A STUDY IN THE APPLICATIONS OF A SMALL-SCALE ELECTRONIC ANALOG COMPUTER FOR SOME INDUSTRIAL ENGINEERING PROBLEMS

By

ROBERT NEIL BRASWELL

A THESIS

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PREFACE

This study is the outgrowth of conversation regarding the potentials that a small electronic analog computer might have as a problem analyzer.for industrial engineering. It entails the experimental evaluation of analog computer solutions of typical industrial engineering problems. The author wishes to express his sincere appreciation to the following for their contributions in the compilation of this thesis:

1. The United States Steel Foundation for the graduate fellowship that made possible the two years graduate program which led to this study,

2. Professor George C. K. Johnson, thesis committee chairman, who graciously consented to the research approach and whose guidance and counciling was always sincere and helpful,

3. Professor Wyllys G. Stanton, who initially conceived the potential of such a study and who followed its progress throughout,

4. Assistant Professor H. Paul Hassel whose optimistic consultation and recommendations were most helpful during the entire study.

(5) Nona, my wife, whose optimistic encouragement and assistance in typing were a necessity.

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CHAPTER I

COMPUTING DEVICES

Introduction

A knowledge of the history of computing devices is very useful in helping to understand the underlying theories and applications of the modern machines of today. The ingenious ideas that were conceived, and the subsequent development of these ideas are vividly portrayed in the many present-day versatile computing machines.

Historical Background[1]

The beginning of civilization can well be considered as the origin of computers. In a somewhat crude manner, the early forms of digital computation were performed by man keeping tab, or accounting for events, by counting on his fingers and toes, or by using pebbles, to mark the events. The devices account for the very existance of our present-day number system, as well as the principle of counting. The earliest known record of digital computation, of course, goes back to the use of pebbles and the toes, while the first digital computer was the abacus. The earliest known

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[1] The general outline of this history came from notes taken in GES 126 - Introduction to Electronic Data Processing, Spring Semester, 1957.

[2] D. D. McCracken, Digital Computer Programming (New York: John Wiley & Sons, Inc., 1957), p. 6.

form of analog computation was a surveying and map making process used in Babylonia in 3800 B. C. for the purpose of taxation. By 1300 B. C. surveying and map making were quite common in Egypt.[3]

The digital computer in its earliest form, a rather crude abacus, consisted of a clay board with grooves in which pebbles were placed. This was used in the Tigress and Euphrates Valley as early as 460 B. C. Later a revised version of the same instrument appeared in Rome, Japan, and China in the form of a crude system of wires with beads mounted to form a compound device. This same computing device can be seen quite frequently in many of the Oriental countries today.[4]

John Napier invented logarithms in 1614, and John Briggs, in collaboration with Napier, converted them to the base ten in 1615. In 1617 Napier devised a method of multiplication utilizing "Napier's Bones." Edmund Genter utilized Napier's logarithms in 1620 to create a slide rule with no moving parts. This was improved in 1632 by William Oughtred's conception of the astrotabee, which was the underlying principle of the slide rule and nomogram with a sliding scale.

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[3] "Maps," Encyclopedia Brittanica, Vol. 21, 1949, pp. 837-38.

[4] George R. Stibitz and Jules A. Larrivell, Mathematics and Computers (New York: McGraw-Hill Book Company, Inc., 1957), pp. 46-47.

In 1642, Blaise Pascal invented the first desk calculator. The calculator used toothed wheels, which limited its operations to addition and subtraction. Moreland made many improvements on Pascal's adding machine in 1666.

G. W. Liebnitz, in 1671, made an important contribution by his improvements to the Pascal machine to facilitate multiplication by repeated addition. Because of the many mechanical imperfections, Liebnitz's machine, which was completed in 1694, was never practical.[5]

Jacquard in 1790 invented an automatic loom operation. The operation received its instructions from punched cards. He was the first to use punched cards, the forerunner for input data for the large scale digital computers.

The planimeter, an analog device first appeared in 1814. This was primarily the work of J. A. Hermann, a Bavarian engineer. Many new and improved types of planimeters appeared between the years 1814 and 1854, at which time Amsler invented the modern polar planimeter.[6]

In 1812, Charles Babbage, the "father" of computers, conceived a digital difference-equation solver which was to be complete with means to print the answers. The English Government supported the construction of this machine until

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[5] _Ibid_., pp. 47-48.

[6] "Mathematical Instruments," _Encyclopedia Brittanica_, Vol. No. 15, 1949, p. 70.

work was finally suspended in 1833. In 1842 the entire project was abandoned after a considerable sum of money had been invested. The only part of Babbage's "Difference Engine" that was actually built was merely an accumulator mechanism. This was primarily attributed to the lack of facilities such as tools, electrical power, etc.

Thomas De Colmar made vast improvements on Pascal's calculator in 1820. This improved version of Pascal's calculator was considered to be the first successful machine for multiplication. Approximately 1500 of the machines were in operation by 1878, a manufacturer in Paris still makes the machine.

About the time that work was abandoned on Babbage's "Difference Engine," he conceived an analytical engine, which was to be much more versatile than his original machine. The principle of this latter machine is said to be the fore-runner of the modern large-scale digital computers. The principles of programming and memory were to be in the form of punched cards, which was a contribution of Jacquard and Hollerith during the first part of the nineteenth century. Hollerith later founded the IBM Company in 1890. The arithmetic section of Baggage's machine was to be performed by tooth wheels, very much like the present adding machines.[7]

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[7] Douglas R. Hartee, Calculating Instruments and Machines (Urbana: The University of Illinois Press, 1949), pp. 69-73.

Scheutz, a Swedish scientist, completed a machine based on Babbage's idea of the "Difference Engine," in 1855. The "Scheutz Differential Calculator" was very much like the "Difference Engine," which was never finished.

In 1836, Dorr E. Felt developed the first practical key-driven adding machine. This came thirty-six years after the first patent was approved for a key-driven adding machine. Felt's machine was called a "Macaroni Box" and was a comptometer; it was completely built by the use of a hammer, a screw-driver, and a chisel.

