Majorana Neutrinos as the Dark Matters in the Cold Plus Hot Dark Matter Model

N. Kitazawa – Tokyo Metropolitan University
N. Okada – Tokyo Metropolitan University
S. Sasaki – University of Tokyo

Citation of published version:

Kitazawa, N., Okada, N, Sasaki, S. (1996): Majorana Neutrinos as the Dark Matters in the Cold Plus Hot Dark Matter Model. *Physics Letters B,* 380(3-4).
DOI: https://doi.org/10.1016/0370-2693(96)00530-8

11 July 1996

PHYSICS LETTERS B

ELSEVIER

Physics Letters B 380 (1996) 324–330

Majorana neutrinos as the dark matters in the cold plus hot dark matter model

Noriaki Kitazawa [a], Nobuchika Okada [a,1], Shin Sasaki [b,2]

[a] *Department of Physics, Tokyo Metropolitan University, Hachioji-shi, Tokyo 192-03, Japan*

[b] *Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113, Japan*

Received 12 February 1996; revised manuscript received 23 April 1996

Editor: H. Georgi

Abstract

A simple model of the Majorana neutrino with the see-saw mechanism is studied, assuming that two light neutrinos are the hot dark matter each with a mass of 2.4 eV in the cold plus hot dark matter model of cosmology. We find that the heavy neutrino, which is the see-saw partner with the remaining one light neutrino, can be the cold dark matter, if the light neutrino is exactly massless. This cold dark matter neutrino is allowed to have a mass in the wide range from 5.9×10^2 eV to 2.2×10^7 eV.

Recently Primack et al. [1] pointed out that the cold and hot dark matter model agrees very well with the observations of the matter distribution in the universe with the total density parameter $\Omega = 1$ and the Hubble constant $h \equiv H_0/100$ km s^{-1} Mpc^{-1} = 0.5. They assumed that two massive neutrinos which have nearly degenerate masses 2.4 eV play the role of the hot dark matter. The hot dark matter, the cold dark matter, and the Baryons occupy 20%, 72.5%, and 7.5% of the total density parameter, respectively.

On the other hand, there are some current data regarding the masses and the flavor mixings of neutrinos. The solar neutrino deficit [2] and the atmospheric neutrino anomaly [3] seem to give indirect evidence of the non-vanishing masses and flavor mixings of the neutrinos in the view of neutrino oscillation. In addition, recent LSND experiment [4] seems to have brought the first direct evidence for neutrino masses and flavor mixings in the $\overline{\nu_\mu} \to \overline{\nu_e}$ oscillation. If some species of neutrinos have masses of order eV, they can be appropriate for the hot components of the dark matter in the cold and hot dark matter model.

In the standard model of the elementary particle physics, three species of neutrinos are exactly massless and there is no particle which can be the cold dark matter. Some extension is needed to include the mass of the neutrinos and the cold dark matter. In this letter, we study the model first introduced by Chikashige, Mohapatra and Peccei [5]. This model is a very simple extension of the standard model, which includes massive Majorana neutrinos.

We introduce three species of right-handed neutrinos and an electroweak-singlet scalar Φ as new particles to the standard model. In order to make neutrinos massive, two kinds of Yukawa couplings are considered. The Yukawa interaction is described by

[1] E-mail: n-okada@phys.metro-u.ac.jp.

[2] Present address: Department of Physics, Tokyo Metropolitan University, Hachioji-shi, Tokyo 192-03, Japan.

PII S0370-2693(96)00530-8

$$\mathcal{L}_{\text{Yukawa}} = -g_{Yij}\overline{\nu_{Li}}\phi\nu_{Rj} - g_{Mij}\overline{\nu_{Ri}^{c}}\Phi\nu_{Rj} + \text{h.c.}, \qquad (1)$$

where ϕ is the electric-charge neutral component of the Higgs field in the standard model, and i and j denote flavors $(i, j = 1, 2, 3)$. The Dirac and the Majorana mass terms appear by the non-zero vacuum expectation values of these scalar fields ($\langle\phi\rangle \neq 0$ and $\langle\Phi\rangle \neq 0$). The mass matrix is given by

