c_{2\beta}^D > c_{2\beta}^*$ , and  $\lambda_\beta^D < \lambda_\beta^*$ . In addition, using (26) the repo rate with respect to investment is shown to be:

$$\frac{\partial r_\alpha}{\partial x} = -\frac{p_B z}{\delta x^2}$$

### Proof that Bailouts are larger in the decentralized case

The collateral liquidation variable is defined by:

$$\frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

The provision of public goods in the bad state is  $\tau - k$ ; where  $k$  once again is the size of the bailout package.

$$\frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta} - (1 - \tau + k - x - \theta c_1)}{p_F \delta x} = \lambda_\beta$$

$$\frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta} - (1 - \tau - x - \theta c_1) - k}{p_F \delta x} = \lambda_\beta$$

Our inequality above would then imply that

$$\frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta}^D - (1 - \tau^D - x^D - \theta c_1^D) - k^D}{p_F \delta x^D} < \frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta}^* - (1 - \tau^* - x^* - \theta c_1^*) - k^*}{p_F \delta x^*}$$

We know from the good state feasibility constraint that:

$$(1 - \theta)\hat{\pi}_\alpha c_{1\alpha} = 1 - \tau - x - \theta c_1$$

$$\frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta}^D - (1 - \theta)\hat{\pi}_\alpha c_{1\alpha}^D - k^D}{p_F \delta x^D} < \frac{(1 - \theta)\hat{\pi}_\beta c_{1\beta}^* - (1 - \theta)\hat{\pi}_\alpha c_{1\alpha}^* - k^*}{p_F \delta x^*}$$

From the inequalities that define the decentralized economy above we know,  $c_{1\beta}^D > c_{1\beta}^*$ ,  $c_{1\alpha}^D < c_{1\alpha}^*$ , and  $x^D < x^*$ .

Therefore, for the inequality to hold, it must be true that

$$k^D > k^*$$

## Proof of Proposition 11

Fragility Proof:

We have shown that  $c_1^D < c_1^*$  and  $c_{2\beta}^D > c_{2\beta}^*$  which together imply that the economy is more fragile in the Planner's case than in the decentralized case.

Stability Proof:

Starting with the stability condition around collateral liquidation.

$$\lambda_\beta < 1 - (1 - \theta)(1 - \hat{\pi}_\beta) \left[ \frac{R \cdot x + p_B(z - x\delta)}{x\delta p_B} \right]$$

The RHS of the inequality is decreasing in  $x$ . Therefore, since the following holds true

$$x^D < x^* \text{ and } \lambda_\beta^D < \lambda_\beta^*,$$

Then, the inequality has more support in the decentralized case.

# CHAPTER III:

## The Macroeconomic Effects of Bank Failures

### 1 Introduction

The U.S. banking system is becoming increasingly more concentrated and interconnected. Consequently, bank failures have an increasingly greater impact on the financial system and the broader economy. Concerned with the potential impact of these failures, the Federal Reserve adopted the capital plan rule in November 2011 set by the Dodd-Frank Act. The rule requires banks with consolidated assets of \$50 billion or more to undergo an extraordinary set of regulatory procedures. Thus, the \$50 billion threshold implies that a failure among this class of banks would cause structural issues to the U.S. economy. However, smaller bank failures still remain part of the FDIC's normal resolution process. In this paper, we examine the macroeconomic effects of failed banks from 1973–2006. Most notably, we show that the effects of bank failures become significant at a threshold far below \$50 billion.

Admittedly, this is not the first paper to study bank failures. For example, Bernanke (1983) provides a seminal study on the propagation of the Great Depression due to bank failures. In his 1983 work, Bernanke states:

“[R]ather than consider the 1929–33 episode outside of its context, I have widened the sample to include the entire interwar period (January 1919–December 1941)” p.268

We frame modern bank failures following Bernanke's methodology. In doing so, our analysis compliments the current literature which has not yet examined the aggregate effects of more recent failures in the United States. Contemporary work by Ramirez and Shively (2012) and

Kupiec and Ramirez (2013) examines the variation in the effects of bank failure effects across different states using pre-Depression data. Using more modern data but applying a global perspective, Boyd, Kwak, and Smith (2005) and Dell’Ariccia, Detragiache, and Rajan (2008) study the response of real output during banking crises. Most in line with the work presented here, Gilbert and Kochin (1989), Clair, O’Driscoll, and Yeats (1994), and Ashcraft (2005) examine the effects of bank failures in the U.S during the Saving and Loan (S&L) Crisis. In contrast to the global perspective, they focus on local economic impacts over limited time frames.

An analysis of the financial sector from a limited viewpoint ignores the effects failures have on the broader macroeconomy. The 2008 Financial Crisis has increased public interest in the aggregate effects stemming from the banking system. Since the collapse of Lehman Brothers in September 2008, more than 500 banks have failed – having a detrimental effect on the macroeconomy and slowing the subsequent recovery. Therefore, understanding these effects is important for the design of regulatory policies targeted towards the banking sector.

We construct a vector autoregressive (VAR) model resembling Christiano, Eichenbaum, and Evans (1996) to appropriately study the aggregate impact of failures. Following Bernanke (1983), we contextualize nationwide bank failures by using a sample extending from January 1973-December 2006. First, we present responses from the broad macroeconomy following a bank failure shock. Aggregate consumption, production and employment all decrease significantly. This result conflicts with some of the previous work mentioned and with some policymakers who believe that bank failures no longer pose a threat to the real economy. Next, we disaggregate the model slightly and examine the variable responses between two markets: durable and non-durable goods. The markets react similarly in direction to the aggregate, however, they vary considerably in magnitude. In contrast, prices for durables decrease while prices for non-durables remain unaffected. When combined with previous literature which studies these markets more generally, our results provide new insight into the economic struggles that occur amid bank stress. These insights are presented in more

detail along with the an analysis into the adaptive response by private saving and credit markets.

The reactions from monetary policy and credit extension have important implications towards the propogation of bank stress onto the economy. Friedman and Schwartz (1963) and Bernanke (1983) offer a historical perspective of these implications as well as a basis for more recent analyses. As expected, monetary policy over our sample reacts accommodatively, observed by a decrease in the Federal funds rate. However, stress in the banking system is also shown by the decrease in nonborrowed reserves. Furthermore, we see this stress borne out in credit markets as significant contractions occur in consumer and business credit – a result consistent with the transmission channels of bank stress proposed by Bernanke and Gertler (1995).

However, not all bank failures are equal. When a bank becomes large enough, its demise can have a profound effect on the economy compared to smaller banks. This concept is at the heart of the Federal Reserve’s capital plan rule. Previous bank failure literature, on the other hand, makes a contrasting assumption. Bank failures shocks have previously been assumed to affect the economy in a linear way. We relax this assumption. When testing for nonlinearity within bank failure shocks, we find significant evidence of a threshold. Surprisingly, shocks that meet or exceed this threshold are relatively common. Specifically, in our sample, shocks above our threshold are at least 4 times more likely to occur than shocks recognized by the capital plan rule. We show that cumulative monthly bank failures of \$3 billion or more cause an overall contraction in the U.S. economy and have considerable contagion effects. A result that provides support for a more nuanced bank supervisory and regulatory system.