The principles of analog devices were studied with interest during Lord Kelvin's time. Kelvin's brother, James Thompson, invented an integrating mechanism prior to 1870. In 1876, Lord Kelvin conceived the idea of putting together the art of analogs and Thompson's integrating devices to solve differential equations. The principles are recorded in Kelvin's "Harmonic Synthesizer" which was built in 1878 to predict tides.[8]

Following Felt's patent of his "Comptometer" in 1887, there were numerous patents issued for improvements to adding machines. Typical of these improvements was Felt's printing addition to his comptometer to enhance its versatility. This latter modification called a comp-

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[8] Walter W. Soroka, Analog Methods in Computation and Simulation (New York: McGraw-Hill Book Company, Inc., 1954), pp. 160-61.

lograph was the first practical printing adding machine. By 1911, both the Monroe and Marchant Calculating Machines had been introduced, and by 1920 electric motor drives were incorporated into calculating machines.[9]

The General Electric Company and the Westinghouse Company were the principal contributors to the network analyzers for the simulation of power networks. The first of these D. C. Network Analyzers was patented in 1925, and was a modern piece of equipment, with a console control unit. This system was a resistive analog; therefore, it was limited to problems which were either purely reactive or purely resistive, as long as they were steady-state. In 1929, an A. C. Network Analyzer, a much more versatile machine, was introduced. Because various components of this analyzer could be exchanged so easily, it could simulate both linear and non-linear problems. This gave it the true flexibility of a general purpose computer.[10]

During World War I, Hannibal Ford improved mechanical integrating devices when he increased the torque output of the common fall-and-disc integrator to make possible a naval gun-fire computer. This was followed by more experimentation in the 1920's at the Massachusetts Institute of Technology. This experimentation led to Dr. Vannevar Bush's

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[9] Stibitz and Larrivell, op. cit., p. 152-53.

[10] "High Speed Computing Devices," Engineering Research Associates, Inc. (New York: McGraw Hill Book Company, Inc., 1950), p. 187.

completion of the first large scale mechanical differential analyzer in 1931. The machine is still being effectively used at Wayne University, Detroit, Michigan.

Harmonic analyzers and simultaneous equation solvers of many types appeared during the 1930's. Some of which are: R. M. Mallock's electrical machine for solving simultaneous equations; Wilbur's Mechanism, a mechanical machine for the same purpose; an adjustor type equation solver developed by Berry and his associates; and the multi-harmonigraph, a mechanical harmonic analyzer developed by Brown.[11]

The first large scale general purpose digital computer was completed at Harvard University, Cambridge, Massachusetts, in 1944. This machine, the Harvard Mark I Calculator, was built jointly by Harvard University and the International Business Machines Company. The Mark I was primarily a relay computer since relays were used for the arithematic section. For the read-in and memory, punched cards were used. The first of all relay computers was built by Bell Telephone Laboratories in 1940. This was called the BTL I. By 1944 the BTL V was completed, which was the first general purpose relay computer. Also, around 1944, the Moore School of Engineering completed an all-electronic digital computer for the Aberdeen Proving Grounds. This computer, the ENIAC was the forerunner of such computers as the SWAG, SEAC, MANIAC, etc.[12]

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[11] Soroka, op. cit., pp. 161-65.

[12] Stibitz and Larrivell, op. cit., pp. 153-55.

The operational amplifier has made possible the development of the general purpose analog computer. The George A. Philbrick Researchers, Inc., was probably the first to develop and use such an amplifier as far back as 1838; however, Lovell of the Bell Telephone Laboratories is generally credited with its introduction during World War II. Such computers as the EASE, BOEING, GAP/R, GEDA, IDA, PACE, DONNER, and REAC are descendants of the operational amplifier. Also, the Heath Analog Computer, with which this thesis will deal, has fifteen such high-gain amplifiers. Many special purpose analog computers such as those for gun directors and automatic aircraft control systems use this amplifier.[13] The UNIVAC, produced by what is now the Sperry Rand Corporation, was the first mass-produced computer and was placed on the market in 1951. It was a decimal machine, which used magnetic tapes and a mercury memory. In 1953, the IBM 701 appeared; it used binary members and electrostatic storage. These new features gave it speed and a very rapid access characteristic.

The Whirlwind I, built by the Massachusetts Institute of Technology, was the first large machine to use magnetic cores for a main memory. At the present such machines as the UNIVAC, IBM 704, 705, and 709, and many others use the magnetic core principles.[14]

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[13] Clarence L. Johnson, Analog Computer Techniques (New York: McGraw-Hill Book Company, Inc., 1956), pp. 1-2.

[14] McCraken, *op. cit.*, pp. 9-10.

The IBM Company, Sperry Rand, Inc., Reeves Instrument Corporation, Goodyear Aircraft Corporation and many other companies have made vast strides in the development and improvement of computers. At the time of this study, the research is so intense that by the time a machine is manufactured, many of its characteristics are obsolete.

Even though one may conceive of the basic idea of computers going as far back as the beginning of civilization, he must remember that the very ground-work of the present systems only go back to just before the days of World War II, which is some twenty years ago.[15]

Classification of Computing Devices

Up to this point, mention has been made of various classes of computers, and the subdivisions of these classes as various machines were conceived and developed. Now it is important to classify and define these computers.

There are two major classes of computing devices. These are the digital computer which deals with discrete numbers, and the analog computer which deals with continuous physical variables. A combination of the two are often referred to as the hybrid, but it is not a major class of computers. The digital computer works directly with integers which are expressed by gear teeth in the desk calculator or electrical impulses in the electronic digital computer.

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[15] Franz L. Alt, <u>Electronic Digital Computers</u> (New York: Academic Press, Inc., 1958), p. 17.

The desk calculator adds by the addition of revolutions or tenths of revolutions of the gear, while multiplication is carried out by an extension of the addition principle. The digital computer has addition as its basic function and all other complex arithematic is described in a logic based on this concept. The power of the digital machine lies in the speed with which the machine will add. To solve more complex problems such as the solution of differential equations, the machine computes by repeated refining of an approximation. The accuracy or precision of a digital computer can be greatly increased by sacrificing speed.[16]

The digital computer can be classified as one of two subdivisions. These are the general purpose and the special purpose machines. Each of these subdivisions can be classified into either mechanical, electrical or electromechanical. (See figure 1, page 11.) The only true all-mechanical general purpose computer is probably the machine made by Charles Babbage called the "Analytical Engine." The electromechanical computers were those modifications of Babbage's idea after more knowledge and concepts of electricity were known and available. The digital machines most frequently referred to today are electrical special purpose digital computers.[17]

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[16] George H. Philbrick and Henry M. Paynter, The Electronic Analog Computer as a Lab Tool (Boston: Industrial Laboratories, May, 1952), p. 1-4.

[17] Jerry Roedel, "An Introduction to Analog Computers," ISA Journal, I (August, 1954), pp. 9-15.