$$\mathcal{L}_{\text{mass}} = -\left(\overline{\nu_L}\ \overline{\nu_R{}^c}\right)\begin{bmatrix} 0 & m_D \\ m_D^T & M \end{bmatrix}\begin{pmatrix} \nu_L{}^c \\ \nu_R \end{pmatrix} + \text{h.c.}, \qquad (2)$$

where m_D (M) is the 3×3 Dirac (Majorana) mass matrix defined by $m_{Dij} = g_{Yij}\langle\phi\rangle$ $(M_{ij} = g_{Mij}\langle\Phi\rangle)$, and ν_L and ν_R are the column vectors of the three flavors. Since the symmetry of the lepton number is spontaneously broken by $\langle\Phi\rangle \neq 0$, a massless Nambu-Goldstone boson called majoron appears. Generally m_D (M) is the 3×3 complex (symmetric) matrix and we should diagonalize the whole 6×6 mass matrix in Eq. (2).

As the first approximation, we assume that the off-diagonal elements of the mass matrices m_D and M are very much smaller than their diagonal elements, namely, the effect of the flavor mixing can be neglected. By this assumption, we can take $m_D = \text{diag}\,[m_1, m_2, m_3]$ and $M = \text{diag}\,[M_1, M_2, M_3]$. Furthermore, we assume the hierarchy between m_D and M, namely $|m_i/M_i| \ll 1$. Then, the see-saw mechanism [6] separately works on each generation.

The six mass eigenstates are described by the weak eigenstates, ν_L and ν_R, as

$$\begin{pmatrix} \nu_\ell \\ \nu_h \end{pmatrix} \simeq \begin{bmatrix} 1 & -\epsilon \\ \epsilon & 1 \end{bmatrix}\begin{pmatrix} \nu_L + \nu_L{}^c \\ \nu_R + \nu_R{}^c \end{pmatrix}, \qquad (3)$$

where $\epsilon = \text{diag}[m_1/M_1, m_2/M_2, m_3/M_3]$, and $\nu_\ell = (\nu_1^\ell\ \nu_2^\ell\ \nu_3^\ell)^T$ and $\nu_h = (\nu_1^h\ \nu_2^h\ \nu_3^h)^T$ are the light and heavy Majorana fields, respectively. The masses for ν_i^ℓ and ν_i^h are $m_i^\ell \simeq -(m_i)^2/M_i$ and $m_i^h \simeq M_i$, respectively.

The couplings of the light and heavy neutrinos with the weak bosons are given by

$$\mathcal{L}_{W\nu} \simeq \frac{e}{2\sqrt{2}s}W^{-\mu}\left[\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell + \bar{\ell}\gamma_\mu(1-\gamma_5)\epsilon\nu_h\right] + \text{h.c.},$$

$$\mathcal{L}_{Z\nu} \simeq \frac{e}{4sc}Z^\mu\left[\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\nu_\ell + \{\overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\epsilon\nu_h + \text{h.c.}\} + \overline{\nu_h}\gamma_\mu(1-\gamma_5)\epsilon^2\nu_h\right], \qquad (4)$$

where ℓ denotes the column vector of three charged leptons, $\ell = (e\ \mu\ \tau)^T$. Here we neglect the flavor mixing also in the charged leptons. The couplings of the heavy neutrinos with the weak bosons are suppressed by the small factor ϵ. These fields ν_ℓ and ν_h also couple with the majoron (imaginary part of the field Φ) [3]. According to Eq. (1), these couplings are given by

$$\mathcal{L}_{\chi\nu} \simeq -\frac{1}{\sqrt{2}}\chi\left[\overline{\nu_\ell}g_M^D\epsilon^2 i\gamma_5\nu_\ell - \{\overline{\nu_\ell}g_M^D\epsilon\, i\gamma_5\nu_h + \text{h.c.}\} + \overline{\nu_h}g_M^D i\gamma_5\nu_h\right], \qquad (5)$$

where the field χ is the majoron field defined by $\chi/\sqrt{2} = \text{Im}\Phi$, and g_M^D is a diagonal matrix, $g_M^D = \text{diag}[g_1^M, g_2^M, g_3^M] = M/\langle\Phi\rangle$. In contrast with Eq. (4), the couplings of the light neutrinos with the majoron are suppressed by the factor ϵ.