The remainder of the paper is as follows. Section 2 describes the data and basic empirical methodology. Section 3 presents the VAR model and offers an analysis of the responses from the broad economy, durable/nondurable goods markets, and credit markets to bank failure shocks. Section 4 presents our threshold tests and examines the differences between “small” and “large” shocks. Section 5 concludes. Figures of results are provided in the Appendix.



## 2 The Basic Model

### 2.1 The Data

To study the effects of bank failures, we use monthly data. Our time frame is January 1973–December 2006, during which the FDIC recorded 3,037 bank failures. Subsequently, our sample captures a wide variety of failures over the 34 year time span. At the same time, the dataset also places the S&L Crisis in an appropriate historical context. The characteristics of the dataset can essentially be broken down into three separate periods: pre-S&L Crisis (1973:01-1979:12), the S&L Crisis (1980:01-1995:12), and post-S&L Crisis (1996:01-2006:12). Table 3.1 shows the general statistics of aggregate bank failures over the three time periods.

**Table 3.1**

Bank Failure Statistics (Normalized Failures in Levels)					
Period	Mean	Median	Std. Dev.	Minimum	Maximum
Pre-S&L Crisis	0.0000950	0.0000000	0.000489094	0	0.004185304
S&L Crisis	0.0017239	0.0005138	0.003151925	0	0.021066717
Post-S&L Crisis	0.0000105	0.0000000	0.000039608	0	0.000285503
Full Sample	0.0008342	0.0000187	0.002327593	0	0.021066717

The start and end date of the full sample are chosen because of two data limitations. First, we normalize bank failures to avoid bias from the growing size of the banking system over time. In turn, the start date is chosen due to the earliest availability of total assets held by commercial banks nationwide – a necessary series for normalizing aggregate bank failures.<sup>29,30</sup>

The second limitation pertains to monetary policy. Conventional monetary policy has been suspended since 2008. Therefore, including this data would be introducing a fundamental shift in the transmission of monetary and lender of last resort policies. In addition, unconventional monetary policy is not to be the intended norm.<sup>31</sup> To that extent, develop-

<sup>29</sup>Normalizing bank failures is standard practice in the bank failure literature. This methods keeps the shocks congruent over the locations/time chosen by the practitioner.

<sup>30</sup>The data for bank failures is obtained from the FDIC.

<sup>31</sup>The Federal Reserve continues to consider raising the Federal funds rate target above the zero lower bound. Thus, they are beginning to normalize monetary policy.

ing a model that successfully captures policy shocks is a necessary condition for properly examining disruptions in the banking system. In turn, the end date is chosen to avoid the unconventional monetary policy that took place during the 2008 Financial Crisis.<sup>32</sup> In this manner, we believe our framing offers policymakers a comprehensive examination of modern, aggregate bank failures under conventional monetary policy.

## 2.2 The Methodology

Friedman and Schwarz (1963) and Bernanke (1983) point out that stress in the banking system can cause considerable disruptions to the real economy. Calomiris (1993), Rockoff (1993), Ashcraft (2005), Anari, Kolari, and Mason (2005), among others, show that the transmission mechanism for bank failures resembles the “credit channel”. The credit channel theory is described in detail by Bernanke and Gertler (1995). In line with this literature, Ramirez and Shively (2012) introduce the idea of a “bank failure channel” – essentially broadening the definition of the credit channel. Similar to contractionary monetary policy, bank failures impact depositors’ wealth, liquidity, and/or supply of loans. The bank failure channel, in this way, specifically builds off of the monetary policy transmission literature.

U.S. monetary policy, by construction, affects the economy through the functions of the banking system. Therefore, information shocks to the banking system, whether they be from the monetary authority or otherwise, flow into the economy similarly. Christiano et al. (1996) construct a VAR model to analyze the impact of monetary policy on the macroeconomy. The authors use a modelling strategy that involves choosing variables for real output, nominal effects, and monetary policy. We expand on this strategy by including variables for the banking sector. Thus, bank failure shocks are modelled to affect the economy similarly to contractionary monetary policy shocks.

The vector of endogenous variables follows the information and ordering of  $Z_t$ .

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<sup>32</sup>The nonborrowed reserves series goes negative for the first time in the history of the dataset at the end of 2007.

$$Z_t : [\text{real, nominal, policy, banking}]' \quad (81)$$

The standard form VAR is

$$Z_t = \mu + \sum_{i=1}^k A_i Z_{t-i} + e_t \quad (82)$$

where  $\mu$  is a vector of constants and trend terms,  $A_i$  is a parameter matrix,  $e_t$  is a vector of serially uncorrelated error terms, and  $k$  is the number of lags. Lag length is determined by examining the Akaike information criterion.<sup>33,34</sup> Rewriting (82) as:

$$\mathbf{A}(L) Z_t = e_t \quad (83)$$

and inverting matrix  $\mathbf{A}(L)$  we obtain the reduced-form equation:

$$Z_t = \mathbf{A}(L)^{-1} e_t \quad (84)$$

We assume the error terms are related to the underlying shocks,  $\epsilon_t$ , by:

$$e_t = \mathbf{C}\epsilon_t \quad (85)$$

After imposing a Choleski factorization, we estimate  $\mathbf{C}$  and the parameter matrix  $A_i$  by applying ordinary least squares equation by equation to (82) and (85). Therefore, the dynamic effects of bank failures in our aggregate model are obtained from:

$$Z_t = \mathbf{B}(L) \epsilon_t \quad (86)$$

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<sup>33</sup>Ivanov and Kilian (2005) show that the AIC is the most accurate criterion when analyzing monthly VAR models.

<sup>34</sup>The AIC is used to determine the appropriate lag length on all the VARs presented in Section 3. For each VAR, we find a lag length of 3 to be the best.

where  $\mathbf{B}(L) = \mathbf{A}(L)^{-1} \mathbf{C}$ . Although we use a Choleski factorization for identification purposes, it should be noted that the system is very robust to different orderings.<sup>35</sup> However, the order that we consider to be the best follows  $Z_t$ .

The proposed variable ordering in  $Z_t$  is consistent with the bank failure channel theory. At least in the short run, bank failures are followed by movements in the real economy. In accordance with the literature on monetary policy, bank failure shocks are orthogonal to the contemporaneous information set of the real economy, nominal effects, and, in our case, monetary policy.