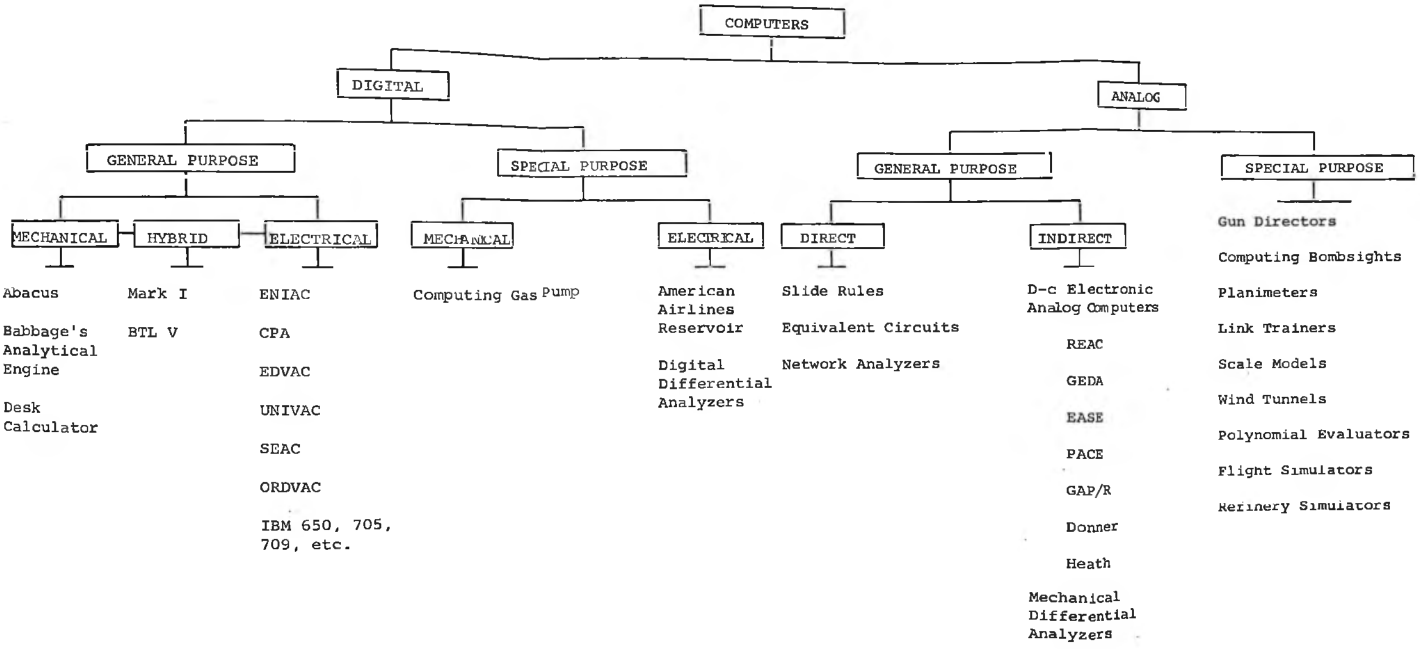

Fig. 1 - Classification of Computers

The electronic analog computer works with continuous physical variables, where one set of units represents another, such as volts representing distance or force. The analog is the dimension or quantity that is represented by the electrical units, usually voltages. At any specific point in the machine solution, the analog is always the physical quantity that is represented by the voltages that are being developed in the machine circuits.

The analog computer is most applicable to the analysis of continuous variables, with time as the independent variable. By the use of potentiometers and high gain d. c. amplifiers with condensers and resistors for input and feedback impedances, the d. c. analog computer can add, subtract, multiply by a constant, integrate, and differentiate.

All of the indirect electronic analog computers operate on the same principle; however, the large analog computers have all the computing components built into the computer. This adds convenience to the operation of problem solving, but at the same time costs are increased, because of the added complexity in manufacturing. Such computers are referred to as large-scale machines, while the computers which have external patch cord connected computing components, thus making the machines cheaper and smaller, are called small-scale machines. The small-scale machines are found in some universities and small companies; while the large-scale machines are found in large centralized computing departments of industrial facilities, government laboratories, other universities, etc.

By the use of scale factors, an analog computer can be made to deal with quantities that far surpass the voltage capacity of the machine, which is usually plus or minus 100 volts. The scale factors are the converting tools for the final answers. The analog at any point in the solution is the voltage at that point, multiplied by the scale-factor. This means that the analog computer must work in real time, which is a characteristic of this machine. An ordinary slide rule is a classical example of an indirect analog type analog computer. The slide rule operates by using linear distances, to represent the logarithms of parameters of a system. By adding the logarithmic distances it multiplies, by subtracting it divides.

The electronic analog computer deals with two classifications of time when scale factors are employed. These time classifications are real, or problem time, and machine time. The problem time is the speed at which the problem variables change in the actual system. The machine time is the time at which the variables are changed in the machine solution. For example, when a scale factor of one is used the problem time equals the machine time, and for a scale factor of one-tenth, the problem time is ten times faster than the machine time. Also, amplitude scale changes must be made in order to keep the maximum and minimum outputs between the voltage limits of the computer. The limits are usually plus or minus 100 volts. In essence, an amplitude scale change is multiplying both sides of the equation by the same number. The magnitude of the number depends on the need for larger or smaller amplitude change. For most computers it is desirable to have a maximum amplitude of plus or minus 50 volts. This gives voltages large enough to measure adequately on the voltmeter and at the same time, is below the maximum limits. The time scale desirable for good solution depends upon the type of recording equipment is used for the output. For an X-Y Recorder, the problem frequency should be approximately one cycle per second, with a horizontal recording speed of one inch per second.

A chart showing the relative costs of digital and analog computers of various sizes is shown in Figure 1, page 11. This shows how computers are classified, and shows some of the computers available for lease or purchase. Also it shows where the machine considered in this thesis fits into the general classification of computers. For example, the Heath computer is an indirect analog computer which comes under the classification of general purpose analog computers.