Some parameters are fixed according to the cosmological model considered by Primack et al. In their model, the mass spectrum of the light neutrinos is constrained as $m_1^\ell \ll m_2^\ell \simeq m_3^\ell \simeq 2.4$ eV. Then, the five parameters are left free in our model, g_1^M, g_2^M, g_3^M, m_1^h and m_1^ℓ $(\ll 2.4$ eV$)$. Since it is very complicated to analyze leaving all of these parameters free, we consider the simplest case, in which masses of all heavy neutrinos are degenerate. Namely, we set $g_1^M = g_2^M = g_3^M \equiv g_M$, therefore $m_1^h = m_2^h = m_3^h \equiv m_h$. Now we have only three free parameters, g_M, m_h, and m_1^ℓ. The value of m_1^ℓ is fixed in the following discussion.

There is a fundamental constraint that the particle has to be stable to be the cold dark matter, namely, its life time must be longer than the age of the universe ($t_U \simeq 10^{17}$ s). The heavy neutrinos decay into the light neutrinos and the majoron through the coupling in Eq. (5). The life time of the i-th heavy neutrino is given by

$$\tau_i = \frac{32\pi}{(g_M)^2 m_i^\ell}. \qquad (6)$$

Unless g_M is very tiny $(g_M < 5.3 \times 10^{-15})$, two neutrinos, ν_2^h and ν_3^h, are unstable because of $m_2^\ell \simeq m_3^\ell \simeq 2.4$ eV. Since such an extremely small coupling constant is unnatural, these two neutrinos cannot be naturally the cold dark matter. Then, the heavy neutrino

[3] These fields ν_ℓ and ν_h also couple with the majoron-Higgs (real part of the field Φ) and the usual Higgs in the standard model. However, we do not consider the effects of these fields assuming that these scalar particles are very heavy.

ν_1^h is the only remaining candidate for the cold dark matter. If we adopt a natural (not so small) value for the Yukawa coupling g_M, the mass of the corresponding light neutrino ν_1^ℓ must be extremely small, for instance, $m_1^\ell < 6.6 \times 10^{-27}$ eV for $g_M \simeq 10^{-2}$. Since we cannot believe that such small value of the mass is explained by some mechanism (the Dirac mass m_1 must be extremely smaller than the weak scale), we assume that ν_1^ℓ is exactly massless by virtue of some symmetries. Note that the vanishing m_1^ℓ, or the Dirac mass $m_1 = 0$, is stable against the radiative correction, since the $U(1)$ symmetry of the phase rotation of the fields ν_{1L} and e forbids the generation of the Dirac mass m_1. Furthermore, it should be stressed that only the mass difference is important (flavor mixings are also needed) for the neutrino oscillation phenomena, but not the absolute value of the mass. There is nothing wrong with the massless ν_1^ℓ. According to this assumption, the heavy neutrino ν_1^h is exactly an electroweak-singlet particle. Therefore, it can be stable cold dark matter. Note that the stability of ν_1^h is ensured, even if the flavor mixing between light neutrinos is introduced, since we are assuming that the mass matrix M is proportional to unit matrix and ignoring CP violation phases in the mass matrix m_D in Eq. (2). There is no higher-order effect which can cause the decay of the cold dark matter neutrino. In the following discussion, our aim is to investigate the cosmologically allowed region of the mass m_h and the coupling constant g_M.

Let us consider the dynamical evolution of the early universe in this model. At very high temperature, we assume that all the particles are in thermal equilibrium. As the temperature cools down, some particles decouple from the thermal equilibrium at each specific temperature called decoupling temperature. It is convenient for considering the evolution of the universe to separate the matter contents into two parts. One is the "heavy neutrino-majoron system" which includes three heavy neutrinos and the majoron. The other is the "electroweak system" in which all the other particles are included. These two systems weakly interact with each other through the couplings suppressed by the see-saw factor ϵ in Eqs. (4) and (5). We assume that at the temperature $T_D^{\rm EW}$ the "heavy neutrino-majoron system" decouples from the "electroweak system", and that the particles in the "heavy neutrino-majoron system" are still in thermal equilibrium after this decoupling. Therefore, the decoupling temperature T_D^X is lower than $T_D^{\rm EW}$. Here T_D^X is the temperature at which the heavy neutrinos and the majoron no longer interact with each other.