### 3 Aggregate Effects of Bank Failure Shocks

In the past half century the American economy has seen two major financial crises. The S&L Crisis and 2008 Financial Crisis had dramatic effects across the United States which contributed to the recessionary conditions and slow recoveries. These effects include reductions in consumption, production, and employment. In addition, bank failures appear to coincide with one another and precede even more. For example, the five years before 2008 saw 10 bank failures. Since 2008, the U.S. has experienced over 500 failures. Similarly, there were 37 failures the five years before 1980 and 2,932 failures from 1980–1995. Yet, some contend that given the advancements in policy and regulation, bank failures may no longer pose a significant threat to the macroeconomy.

#### A. Building the Framework

In this section, we analyze the effects that bank failures have on the United States as a whole. The analysis begins with our basic extension of the Christiano et al. structure. The endogenous variables in  $Z_t$  are specified as

$$Z_t : [Y_t, EMP_t, P_t, PCM_t, NTR_t, FFR_t, HP_t, BF_t]' \quad (87)$$

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<sup>35</sup>The results are robust to the ordering of the variables. Details are available upon request.

with the vector including the log levels of industrial production ( $Y$ ), payroll employment ( $EMP$ ), core-CPI ( $P$ ), crude materials ( $PCM$ ), and real median housing prices ( $HP$ ). The ratio of non-borrowed reserves to total reserves ( $NTR$ ) and the ratio of assets in failed banks to total assets ( $BF$ ) are also included along with the effective Federal funds rate ( $FFR$ ). To build confidence in the structure, we compare the system responses to previous literature.

The impulse response functions in Figure 1 are consistent with that of Christiano et al. (1996).<sup>36</sup> In addition, bank failures and our control variable for bank assets ( $HP_t$ ) respond as predicted. Contractionary monetary policy contributes to bank failures and depresses median housing prices. The latter is a particularly important result for our system as housing prices are meant to capture the strength of bank balance sheets.<sup>37</sup>

Next, we inspect the system response to bank failure shocks. In Figure 2, exogenous bank failures of a size equal to 0.175% of total bank assets or \$26 billion create an unambiguously negative reaction in the economy.<sup>38</sup> Along with contractions in the real economy, deflationary pressures also arise. These reactions from the real economy and the price level mirror the theories set forth by Friedman and Schwartz (1963) and Bernanke (1983).

With confidence our basic system interactions function as predicted, we move on to expanding the model further. In particular, a 9-variable system is created by including personal consumption expenditures ( $CON_t$ ) to capture the demand-side response.

$$W_t : [CON_t, Y_t, EMP_t, P_t, PCM_t, NTR_t, FFR_t, HP_t, BF_t]' \quad (88)$$

Figure 3 shows that the system continues to maintain its integrity. Consumption expenditures react negatively to contractionary FFR shocks while the previously included variables remain consistent with those of the 8-variable system.

With respect to bank failure shocks, impulse responses from this highly aggregated 9-

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<sup>36</sup>As in Christiano et al., we observe a price puzzle, where core-CPI rises for roughly a year and a half.

<sup>37</sup>Mortgage loans comprise a large proportion of the assets in bank balance sheets.

<sup>38</sup>All dollar amounts referenced with regards to bank failures are obtained by taking the percent of failed assets and multiplying it by total current assets. Total assets held by commercial banks at the end of 2014 equaled \$15 trillion.

variable system provide a very interesting narrative. Bank failures remain significant for roughly one year after the initial shock in Figure 4. Therefore, bank failures display a contagion effect causing persistent adversity to the economy well-beyond the initial shock. Although the Federal Reserve reacts by lowering interest rates seven months afterwards, significant contractions in production and employment occur almost immediately.

## B. Durable and Non-Durable Goods Markets

The macroeconomic variables are now deconstructed to analyze a particular sector of the economy – consumer goods. Consumer goods are comprised of two major sub-sectors: durable goods and nondurable goods. As we transition the model to measure the impact of failure shocks on these sub-sectors, we continue to follow the general structure in (88). The sub-sectors are analyzed using

$$W_{s,t} : [CON_t(s) , Y_t(s) , EMP_t(s) , P_t(s) , PCM_t , NTR_t , FFR_t , HP_t , BF_t]' \quad (89)$$

where  $s$  represents either sector.<sup>39,40,41</sup>

The impulse response functions in Figure 5 and Figure 6 provide a substantial amount of insight into the reactions of both markets to nationwide bank stress. In both sectors, consumption decreases in the near term yet recovers quickly. However, the point estimates for durable goods consumption displays a much larger decrease. While the decrease in demand is larger for durable goods, production in non-durables reacts negatively for a much longer duration. Production in non-durables does not recover for nearly a year and a half. Durables, on the other hand, show a significant decrease in months 6-8 after the banking

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<sup>39</sup>Industrial production data is available through the Federal Reserve Board of Governors. Personal expenditure data is available through the BEA. Employment and price level data for either sector is available through the BLS.

<sup>40</sup>All of the following are log level variables in the nondurable goods market: NON is personal consumption expenditures, YNON is the industrial production index, EMPN is employment, and PN is the CPI.

<sup>41</sup>All of the following are log level variables in the durable goods market: DUR is personal consumption expenditures, YDUR is the industrial production index, EMPD is employment, and PD is the CPI.

shock.

Turning to employment, we see where the reaction to the durable goods market is concentrated. Point estimates for employment in durable goods show a decrease of .21% at the 12 month mark. This is an alarming result considering that a decrease of this size equates to over 16,000 jobs based upon 2015 employment data. Accumulating the point estimates results in a total decrease of 1.5% of the work force for durable goods in the first 12 months. Put in current terms, this equates to a loss of 117,524 jobs. By comparison, employment in non-durable goods drops by 0.83% 12 months after the shock – or a total payroll decrease of 37,516 employees. Another point of difference between the two sectors is the reaction from prices. Non-durable goods hold their prices steady compared to durable goods which see persistent deflation.

### C. Interest Rates and Credit Extension

In accordance with the credit channel, and subsequently the bank failure channel, we investigate parts of the market for loanable funds. To do so, we expand on (88) one step further by incorporating  $X_t$  which represents either an interest rate, borrowing, or savings variable.

$$W_t : [CON_t, Y_t, EMP_t, P_t, PCM_t, NTR_t, FFR_t, HP_t, BF_t, X_t]' \quad (90)$$

Common interest rates reactions in Figure 7 and Figure 8 are as predicted accompanying a decrease in the Fed Funds rate. It should be noted that there is a slight uptick in the 30-year mortgage market following a banking shock. This could be due in part to the initial uncertainty in the market before the participants foresee an accommodative response from the Fed. However, the reduction in interest rates does not offer a complete stay of confidence in the private sector.