The analog computer's accuracy is in direct proportion to its cost, while the cost of digital computers does not vary much as the accuracy is increased. On the other hand, analog computers are initially extremely cheaper, (see Figure 2, page 15 for a relative cost versus accuracy of analog and digital computers.) Both machines are only as

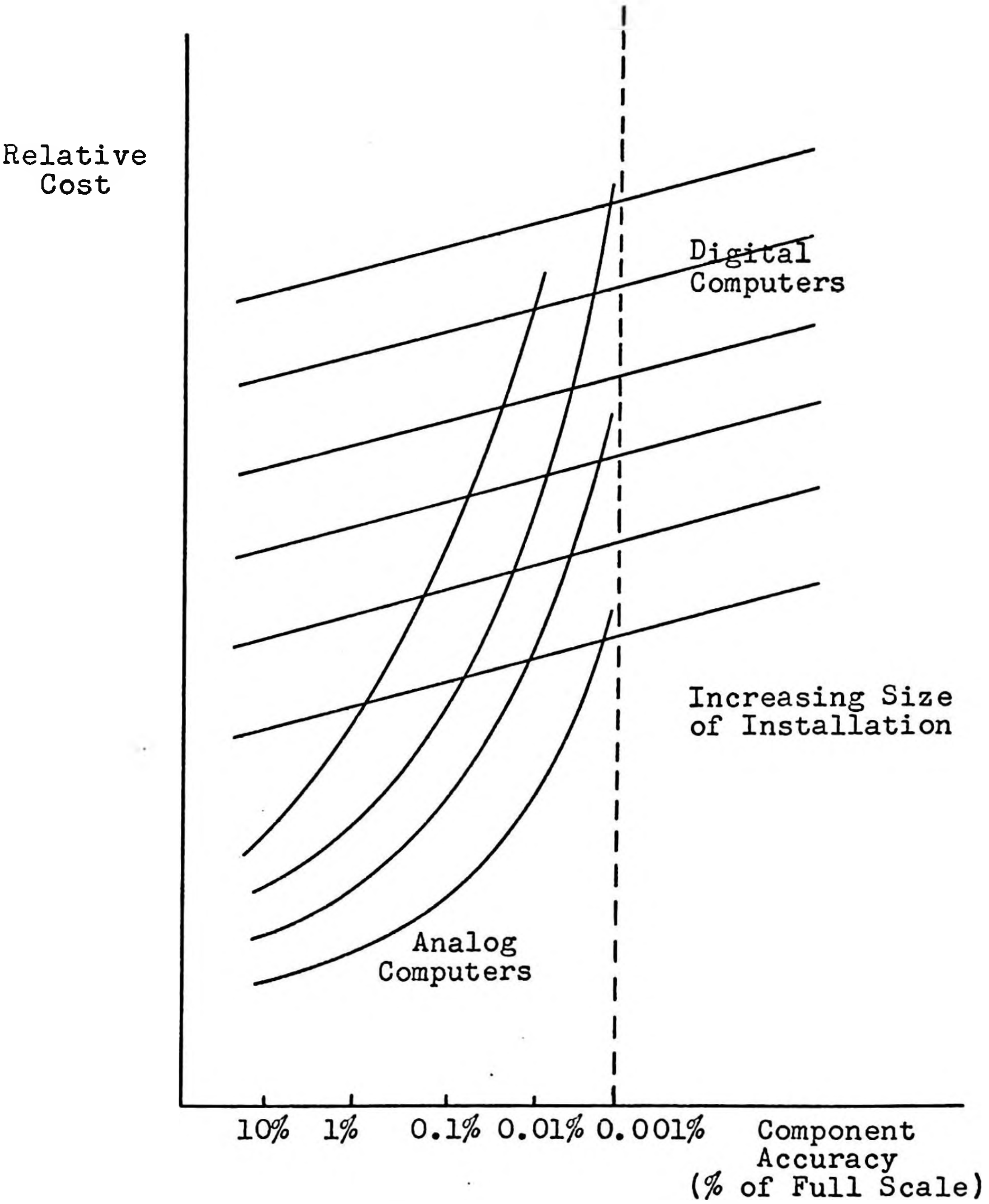

Fig. 2 - Relative Cost vs. Accuracy of Analog and Digital Computers for Different Sizes of Installations.

SOURCE: G. A. Korn and T. M. Korn, <u>Electronic Analog Computers</u> (New York: McGraw-Hill Book Company, Inc., 1956), p. 5.

accurate as their input data or calibration; thus, the digital machine can be made much more accurate by increasing accuracy of input data, while the analog machine would rapidly approach its accuracy limit, under the same conditions, with more accurate calibrations.[18]

Objective of This Study

The objective of this study is to determine the applicability of a small-scale or a relatively inexpensive electronic analog computer to industrial engineering problems. The study is not meant to include a series of recondite problems, but it is meant to consist of a selection of general problems that link together to make up the general tools of the industrial engineering profession. The study is meant to determine what such a computer can do for industrial engineering analysis. The problems are defined, discussed numerically, and programmed if possible for analog computer solution. The programs will concur with programming techniques for analog computers that use patch components, as does the machine considered herein. After the problems are experimentally tested for analog computer solution, a discussion of the advantages and/or disadvantages of such solution methods is presented. The study stays within the parameters of the existing facilities, in terms of equipment, at all times.

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[18] Granine A. Korn and Theresa M. Korn, Electronic Analog Computers (New York: McGraw-Hill Book Company, Inc., 1956), pp. 5-10.

The study is also to instill an interest in analog computers, by making available to the industrial engineering department a small-scale electronic analog computer, and through the efforts of this research, make recommendations of auxiliary equipment that can be adequately linked with the available facilities, so that future inabilities of the industrial engineer to solve his problems, will be due to the inherent capabilities and limitations of the electronic computer and not to the lack of equipment.

CHAPTER II

INDUSTRIAL APPLICATIONS OF ELECTRONIC ANALOG COMPUTERS

Introduction

Electronic analog computers have found many applications in industry. The main advantages of using the analog computer over manual computational methods are:

1. Speeds up the solution of problems. Voltages can be forced as directed by the programmed components in seconds; thus, the only time consumed is the set-up time.

2. Takes into account more variables. When solving mathematical and engineering problems, many formulae must be greatly simplified before they can be solved by normal conventional method. By using the analog computer, complex variables are almost as easy to simulate as the simpler ones.

3. Makes available many solutions that would not otherwise be economical or feasible. Some problems are so lengthy and involved that it is not feasible for an engineer to solve them; the computer may yield such solutions at a low unit cost.

4. Does "experiments" without the necessity of model-building. It is possible to simulate systems on the analog computer to determine which variables to stabilize or vary, or which combination of the variables to use, without the necessity of building an experimental model.[1]

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[1] McCraken, op. cit., pp. 11-12.

The foregoing advantages lend themselves to many types of industrial problems. Some of which have already been proven feasible and simulated for analog computer solution are:

1. Fluid flow problems.[2]
2. Dynamic heat transfer problems.[3]
3. Bomb-trajectory problems.[4]
4. Problems in atmospheric diffusion of particles.[5]
5. Automatic control systems.[6]
6. Automobile suspension systems.[7]
7. Study of escillator circuits.[8]
8. Solution of finite difference equations.[9]

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[2] John E. Nolan, "Analog Computers and their Applications to Heat Transfer and Fluid Flow," Computers and Automation, December, 1954.