Note that the temperatures of these two systems are different after the decoupling at $T_D^{\rm EW}$. The temperature of the heavy neutrino as the cold dark matter in the present universe $T_{\rm CDM}$ is different from the temperature of the photon at present, $T_r = 2.7$K. The reheating factor α_R is estimated by considering the reheating of photon caused by the charged particle and anti-particle annihilation. In addition to this usual factor α_R, the cooling factor R should be introduced in our model in order to include cooling effect by the decaying unstable neutrinos. As soon as the "heavy neutrino-majoron system" decouples from the "electroweak system" at $T_D^{\rm EW}$, two unstable neutrinos ν_2^ℓ and ν_3^ℓ decay into the light neutrinos and the majoron. Then, the "heavy neutrino-majoron system" cools down and the "electroweak system" is heated up, because the energy of the "heavy neutrino-majoron system" flows to the "electroweak system" by the emission of the light neutrinos. Since the degrees of freedom of the "electroweak system" is far larger than that of the "heavy neutrino-majoron system", we can ignore the heating effect of the "electroweak system". The cooling factor R is defined by

$$R = \frac{T_D^X}{T_{\rm EW}^X}\,, \tag{7}$$

where $T_{\rm EW}^X$ is the temperature of the "electroweak system" when the heavy neutrinos decouple from the majoron. Since $T_D^X < T_D^{\rm EW}$ as assumed above, the factor R is less than unity. The temperature $T_{\rm CDM}$ is written as

$$T_{\rm CDM} = \alpha_R\, R\, T_r\,. \tag{8}$$

by using the factors α_R and R.

Now we can write down the condition for the heavy neutrino ν_1^h to be the cold dark matter, $\rho_{\nu_1^h} = \rho_{\rm CDM}$. Here $\rho_{\rm CDM} = 1.9 \times 10^{-6}$ GeV/cm^3 is the energy density of the cold dark matter in the present universe based on the model considered by Primack et al. The present energy density of the cold dark matter neutrino ν_1^h is given by

$$\rho_{\nu_1^h} = m_h\, n(T_D^X)\, \left(\frac{T_{\rm CDM}}{T_D^X}\right)^3 , \tag{9}$$

where $n(T_D^\chi)$ is the number density of the cold dark matter at the decoupling temperature T_D^χ. This number density is given by

$$n(T_D^\chi) = \frac{1}{\pi^2}\int_0^\infty \frac{p^2 dp}{\exp[\frac{E}{T_D^\chi}]+1} = f(x_D)\,(T_D^\chi)^3, \tag{10}$$

where $E = \sqrt{p^2+m_h^2}$, $x_D = T_D^\chi/m_h$, and $f(x_D)$ is defined by

$$f(x_D) = \frac{1}{\pi^2}\int_0^\infty \frac{y^2 dy}{e^{\sqrt{y^2+x_D^{-2}}}+1}. \tag{11}$$

Considering that the cold dark matter should decouple in the non-relativistic regime, we provide the upper bound of x_D as $x_D \leq 1$. By using the condition, $\rho_{\nu_1^h} = \rho_{\rm CDM}$, and Eqs. (8)–(10), the mass m_h is given by

$$m_h(x_D, R) = 1.1\times 10^{-9}\,\alpha_R^{-3}\,f(x_D)^{-1}\,R^{-3}\ \text{GeV} \tag{12}$$

as the function of x_D and R.

On the other hand, the coupling constant g_M is obtained from the definition of the decoupling temperature T_D^χ. The decoupling temperature is defined by [7]

$$n(T_D^\chi)\,\langle\sigma|v|\rangle_{T_D^\chi} = H, \tag{13}$$

where $\langle\sigma|v|\rangle_{T_D^\chi}$ is the average value of the annihilation cross section of a heavy neutrino times relative velocity, and H is the Hubble parameter. For the non-relativistic heavy neutrino, we obtain

$$\langle\sigma|v|\rangle_{T_D^\chi} = \frac{(g_M)^4}{128\pi}\,\frac{T_D^\chi}{(m_h)^3}, \tag{14}$$