Figure 9 and Figure 10 show significant decreases in business lending and consumer lending. These results support the notion that bank failures affect the economy through a

credit channel mechanism. After an initial reduction, consumer lending recovers in only 4 months. Business lending, on the other hand, experiences significantly large decreases for over a year – reaching over \$2 billion in reductions 15 months after the bank failure shock.<sup>42</sup>

#### **D. Private Saving**

In line with conventional views on market reactions during economic stress, Figure 11 shows the amount of saving in the economy increases following a bank failure shock. Although overall private saving increases, the types of saving decisions of market participants is particularly interesting. Small time deposits, as shown in Figure 12, increase. Given that the stress is originating from the banking sector, one imagines that agents would shift their savings out of the banking sector of the financial system. What does appear to happen, however, is a shift away from liquid assets within the banking sector, seen in Figure 13 and Figure 14. Demand deposits show some reductions along with large decreases in the amount of cash held by banks. We believe the latter may be directly caused by the bank failures experienced. As banks close, their assets are received by the FDIC and thus the level of cash balances held by all banks is affected.

The banking system is unique with regard to public sentiment. Typically, stress in one area of the economy causes significant outflows of investment. This is not the case for the banking sector. Instead of funds shifting out of the banking sector due to unanticipated failures, savers simply shift between assets within the sector. We believe this occurs for two possible reasons. First, and most obviously, the majority of commercial bank deposits are guaranteed by the FDIC. Because of deposit insurance, small time deposits remain one of the safest assets to own during economic stress. Second, the potential adverse effects banking shocks have on assets in the real economy are of greater concern than the risk of assets within the banking sector. Thus, banks in the modern economy are considered a safe haven even during banking crises.

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<sup>42</sup>The levels in the business and consumer loan responses are converted into real year 2000 dollars.



## 4 Asymmetric Shocks

The concept of a “systemic failure” inherently projects the idea that larger failures have a markedly different effect on the economy than smaller ones. This threshold is highlighted in the Dodd-Frank Act by requiring all bank holding companies with consolidated assets of \$50 billion or more to submit resolution plans to the Federal Reserve and the FDIC. However, this procedure is not required by all financial institutions. Therefore, policymakers are implicitly suggesting that below the \$50 billion threshold all institutions can be treated similarly. In essence, failures among these lower-ranked institutions can be generally resolved by following the FDIC’s Resolution Handbook. In this section we test for thresholds within bank failure shocks. Interestingly, we find a threshold far less restrictive than the one set forth by Dodd-Frank. As a matter of fact, the threshold is an order of magnitude less.

### 4.1 Threshold Testing

Finding the threshold for these shocks is somewhat straightforward. However, there is one important difference from a standard threshold analysis that is to be noted. The threshold value being searched for lies in the exogenous shocks, not the level bank failure data. Thus, the regime change does not necessarily occur at a specific, pre-defined bank failure size – although the raw failure would necessarily be in excess of the shock threshold. Before searching for the threshold, it is necessary to perform a test for nonlinearity within the residuals.

Obtaining the vector of shocks ( $\epsilon_t$ ) from the VAR model outlined in Section 3,

$$\epsilon_t = \begin{bmatrix} \widehat{\epsilon}_{CON} \\ \widehat{\epsilon}_Y \\ \dots \\ \widehat{\epsilon}_{BF} \end{bmatrix} \quad (91)$$

we then extract the series of shocks associated with bank failures. Since the ordering of the series within the shock vector follows the ordering of (88), extracting the series is simple. In our case, the series of interest is denoted as ( $\widehat{\epsilon}_{BF}$ ) above. The series of shocks can be seen

in Figure 15.

To test for nonlinearity, a Lagrange multiplier (LM) test is employed.<sup>43,44</sup> The test is performed using the following regression equation:

$$\widehat{\varepsilon}_{BF,t}^2 = \theta_0 + \sum_{i=1}^3 \theta_i \cdot \widehat{\varepsilon}_{BF,t-i}^2 + \xi_t \quad (92)$$

The joint test in equation (92) uses a test statistic equal to  $TR^2$ . Since the sample value  $TR^2$  exceeds the critical value given by a  $\chi^2$  distribution, the null hypothesis of linearity is rejected in our case. Although the test does not tell us the exact nature of the nonlinearity, it provides confidence that further steps can be taken. More specifically, we are confident that the error terms have some characteristic size or variance.

Our conjecture is that the bank failure residual series contains a characteristic size or threshold. Therefore, we test for a threshold using a method similar to Chan (1993). This method searches for a threshold by finding a value that minimizes the residual sum of squared errors (RSS). If a threshold does not exist then it would be clear when observing the scatter plot of the RSS. In essence, no single value would provide more explanatory power than those above or below it. However, in our case, the scatter plot in Figure 16 does reveal such a value. The lowest RSS is obtained at the value  $\tau = 0.02077$ . In order to confirm the significance of this threshold, we test the impact bias using

$$\widehat{\varepsilon}_{BF,t} = \phi_0 + \phi_1 \cdot I_{t-1} + \xi_t, \quad (93)$$

where

$$I_{t-1} = \begin{cases} 1 & \text{if } \widehat{\varepsilon}_{BF,t-1} \geq \tau \\ 0 & \text{if } \widehat{\varepsilon}_{BF,t-1} < \tau \end{cases} \quad (94)$$

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<sup>43</sup>This is the LM test pioneered by McLeod-Li (1983).

<sup>44</sup>Brüggemann, Lütkepohl, and Saikkonen (2006) show that an LM test is appropriate to test for residual autocorrelation when the VAR is I(1) with possible cointegrated variables.

If the response of the shock is asymmetric, then  $\phi_1$  will be statistically significant.<sup>45</sup> The significance of  $\phi_1$  is shown in the Table 3.2.

**Table 3.2**

F-Test on $\phi_1$			
Variable	Coeff.	Std. Err.	T-Stat
Constant	-0.0163	0.0112	-1.4514
$\phi_1$	0.0660	0.0225	2.9208

The significance of the threshold provides insight into the contagion effect present in larger bank failure shocks. Shocks greater than 0.02% in month  $t-1$  predict ‘large’ shocks in month  $t$ . In terms of current assets held by commercial banks, this threshold is equivalent to \$3 billion. On the other hand, shocks below the threshold offer little, if any, predictive power as seen by the t-statistic on the constant term. Bank failure shocks are now broken into two separate series – “small” and “large” shocks. The two series are seen in Figure 17.

## 4.2 Small vs. Large Shocks: The Macroeconomic Effects

In this section, we provide an analysis of the macroeconomy in the wake of a “small” or “large” bank failure shock. Before now, policymakers have only considered “systemic” failures – those that would bring about major consequences.<sup>46</sup> While empiricists, on the other hand, have previously considered all failures to have the same proportionate impact on the macroeconomy. However, analyzing bank failures either way is arguably inappropriate.

As a reference point for policymakers, the Dodd-Frank definition of a systemic failure has occurred only 35 times within our sample.<sup>47</sup> This represents merely 1% of all bank failures. Thus, it is important that we define a tier system that is more representative of the U.S.

<sup>45</sup>The test is similar to the “sign bias test” pioneered by Engle and Ng (1993).

<sup>46</sup>Previous literature has also used the Caprio and Klingebiel (1997, 1999) definition. I.e., “systemic crises” are large banking crisis that result in the loss of all equity capital of commercial banks.