[3] G. D. McCann, Jr., and C. H. Wilts, "Application of Electric-Analog Computers to Heat-Transfer and Fluid Flow Problems," Journal of Applied Mechanics, September, 1949, pp. 247-58.

[4] Johnson, op. cit., pp. 83-86.

[5] Walter J. Karplus, Analog Simulation (New York: McGraw-Hill Book Company, Inc., 1958), pp. 361-65.

[6] D. McDonald, "Analog Computers for Servo Problems," Review of Scientific Instruments, Vol. 21, No. 2, February, 1950, pp. 154-57.

[7] "Geda Analysis of a Standard Automobile Suspension System," Goodyear Aircraft Corporation Report No. GER-5262. March 12, 1953.

[8] Han Chang, R. C. Lathrop and V. C. Rideout, "The Study of Oscillator Circuits by Analog Computer Methods," Proceedings of the National Electronics Conference, 1950.

[9] Walter J. Karplus, op. cit., pp. 260-64.

By the use of function multipliers, servo-mechanisms, function generators, and various other auxiliary equipment, the linear and non linear ordinary differential equations and partial differential equations which describe the above problems have been simulated. The techniques of simulating problems on the analog computer requires a good understanding of how the problem would be solved by conventional methods, and perhaps, an estimate of approximately what the solution would be. The latter is not completely necessary, but it adds confidence to the machine solutions. An engineer can learn the fundamentals of analog computers without major difficulty.

The first requisite for the analog computer simulation of mathematical models is that the model possess the flexibility that allows it to be adapted to the mathematics that is applicable to the computer. The most applicable mathematics for analog computer solution is in the form of a ordinary differential equation with constant coefficients.[10] The physical problems must be stated in the form of an equation or continuous mathematical model.

Basic Theory of the D-c Electronic Analog Computer Applications

Analog computers represent the problem variables by corresponding physical quantities, such as continuously variable voltages in the case of the electronic analog

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[10] Johnson, op. cit., p. 181.

computer. The voltages are made to obey mathematical relations analogous to the problem, by their being found to vary according to the programmed circuits of the computer. In the case of the d-c analog computer, the machine variables are d-c voltages which may vary with time. Time is usually the independent variable, especially in the solution of ordinary differential equations.

The d-c analog computer can perform the functions of addition, subtraction, sign change, integration, differentiation, multiplication and division by a constant, and by the use of a function generator it can generate variables. A problem circuit is set-up on the computer patch panel by interconnecting the operational amplifier jacks with selected plug-in components and establishing bias voltages by the use of potentiometers, equivalent to the constants and initial conditions in the problem to be solved. The theory of these set-ups will be explained below, while some actual problems will be solved in Chapter IV.

The output voltage, e_o, of the following problem circuits must not exceed the voltage limits of the high-gain amplifier. When the input data does exceed the amplifier limits, the use of scale factors to reduce the physical magnitude of each input is the usual technique. This is often called an amplitude check and adjustment. The maximum limits of the Heath computer are plus and minus 100 volts.

Addition

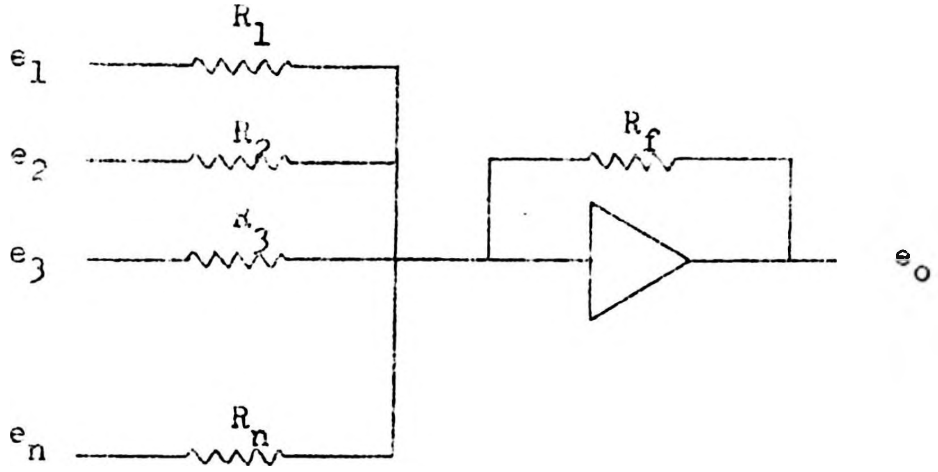

Fig. 3 - Schematic Diagram for Addition

The mathematical operations for the addition schematic are as follows:

$$e_o = -\left(e_1 \frac{R_f}{R_1} + e_2 \frac{R_f}{R_2} + e_3 \frac{R_f}{R_3} + \ldots + e_n \frac{R_f}{R_n}\right)$$

$$e_o = -\sum_{i=1}^{N} R_f \frac{e_i}{R_i}$$

The output voltage is equal to the sum of the input voltages. To add on the analog computer, both the input impedance and the feed back impedance have to be resistors. For the relationships between the feed back impedance, R_f, and the input impedances R_1k R_2, R_3, -- R_n, see Figure 5. The amplifier changes the sign of the input voltages, which accounts for the negative output.

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[11] *Ibid.*, p. 12.

Subtraction

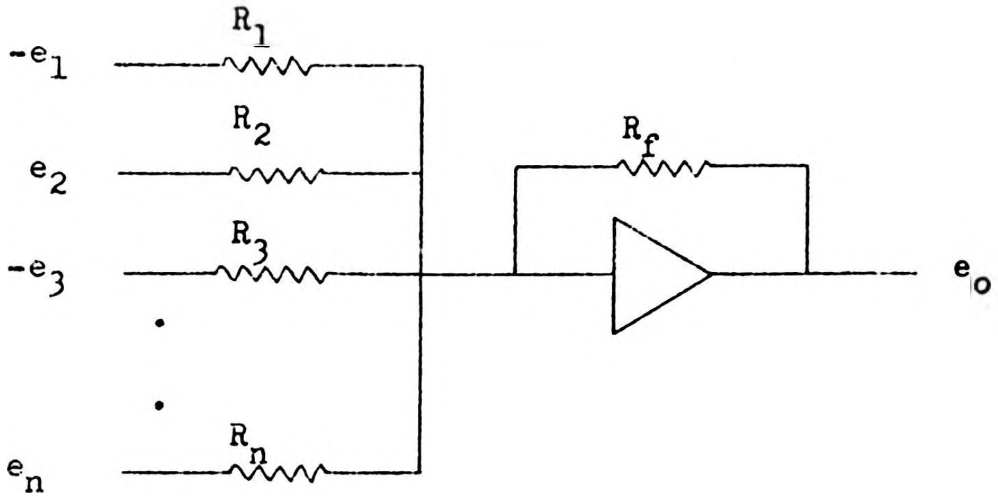

Fig. 4 - Schematic Diagram for Subtraction[12]