by considering the heavy neutrino annihilation process, $\nu_1^h\nu_1^h \to \chi\chi$. The Hubble parameter H is given by

$$H = \left(\frac{8\pi^3}{90}\right)^{1/2}\sqrt{g_*}\,\frac{T^2}{M_P}, \tag{15}$$

where $M_P \simeq 1.2\times 10^{19}$ GeV is the Planck mass, and g_* is the total degrees of freedom of the all particles in thermal equilibrium. Note that T is not equal to T_D^χ. Since the degrees of freedom of the "electroweak system" are far larger than that of the "heavy neutrino-majoron system", the expansion rate of the universe, or the Hubble parameter, is approximately controlled only by the "electroweak system". Therefore, we can set T and g_* the temperature $T_{\rm EW}^\chi$ and the degrees of freedom of the "electroweak system", respectively. From the definition of R in Eq. (7), the Hubble parameter is rewritten as

$$H = \left(\frac{8\pi^3}{90}\right)^{1/2}\sqrt{g_*}\,\frac{(T_D^\chi)^2}{M_P R^2}. \tag{16}$$

Substituting Eqs. (10), (14) and (16) into Eq. (13) and eliminating m_h by using Eq. (12), g_M is given by

$$g_M(x_D, R) = 5.0\times 10^{-7} \times g_*^{1/8}\,\alpha_R^{-3/4}\,x_D^{-1/2}\,f(x_D)^{-1/2}\,R^{-5/4}. \tag{17}$$

as the function of x_D and R. Since we obtain both m_h and g_M as functions of x_D and R, one line is drawn in the m_h-g_M plane for one fixed values of R (≤ 1) varying x_D from zero to unity. The allowed region of m_h and g_M is very large, if the value of R is absolutely free.

Next we estimate the cooling factor R by using the "sudden-decay" approximation for two unstable heavy neutrinos ν_2^h and ν_3^h. We approximately consider that all the unstable neutrinos decay and disappear at once when the age of the universe is equal to their life time, τ ($= \tau_2 \simeq \tau_3$) [4]. In addition to this approximation, we assume that the disappeared ν_2^h and ν_3^h are quickly supplied by the majoron annihilation, and the thermal equilibrium is recovered. The same situation is expected to occur also at the age $t = 2\tau, 3\tau$ and so on, until the temperature of the "heavy neutrino-majoron system" cools down to the decoupling temperature T_D^χ. According to these approximations, the ratio $\tilde{T}_\chi/T_\chi$ can be estimated, where $\tilde{T}_\chi$ is the temperature of the "heavy neutrino-majoron system" just after "quick supplement" and T_χ is the one just before the "sudden-decay".

The energy density of the "heavy neutrino-majoron system", $\rho_{\rm hm}$, is described in two different ways. Since

[4] The unstable neutrinos ν_2^h and ν_3^h do not start decaying from $t = 0$, but $t = t_D^{\rm EW}$ at which they decouple from the "electroweak system". However, we can approximately set $t_D^{\rm EW} = 0$, because $t_D^{\rm EW} \ll \tau$ is satisfied in our final result.

the heavy neutrinos and the majoron are in thermal equilibrium just after "quick supplement", we obtain

$$\rho_{\rm hm} = \frac{\pi^2}{30}(1+\tfrac{7}{4}\times 3)\,\tilde{T}_\chi^4 . \tag{18}$$

On the other hand, just after "sudden-decay" (before "quick supplement"), $\rho_{\rm hm}$ is given by

$$\rho_{\rm hm} = \frac{\pi^2}{30}(1+\tfrac{7}{4}\times 3)\,T_\chi^4 - \frac{\pi^2}{30}(\tfrac{7}{4}\times 2\times\tfrac{1}{2})\,T_\chi^4 . \tag{19}$$

The second term denotes the loss of the energy density due to the emission of the light neutrinos. From these two expressions of $\rho_{\rm hm}$, we can obtain $\tilde{T}_\chi/T_\chi = (18/25)^{1/4}$. Since the "sudden-decay" and the "quick supplement" recurrently occur, we obtain $T_{\rm hm}/T_{\rm EW} = (18/25)^{n/4}$ at the age $t = n\tau$, where $T_{\rm hm}$ and $T_{\rm EW}$ are the temperature of the "heavy neutrino-majoron system" and the "electroweak system" at the same age, respectively, and n is the positive integer. This ratio is translated to the smooth function of the age of the universe t: $T_{\rm hm}/T_{\rm EW} = (18/25)^{t/4\tau}$. Therefore, the cooling factor R is given by

$$R = \left(\tfrac{18}{25}\right)^{t_D^\chi/4\tau} , \tag{20}$$

where t_D^χ is the age of the universe at which the heavy neutrino decouples from the majoron.