<sup>47</sup>Although the definition set by Dodd-Frank states that the threshold is “consolidated assets within a single institution”, for the sake of comparison we relax this definition. In this way, the Dodd-Frank threshold is re-defined to represent “consolidated assets within a single month.”

experience. For empiricists, restricting failures to a linear impact ignores recent crisis events and, at best, over-emphasizes the aggregate effects of small failures.

We analyze the shocks separately by inserting the small/large series as deterministic variables into the VAR model following the processes outlined in Cover (1992) and Jones, Olsen, and Wohar (2015).

$$W_t = a_0 + \sum_{i=0}^k b_i s_{t-i} + \sum_{i=0}^k c_i l_{t-i} + \sum_{i=1}^k d_i W_{t-i} + e_t \quad (95)$$

where  $s$  = small shocks, and  $l$  = large shocks.

Taking a look first at the effects of a small shock, we find that there is a completely ambiguous impact on the aggregate economy. More specifically, the results from the VAR following a one standard deviation shock to the small failure series yields no conclusive set of impulse response functions. Turning to the system response from the large shock series, Figure 18 presents a definitive set of responses following a shock to the large failure series. Moreover, the effects of large shocks are clearly what drive the results presented in Section 3. Importantly, large shocks cause persistent bank failures every month for up to a year.

## 5 Conclusion

This paper characterizes the impact bank failures have on the U.S. macroeconomy. First, we construct a VAR model consistent in theory and response with the previous literature. After doing so, we examine the effects bank failures have on various economic aggregates. In contrast to the notion that bank failures have little economic impact, our findings show broad measures of consumption, production, employment, and prices decrease significantly following an aggregate bank failures shock. To this point, we find that the impact on employment is the most notable.

The reaction of payroll employment to exogenous bank failures is immediate and persistent. This result also continues to hold when the data is broken down into the durable and non-durable goods markets. We conjecture that firms interpret bank failures as a definitive

sign of impending economic adversity. Although, interestingly, we do not find evidence of considerable reductions in demand. However, we do find that business lending decreases. Therefore, it appears that the strongest effect bank failures have on the macroeconomy is the reduction in employment through a modest decrease in demand coupled with the a constriction in credit markets.

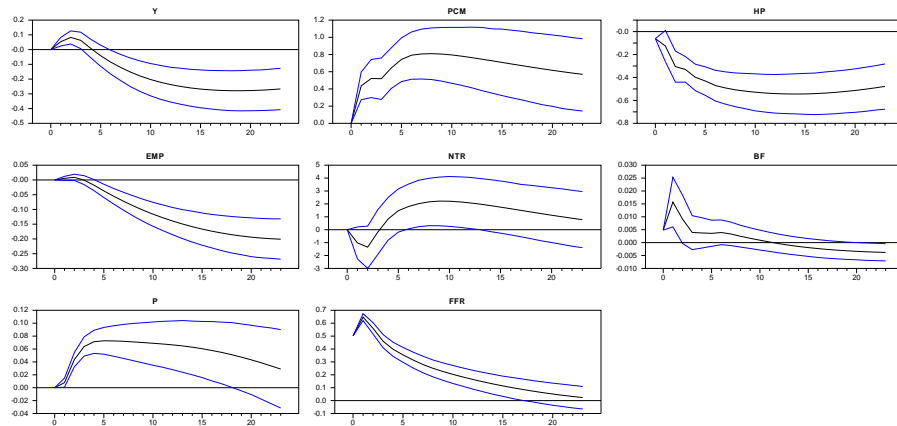
To further understand these effects, we relax the assumption of linearity within the bank failure shocks. This line of thought follows the capital plan rule recently adopted by the Federal Reserve System. Upon analyzing the data, we find the presence of nonlinearity. More specifically, bank failure shocks contain a natural threshold. A failure shock to current assets of \$3 billion causes significant harm to the macroeconomy and also has contagion effects – i.e., leads to additional bank failures. On the other hand, shocks below \$3 billion have a completely ambiguous effect. This finding is of particular interest since the current threshold identifying “systemically important financial institutions” utilized in the capital plan rule is \$50 billion in consolidated assets. To that extent, we do not necessarily contend that \$3 billion should be the new threshold. However, our result offers support for additional tiers in regulatory oversight thresholds.

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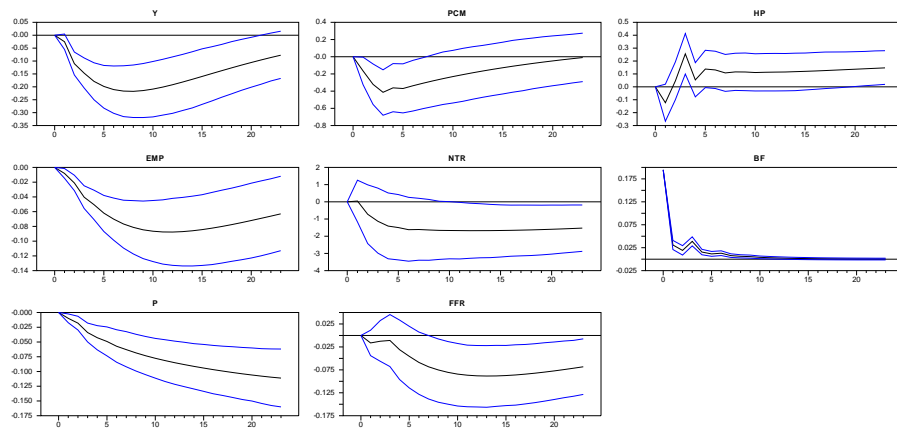
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# Appendix



Responses to FFR

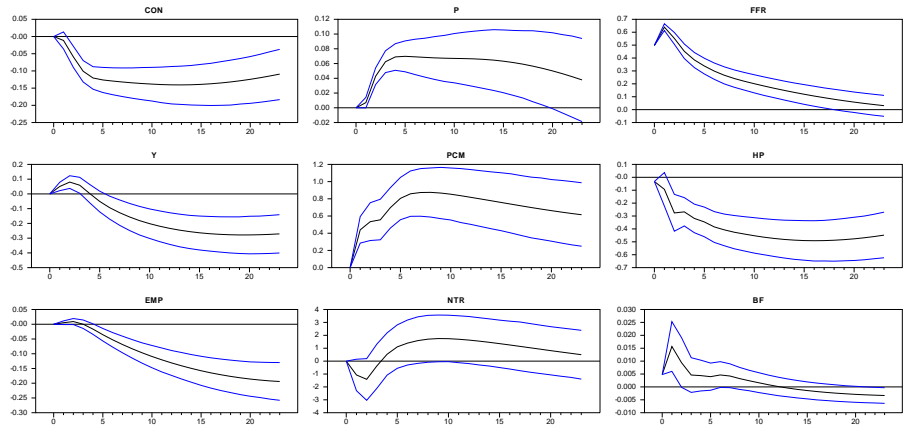
**Figure 3.1: Aggregate 8-Variable System Response to a FFR Shock**