The mathematical operation for subtraction schematic are as follows:

$$e_o = -\left(-e_1 \frac{R_f}{R_1} + e_2 \frac{R_f}{R_2} - e_3 \frac{R_f}{R_3} + \cdot \cdot + e_n \frac{R_f}{R_n}\right)$$

$$e_o = e_1 \frac{R_f}{R_1} - e_2 \frac{R_f}{R_2} + e_3 \frac{R_f}{R_3} - \cdot \cdot \cdot - e_n \frac{R_f}{R_n}$$

The output voltage is equal to the algebraic sum of the input voltages. The circuit is essentially the same as the addition circuit, the negative quantities are algebraically added as in Figure 3. The effect of these input quantities on the output can be varied by varying the input and feedback impedances, as shown in Figure 5.

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[12] Ibid.

Multiplication and Division by a Constant and Sign Change

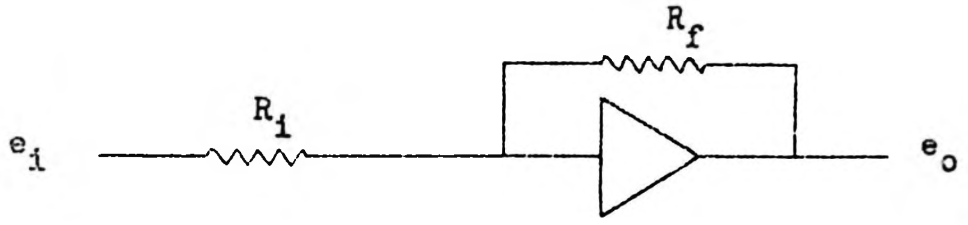

Fig. 5 - Schematic Diagram for Multiplication and Division by a Constant[13]

The mathematical operations for multiplication and division by a constant and sign change are as follows:

$$e_o = - e_i \frac{R_f}{R_i} = - K_1 e_i$$

If R_f is greater than R_i, K_1 is greater than one, or if R_f is less than R_i, K_1 is less than one. When K_1 is greater than one the operation is multiplication by a constant greater than unity, and when K_1 is less than one, the operation is division by a constant greater than unity. The output voltage of a high gain feedback amplifier always has a sign opposite to the input voltage, thus accomplishing the sign change.

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[13] *Ibid.*

Use of a Coefficient Potentiometer

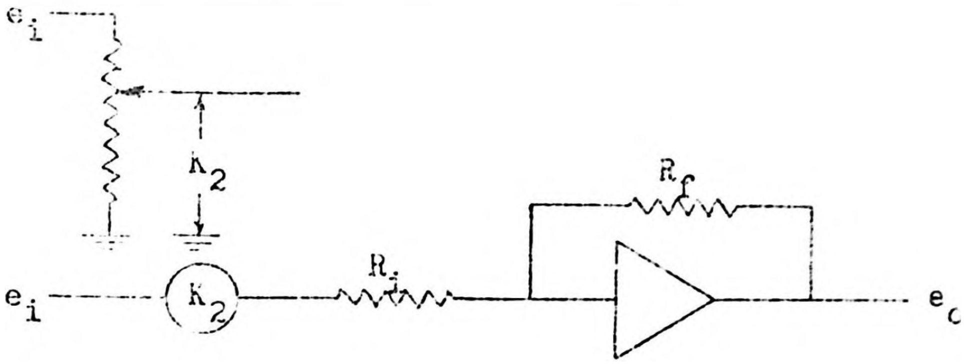

Fig. 6 - Schematic Diagram Showing the Use of a Coefficient Potentiometer[14]

The mathematical operations for the use of a Coefficient potentiometer are as follows:

$$e_o = - K_2 e_i \frac{R_f}{R_i}.$$

$$e_o = - K_1 K_2 e_i = - K_3 e_i$$

The output voltage of the variable resistor or coefficient potentiometer will be some fractional part of the input voltage, where the fraction K_2, can be varied from zero to one. The K_1 variable can be varied to value as large as the patch components permit, usually from 100 to 0.01. Then the variable, K_3, can be made to vary within any combination of the two. By using both features, the analog computer is capable of multiplying and dividing by any constant within the component or computer range. The coefficient potentiometer can be put in any of the schematics representing the basic theory of analog computer applications.

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[14] Ibid.

Integration

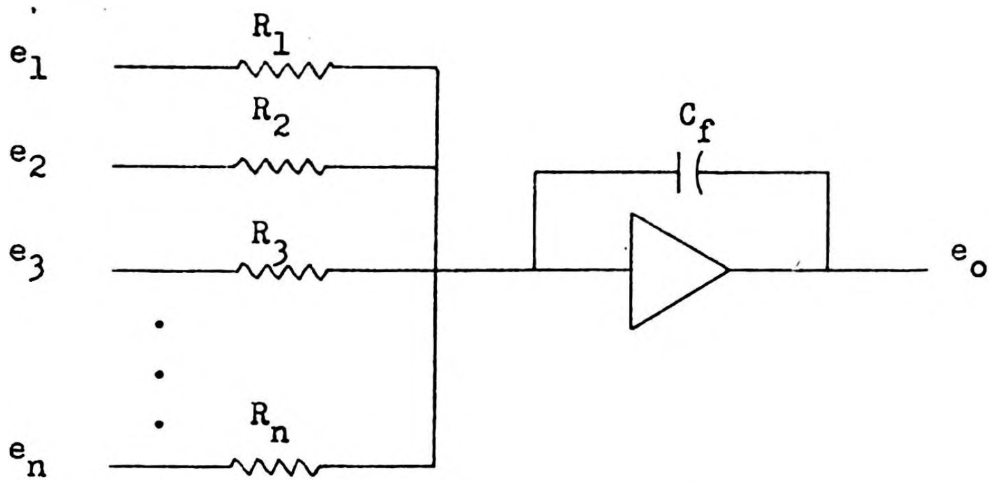

Fig. 7 - Schematic Diagram for Integration[15]

The mathematical operations for integration are as follows:

$$e_o = -\left(\frac{1}{R_1 \; C_f} \int_0^t e_1 dt + \frac{1 \; 1}{R_2 \; C_f} \int_0^t e_2 dt + \frac{1}{R_3 \; C_f} \int_0^t e_3 dt + \dots \right.$$

$$\left. + \frac{1}{R_n \; C_f} \int_0^t e_n ft \right)$$

$$e_o = - \int_0^t \sum_{i=1}^{N} \frac{1}{R_i \; C_f} \; e_i dt$$

The output voltage is equal to the sum of the integrals of the input voltages multiplied by the RC constant. For integration the feedback impedance is a capacitor. The integrator starts off at zero time and integrates to some time t. For this reason, the independent variable in the problem to be integrated must be expressed as a function of time.