On the other hand, the definition of R in Eq. (7) is rewritten as

$$R = \frac{T_D^\chi}{T_{\rm EW}^\chi} = \frac{m_h x_D}{T_{\rm EW}^\chi} . \tag{21}$$

Since $T_{\rm EW}^\chi$ is described by t_D^χ by using the relation between the Hubble parameter and the age of the universe:

$$H = \left(\frac{8\pi^3}{90}\right)^{1/2} \sqrt{g_*}\frac{(T_{\rm EW})^2}{M_P} = \frac{1}{2t} , \tag{22}$$

we obtain

$$R = m_h x_D \left(\frac{90}{8\pi^3 g_*}\right)^{-1/4} \sqrt{\frac{2t_D^\chi}{M_P}} . \tag{23}$$

By using Eqs. (6), (21), and (23), we can obtain a relation among m_h, g_M, x_D, and t_D^χ as

$$\left(\tfrac{18}{25}\right)^{t_D^\chi/4\tau(g_M)} = \frac{m_h x_D}{T_{\rm EW}^\chi(t_D^\chi)} . \tag{24}$$

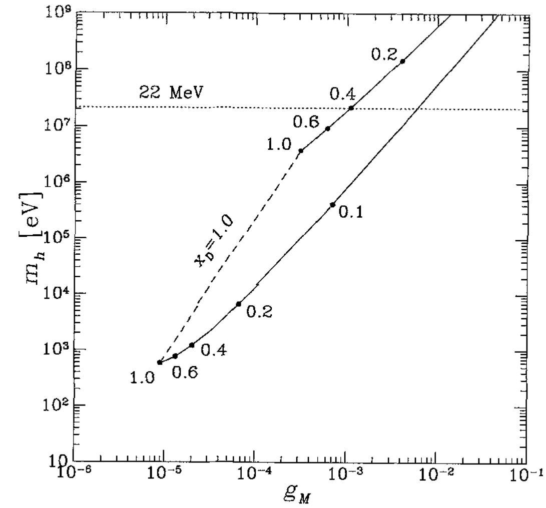

Fig. 1. The allowed region of the mass of the cold dark matter m_h and the coupling to the majoron g_M. The dashed line is the line of $x_D = 1$ for various values of R. The upper solid line is the upper bound for the allowed region. The dots on this line correspond to $x_D = 1, 0.6, 0.4, 0.2$ from below, respectively. The lower solid line is the lower bound for the allowed region. The dots on this line correspond to $x_D = 1, 0.6, 0.4, 0.2, 0.1$ from below, respectively. The dotted horizontal line is the upper bound on the mass, $m \leq 22$ MeV. The region among these four lines is allowed in our analysis.

Now we obtain three independent relations, Eqs. (12), (17), and (24) for m_h, g_M, x_D and t_D^χ. Therefore, we can draw one line in the m_h-g_M plane. The result of numerical calculations for these relations is shown in Fig. 1 (upper solid line for various values of $x_D \leq 1$).

However, note that our approximations underestimate the value of R, since the correct amount of the decaying ν_2^h and ν_3^h is clearly smaller than that estimated by "sudden-decay" approximation. Furthermore we provide decaying neutrinos by the "quick supplement" approximation, although the disappeared ν_2^h and ν_3^h are not so quickly supplied. Since both m_h and g_M are decreasing functions of R as can be seen in Eqs. (12) and (17), the upper solid line in Fig. 1 is interpreted as the upper bound of the allowed region.