Responses to BF

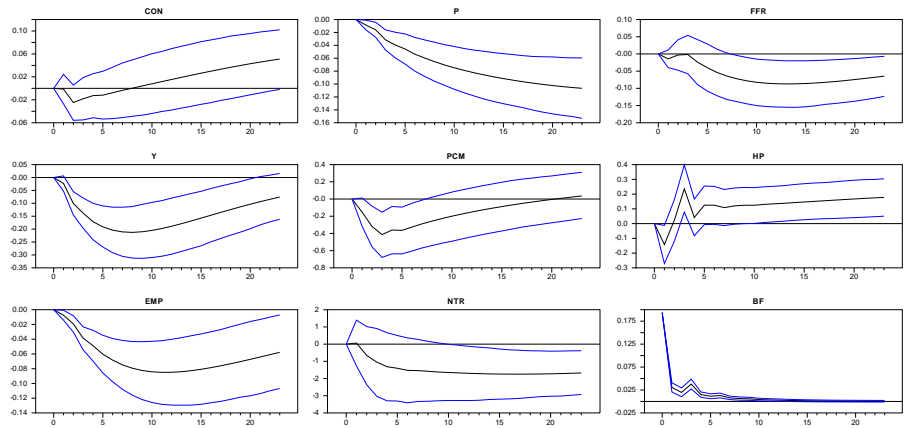
**Figure 3.2: Aggregate 8-Variable System Response to a BF Shock**