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[15] *Ibid.*, p. 16.

Differentation

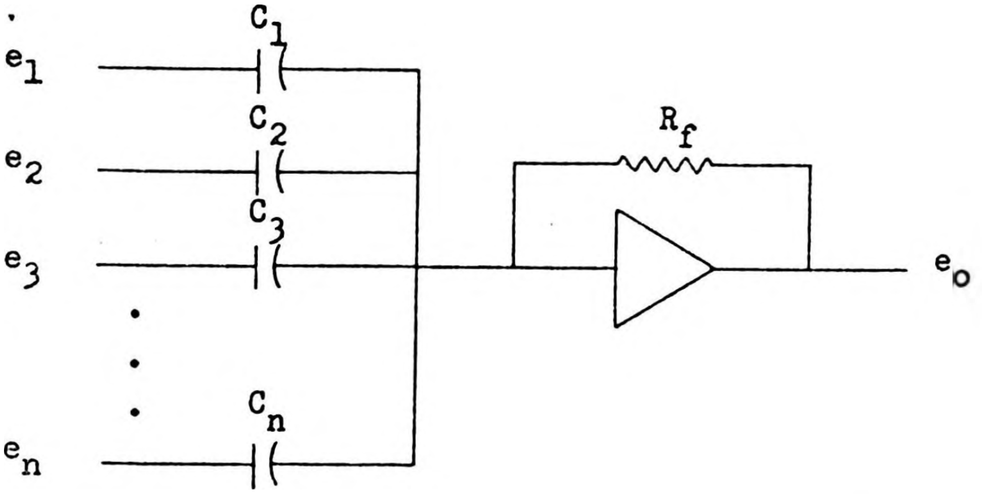

Fig. 8 - Schematic Diagram for Differentation[16]

The mathematical operations for the differentation schematic are as follows:

$$e_o = -\left(R_f\, C_1 \frac{de_1}{dt} + R_f\, C_3 \frac{de_3}{dt} + \ldots + R_f\, C_n \frac{de_n}{dt}\right)$$

$$e_o = -\sum_{i=1}^{N} C_i\, R_f \frac{de_i}{df}$$

The output voltage is equal to the sum of the time derivatives of the input voltages, each multiplied by the RC constant. For differentation, the input impedance is a capacitor, and the feed-back impedance is a registor. Differentation is not recommended at any time if possible, because of its noise-amplifying characteristics.

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[16] *Ibid.*, p. 11.

Many times, by simple mathematical manipulation the characteristic equation can be changed to one that must be integrated.[17]

An important part of analog computer simulation is the selection of the correct scale factor. The scale factor is the relationship of the machine voltage to the problem variable. In essence, it is the voltage divided by the problem variable. Whenever the output voltage in any of the foregoing schematic diagrams, exceeds the maximum or minimum limits, scale factors can be used to step-up or step-down the input votages to make the machine operate within the most accurate ranges. When this is done, the output voltage has to be converted to the real answer by multiplying by the scale factor constant.

The principle on which the high gain d-c amplifier works is the voltage through the amplifier is zero or some finite value because of the large gain of the amplifier. The gain is the output divided by the input. This feature routes the input voltage to the amplifier through a feed-back impedance and across the amplifier. The output voltage of an amplifier is then equal to the input voltage multiplied by the R-C constant of the circuit.

For versatile applications of the d-c analog computer, a variety of auxiliary equipment is necessary. This

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[17] *Ibid.*, pp. 49-50.

equipment requires the use and understanding of diodes, differential relays, servo multipliers, function multipliers, function generators, etc. The basic theory of this equipment, is beyond the scope of this study.[18]

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[18] The texts listed in the bibliography of this thesis, give a detailed discussion of all such characteristics of electronic analog computer components and the use of many kinds of auxiliary equipment.

CHAPTER III

RESEARCH PROCEDURE

Introduction

The objective of this study was first conceived by Professor Wyllys G. Stanton, when the author went to him for consultation on another prospective problem. A comprehensive literature survey revealed that there had not been anything published on the applications of analog computers to industrial engineering problems. The next problem was the difficulty that would be encountered in substantiating such a study without experimental data. The necessity of these data required an investigation into the possibility of acquiring a small-scale analog computer, since there was not one available on the campus. The thesis advisory committee gave the author the freedom to make a survey on the available electronic analog computers that could fill the requirements, yet remain within a $1500.00 budget that was set up by the United States Steel Foundation for equipment to be used in thesis research by the recipient of their scolarship.

In making the survey for the selection of the computer, many things were carefully considered because of the limitations in justifying a computer that was cheap enough, yet flexible enough to give adequate grounds for a decision. The main considerations were (1) cost, (2) accuracy, (3) convenience, (4) flexibility. The considerations are defined as follows:

1. Cost has a relationship to quality of components, complexity of design and size of facility.

2. Accuracy depends upon the quality of the components, quality of units such as power supplies, reference standard and passive R-C components. Accuracy may be increased by proper calibration before each run, but this makes the unit less convenient.

3. Convenience in general is inversely proportional to the time necessary to set up and reach a solution to a particular problem. A computer is considered more convenient with features such as predetermined integrators, summers, pushbutton potentiometer setting and removal patch board.

4. Flexibility refers to the availability of these computing elements. A computer is more flexible if the passive resistors and capacitors are not associated with a certain amplifier and may not be used as any computing impedance.[1]

Of the foregoing consideration, cost was the limiting factor. Of course, the other factors were evaluated within the budget parameter. Since the manufacturers of computers have been striving to reduce cost and increase accuracy, this proved to be advantageous in the selection of an accurate, low-cost, analog computer.