There exists an upper bound on m_h, since we assumed that all particles are in thermal equilibrium at very high temperature. Then there should exist a value of temperature $T(\neq 0)$ which can satisfy the condition as follows:

$$\frac{n\,\langle\sigma|v|\rangle}{H} \geq 1 . \tag{25}$$

Here n is the number density of the heavy neutrino and $\langle\sigma|v|\rangle$ is the average value of the annihilation cross section of the heavy neutrino times relative velocity. Considering the annihilation processes $\nu_h\nu_h \to \nu_\ell\nu_\ell$, $\ell\bar{\ell}$, or $q\bar{q}$ (q denotes a quark) according to the weak interaction of Eq. (4), we obtain

$$n\,\langle\sigma|v|\rangle = \frac{4NG_F}{3\pi^2}\left(\frac{m_\ell}{m_h}\right)^2 \times \int_0^\infty \frac{p^4 dp}{e^{E/T}+1}\frac{M_Z^4}{(4E^2-M_Z^2)^2+M_Z^2\Gamma_Z^2}, \tag{26}$$

where $E=\sqrt{p^2+m_h^2}$, $m_\ell \simeq 2.4$ eV, $G_F \simeq 1.17 \times 10^{-5}$ GeV^{-2} is the Fermi constant, M_Z is the mass of the Z boson, and $\Gamma_Z = 2.5$ GeV is the total decay width of the Z boson[5]. The factor N is defined by $N = (I_3 - Q\sin^2\theta_W)^2 + (I_3)^2$, where I_3 and Q are the third component of the weak isospin ($\pm\frac{1}{2}$) and the electric charge of the final state fermion, respectively. Summing N for all the possible final state fermions, we obtain $N \simeq 7.3$. By numerical calculation of Eqs. (25) and (26), we obtain the upper bound $m_h \le 22$ MeV.

Finally, we consider the constraint from the big bang nucleosynthesis (BBN). The number of species of the light neutrinos is constrained as $N_\nu \le 3.04$ [8]. The contribution of new particles (three heavy neutrinos and the majoron) to the energy density in the BBN era ($\simeq 1$ MeV) have to be small enough in comparison with

$$\rho_{\Delta N} = \frac{\pi^2}{30}\,\frac{7}{4}\,\Delta N \times (1\ \text{MeV})^4, \tag{27}$$

where $\Delta N = \max(N_\nu) - 3 = 0.04$. The energy density of the new particles is given by

$$\rho_{\text{new}} = \frac{\pi^2}{30}\,(1+\tfrac{7}{4}\times 3)\ \times (\tilde{\alpha}_R R)^4\,(1\ \text{MeV})^4, \tag{28}$$

where $\tilde{\alpha}_R$ is the ordinary reheating factor at the BBN era, $\tilde{\alpha}_R = \left(g_*(1\ \text{MeV})/g_*(T_D^{\text{EW}})\right)^{1/3}$. Therefore, we can obtain the upper bound of R:

$$R \le \left(\tfrac{7}{25}\times\Delta N\right)^{1/4}\tilde{\alpha}_R^{-1}. \tag{29}$$

[5] The annihilation cross section intermediated by the majoron is far smaller than that intermediated by the gauge boson in the region of m_h and g_M in our final result. Then, it can be ignored.

This provides the lower bond on the allowed region of m_h and g_M, since both m_h and g_M are the decreasing functions of R. The bound[6] is shown in Fig. 1 as the lower solid line for various values of x_D.

Our final result for the mass of the cold dark matter and the coupling to the majoron is shown in Fig. 1. The region among the dashed line, the upper solid line, the lower solid line, and the horizontal line of the upper bound on m_h is allowed in our analysis. The allowed region of m_h and g_M covers over about five and three orders of magnitude, respectively. The mass matrices of the see-saw type are realized in this allowed region of m_h. The right hand side of the dashed line in Fig. 1 satisfies the constraint that the cold dark matter should decouple from the majoron in non-relativistic regime. Requiring that the heavy neutrino has been in thermal equilibrium of the "electroweak system" once, the upper bound on m_h (≤ 22 MeV) is obtained. The "sudden-decay" and "quick supplement" approximations provide the upper bound of the allowed region (upper solid line in Fig. 1). The lower bound of the allowed region is obtained by the BBN constraint (lower solid line in Fig. 1).