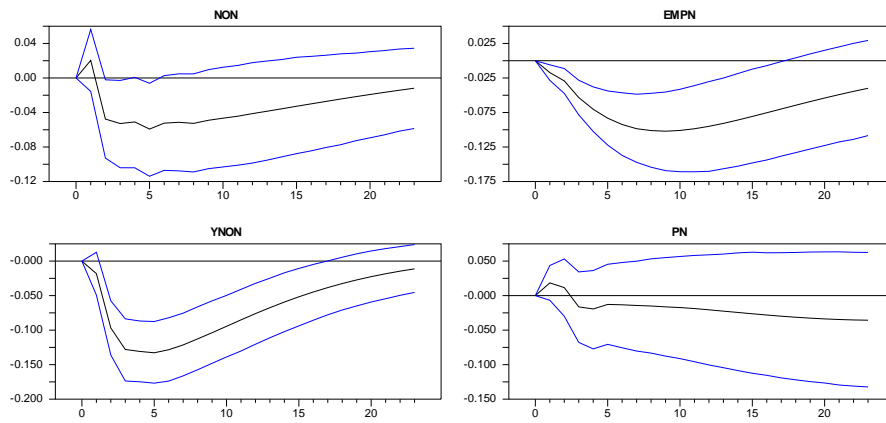
Responses to FFR

**Figure 3.3: Aggregate 9-Variable System Response to a FFR Shock**



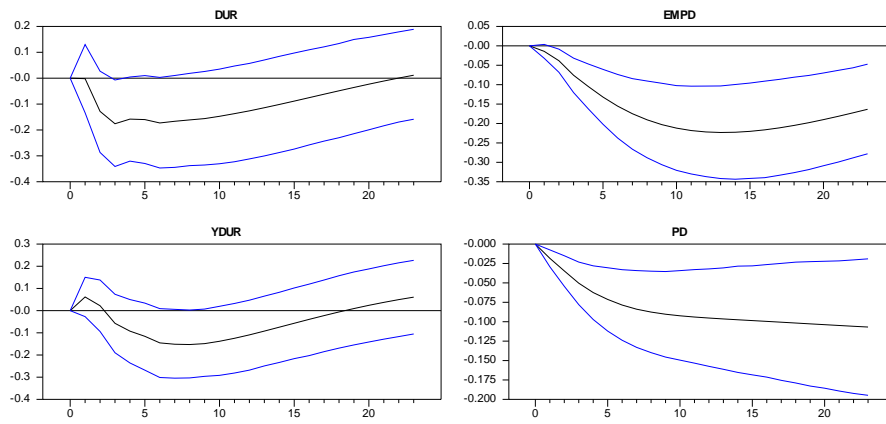
Responses to BF

**Figure 3.4: Aggregate 9-Variable System Response to a BF Shock**



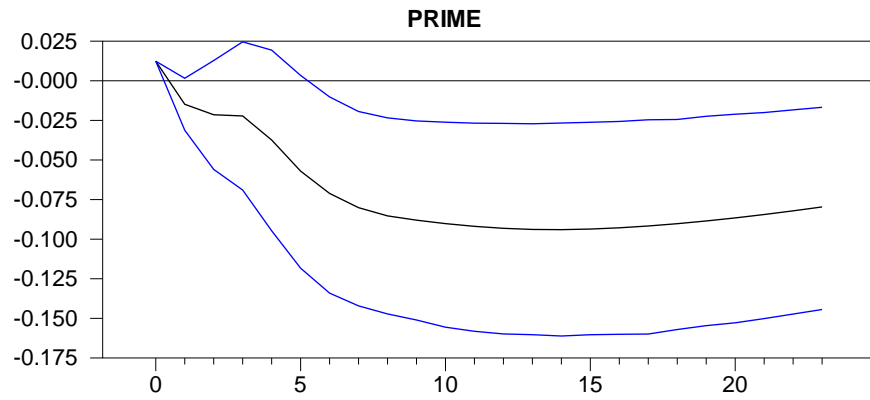
Responses to BF

**Figure 3.5: System Response for the Nondurable Goods Market to a BF Shock**



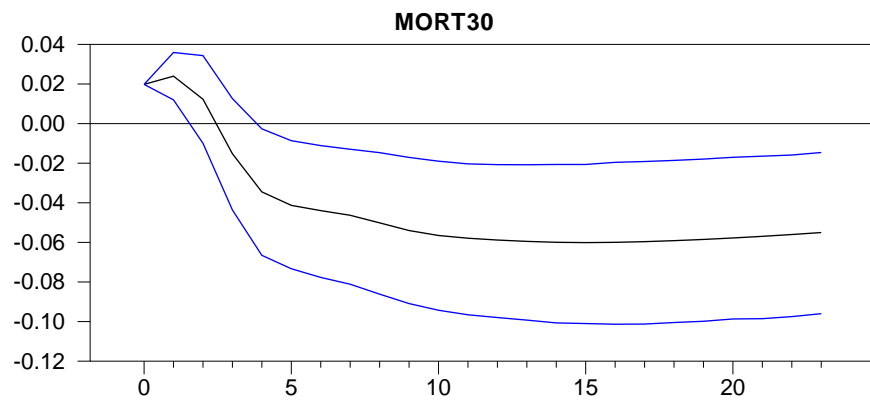
Responses to BF

**Figure 3.6: System Response for the Durable Goods Market to a BF Shock**



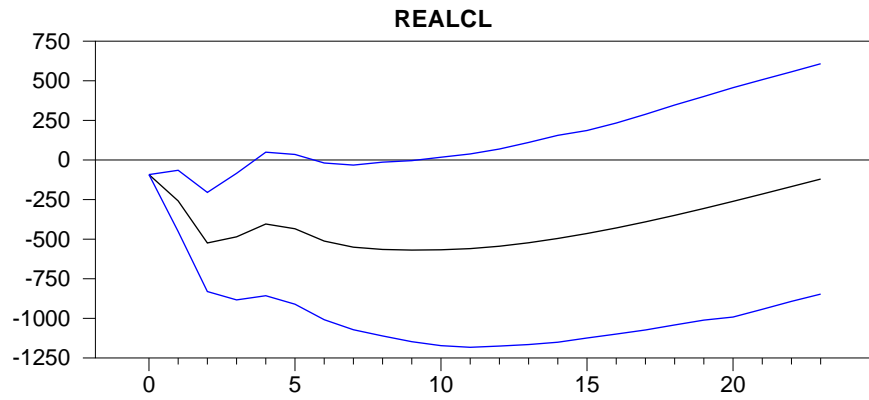
Responses to BF

**Figure 3.7: Response of the Prime Interest Rate to a BF Shock**



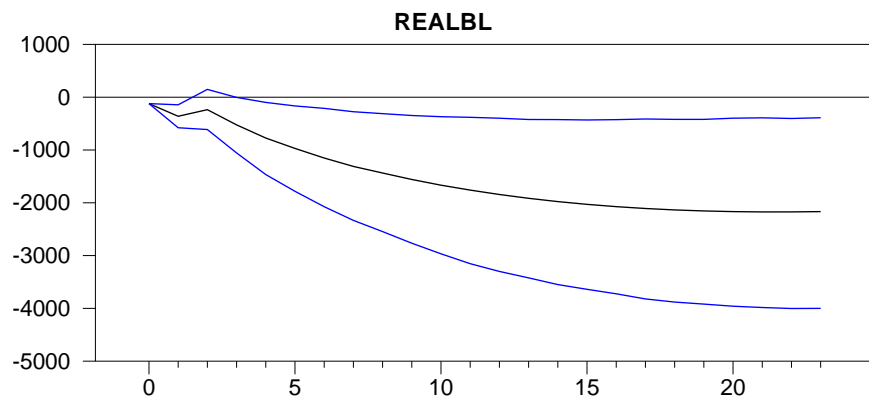
Responses to BF

**Figure 3.8: Response of the 30-yr Mortgage Rate to a BF Shock**



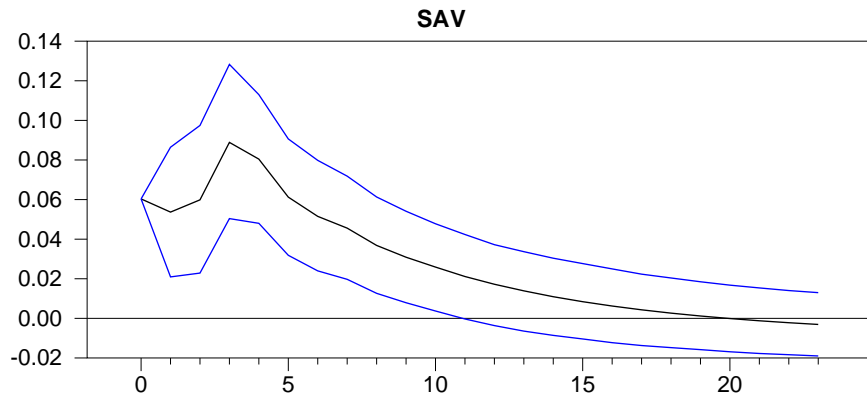
Responses to BF

**Figure 3.9: Response of Consumer Loans to a BF Shock**



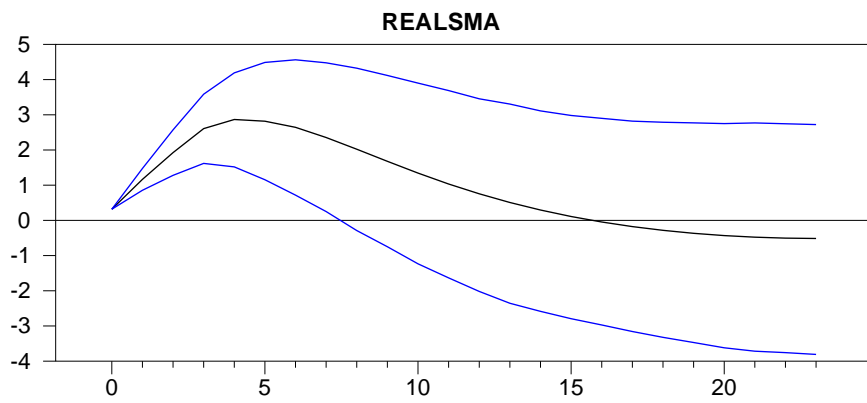
Responses to BF

**Figure 3.10: Response of Business Loans to a BF Shock**



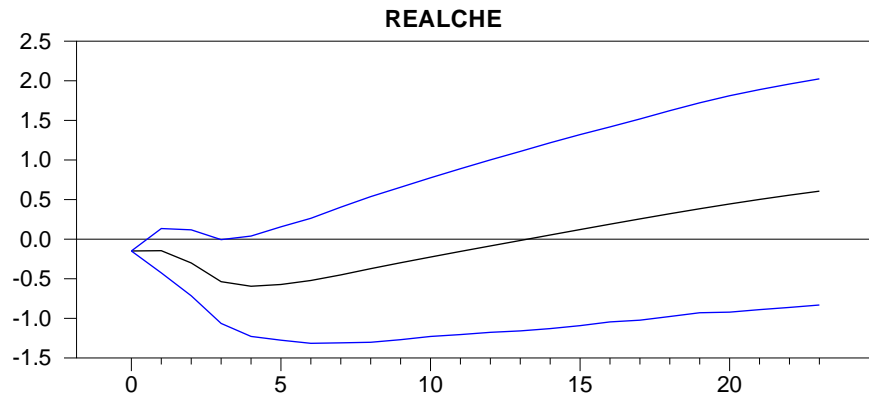
Responses to BF

**Figure 3.11: Response of Private Savings to a BF Shock**



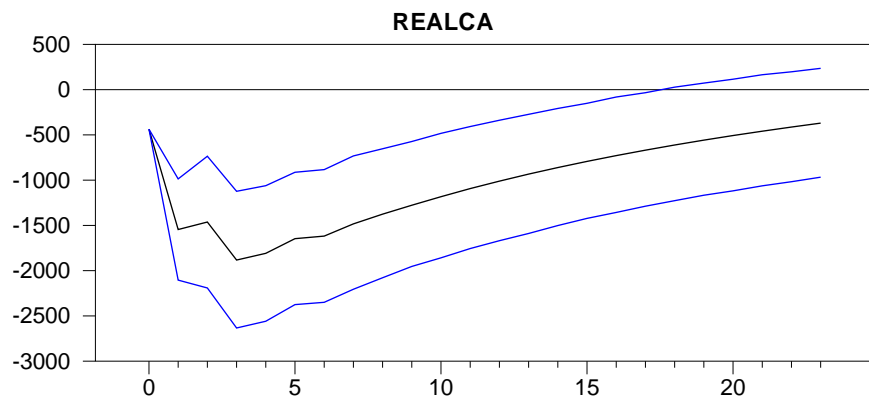
Responses to BF

**Figure 3.12: Response of Small Time Deposits to a BF Shock**



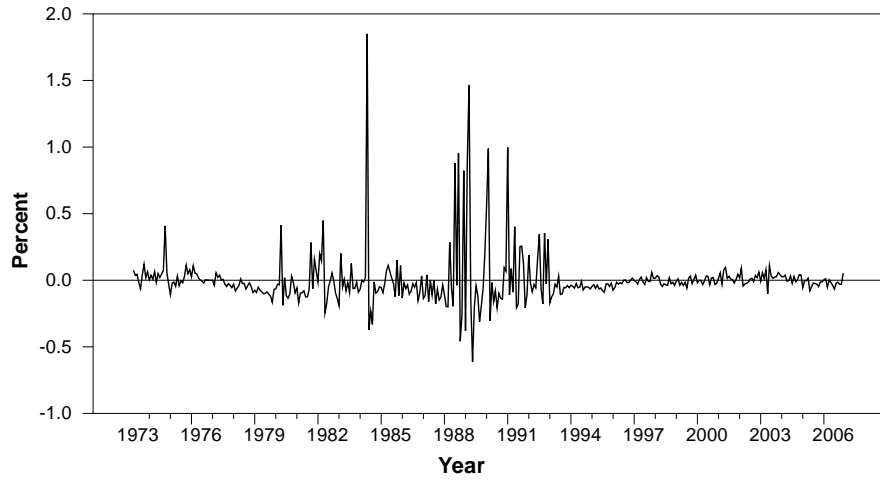
Responses to BF

**Figure 3.13: Response of Demand Deposits to a BF Shock**

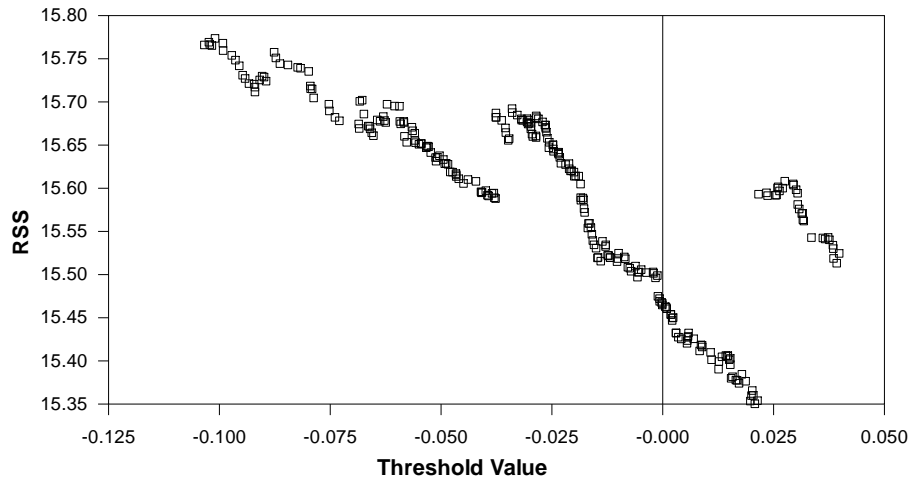


Responses to BF

**Figure 3.14: Response of Bank Cash Holdings to a BF Shock**



**Figure 3.15: Bank Failure Shock Series**



**Figure 3.16: Plot of the Residual Sum of Squares**

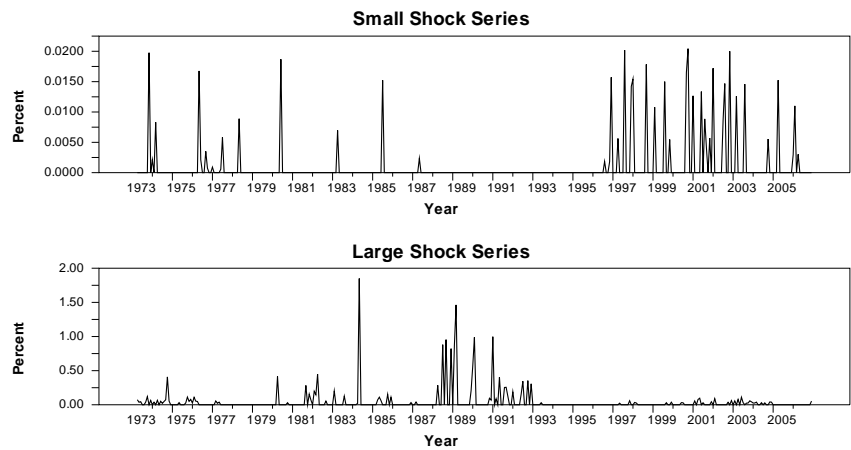


Figure 3.17: Small/Large Shock Series

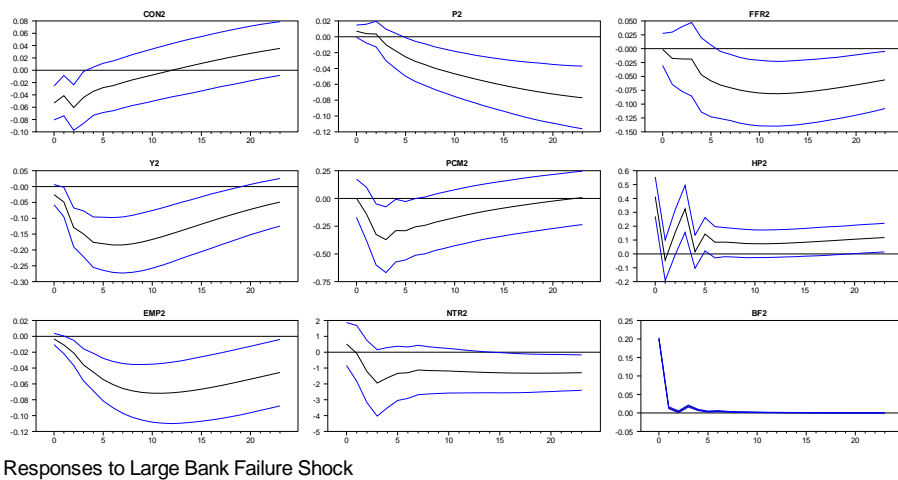


Figure 3.18: Aggregate System Response to a Large BF Shock