Here we would like to mention the characteristic mass scale of the free streaming of the cold dark matter. The free streaming length is roughly estimated as [7]

$$\lambda_{\text{FS}} \simeq \left(\frac{1\ \text{keV}}{m_h}\right)\left(\frac{T_{\text{CDM}}}{T_r}\right)\ \text{Mpc} = \left(\frac{1\ \text{keV}}{m_h}\right)\alpha_R R\ \text{Mpc}, \tag{30}$$

and we obtain the characteristic mass scale of the free streaming:

$$M_{\text{FS}} = \tfrac{4}{3}\pi\lambda_{\text{FS}}^3\,\rho_{\text{CDM}} \simeq 2.1\times 10^{20} M_\odot \left(\frac{\alpha_R R}{m_h(\text{eV})}\right)^3, \tag{31}$$

where $M_\odot$ is the solar mass. This scale means the lower limit of the scale of structure which can be formed by the effect of the cold dark matter. Using the lower limit on the mass of the cold dark matter,

[6] If we refer the recent conservative bound, $N_\nu \le 3.9$, considered by Copi, Schramm, and Turner [9], the upper bound on R becomes a little larger, and the allowed region becomes a little larger. However, our model cannot be consistent with the bound $N_\nu \le 2.6$ by Hata et al. [10], since we have three light neutrinos.

$m_h = 590$ eV, in Fig. 1, we obtain a galactic mass scale [7], $M_{FS} = 10^{10} M_\odot$.

In conclusion, we studied whether the heavy Majorana neutrino can be the cold dark matter or not in the cold plus hot dark matter model considered by Primack et al. The model of the Majorana neutrino first introduced by Chikashige, Mohapatra, and Peccei was considered as the simple extension of the standard model. We found that if a light neutrino is exactly massless, the heavy neutrino, which is the see-saw partner of the massless neutrino, can be the cold dark matter, provided that the other two light neutrinos play the role of the hot dark matter. Therefore, both the hot and cold dark matters are Majorana neutrinos. We obtained a wide allowed region in the m_h–g_M plane by considering cosmological arguments.

[7] If we require that a mass scale of globular clusters ($10^6 M_\odot$) is formed by the cold dark matter, the lower bound on m_h becomes larger about an order of magnitude: $m_h > 6.3 \times 10^3$ eV.

References

[1] J.R. Primack, J. Holtzman, A. Klypin and D.O. Caldwell, Phys. Rev. Lett. 74 (1995) 2160.
[2] B.T. Cleveland et al., Nucl. Phys. B (Proc. Suppl.) 38 (1995) 47;
Y. Suzuki, Nucl. Phys. B (Proc. Suppl.) 38 (1995) 54;
J.N. Abdurashitov et al., Nucl. Phys. B (Proc. Suppl.) 38 (1995) 60;
P. Anselmann et al., Phys. Lett. B 327 (1994) 377; B 285 (1992) 376;
K.S. Hirata et al., Phys. Rev. D 44 (1991) 2241;
A.I. Abazov et al., Phys. Rev. Lett. 67 (1991) 3332.
[3] K.S. Hirata et al., Phys. Lett. B 205 (1988) 416; B 280 (1992) 146.
[4] C. Athanassopoulos et al., Phys. Rev. Lett. 75 (1995) 2650;
J.E. Hill, Phys. Rev. Lett. 75 (1995) 2654.
[5] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Phys. Lett. B 98 (1981) 265.
[6] T. Yanagida, in: Proc. of the Workshop on the Unified Theory and Baryon Number in the Universe, eds. O. Sawada and A. Sugamoto (KEK report 79-18, 1979), p. 95;
M. Gell-Mann, P. Ramond and R. Slansky, in: Supergravity, eds. P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979) p. 315.
[7] E.W. Kolb and M.S. Turner, The Early Universe (Addison-Wesley Publishing Co., California, 1990).
[8] P. Kernan and L. Krauss, Phys. Rev. Lett. 72 (1994) 3309.
[9] C.J. Copi, D.N. Schramm and M.S. Turner, Phys. Rev. Lett. 75 (1995) 3981.
[10] N. Hata et al., Phys. Rev. Lett. 75 (1995) 3977.