Selection of Equipment

After a literature survey on the types of analog

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[1] Chalmer E. Jones, "The Functional Considerations for a Low Cost Electronic Analog Computer," Heath Company Paper No. 55-25-3, p. 1.

computers that were available, two manufacturers were chosen as having the machines that fulfiled the requisites of this study. These manufacturers were the Heath Company, Bonton Harbor, Michigan, and the Donner Scientific Company, Concord, California. Each of these manufacturers' machines had their own advantages and disadvantages. The models finally taken into consideration were the Heath Electronic Analog Computer, Model ES-400 (Group C), and the Donner Analog Computers Model 3400 with Models 3430 problem board, 3011 - cooling fan unit, 3073 - potentiometer strip, and 3750 - function generator, to make up the full computer group. Each computer group required an auxiliary selection of electrical components for the external patch equipment.

The Heath Computer kit group listed for $945.00 and contained approximately the same advantages as did the Donner group for approximately $2,000.00.[2] The components that were available for the Donner group cost $452.00, while adequate components for the Heath group would cost $75.00. This difference was in the precision of the components.

A comparison of the Heath versus the Donner electronic Analog Computer is as follows:

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[2] Since the Heath computer was in kit form the labor and overhead of its assembly was not a part of the Company's price.

1. Fifteen amplifiers vs. ten amplifiers.
2. Thirty-two potentiometers vs. ten potentiometers.
3. Eight diodes vs. two diodes.
4. Unstable amplifiers vs. stable amplifiers.
5. One thousand dollars vs. two thousand dollars.
6. Six integrators vs. five integratros.[3]

Primarily because of the cost considerations, the Heath Computer was selected. The disadvantage of selecting this computer was the time required to assemble it. This assembly is covered in the next section.

The Heath Computer has many outstanding features, such as its ability to be calibrated at high accuracy. By the use of a null meter the following functions may be accurately measured:

1. Overall gain from input to output of amplifier.
2. Coefficient potentiometer setting.
3. Initial condition setting.
4. Bias diode setting.
5. Function generator setting.
6. Operational relay setting by means of throw voltages.[4]

See Appendices I and II for more questions, answers and particular features of the Heath Analog Computer.[5]

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[3] Letter from the Heath Company, March, 1958.

[4] Ibid.

[5] Appendices I and II were compiled from correspondence and brochures from the Heath Company, April, 1958.

Many of these questions and answers apply to any small-scale electronic analog computer.

Assembly of Equipment

As in the case with any electronic kit, the Heath Analog Computer had to be assembled. The assembly instructions were straight-forward and systematic, and required little knowledge of either electronics or the fundamental operations of the component parts assembled. The instruction manuals accompanying each component part of the computer included a parts check list. The parts check preceded the assembly of each major component and served as an introduction to the electrical terminology of the kit prior to its assembly.

A list of the major components assembled, together with their functions and features are presented in Appendix II, page 71. The assembly of these components, including their subsequent integration and check-out with each other, took the author approximately 300 hours. An experienced technician with ample resources and adequate consecutive time could probably have done the same job in in approximately 225 hours.

The most apparent weakness of the entire assembly procedure was the lack of instructions on the precedent each component part should take in the assembly series.

Procedure in Selecting Experimental Problems

Since the objective of this research was to determine the applicability of the electronic analog computer to problems in industrial engineering, the procedure in selecting the experimental problems for analysis was to determine the fields of industrial engineering encountered most frequently, then choose general problems from these fields as experimental problems. These areas of industrial engineering are explicitly presented, with the selected problems defined.

The broad scope of industrial engineering has various meanings--to non-industrial engineers as well as to well trained industrial engineers. A definition selected and accepted by the American Institute of Industrial Engineers is as follows:

> Industrial engineering is concerned with the design, improvement, and installation of integrated systems of men, materials and equipment; drawing upon specialized knowledge and skill in the mathematical, physical, and social sciences together with the principles and methods of engineering analysis and design, to specify, predict, and evaluate the results to be obtained from such systems.[6]

Some of the fields of industrial engineering that are most frequently encountered are:

1. Methods engineering.
2. Statistical analysis.

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[6] Official definition adopted by the American Institute of Industrial Engineers.

3. Production and inventory control.
4. Job evaluation.
5. Plant layout.
6. Operations research.

From these major fields, typical problems are selected, defined, and simulated, if possible, for analog computer solution. This research is shown in Chapter IV.

Procedure in Analyzing Experimental Data

The conventional solution procedures of the selected problems are discussed and in some cases they are partially solved, so that the numerical solution procedure can be seen. Many of the complete numerical solutions are voluminous and trivial to the industrial engineer, and would not serve any advantage as detailed solutions. For the problems that can be simulated on the computer, the numerical procedure will be discussed along with the analog computer problem schematic for the problem simulation.

By a comparison of what was involved in solving the experimental problems and what the small-scale electronic analog computer was capable of solving, each problem is discussed in relationship to its applicability to the computer. The main objective at all times is to find what such a computer can do as an analysis tool for industrial engineering problems. The conclusions of the research are presented with each analysis.

Method of Solution

In solving any industrial engineering problems, a systematic procedure should be followed. The procedure that has proven satisfactory for the solution of problems on the analog computer is tabulated below. This procedure not only enhances organization in the computer solution, but it also leaves that kind of solution that can be easily filed for future reference. In actual practice it is suggested that such a procedure be followed:

1. Problem title.
2. Description of problem (include diagrams or schematics that help in describing problem).
3. The system equation or mathematical model of the system.
4. A check solution.
5. The range of the problem variables.
6. The range over which the problem variables will be varied (this is often an estimation).
7. The time-scale change to be made.
8. The new mathematical model after the time-scale has been made.
9. The amplitude scale adjustment to prevent overloading the amplifiers.
10. The transformation of the problem to machine language (circuit diagram).
11. Tabulation of potentiometer settings and initial conditions parameters.
12. Tabulation of machine runs or solutions.
13. Discussion and interpretation of results.[7]

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[7] Johnson, op. cit., p. 42.

The author found that every experimental problem had its own unique characteristics, but the solution procedure was helpful as a constant check during the problem simulation.

The one thing that is laborious is the task of getting each problem into the mathematical model that fits the computer. Often this will entail more than manually working the problem by conventional means. Any advantage in this case would depend on the quantity of problems that needed to be solved using the same model, with varying parameters or input data.

CHAPTER IV

ANALYSIS OF EXPERIMENTAL PROBLEMS

Introduction

The purpose of this analysis is to take each problem in its applicable field and discuss the possibility of solving it on the small-scale electronic analog computer and any resulting advantage of such a method of solution. The first requisite for the analog computer simulation of a problem is for the problem equation to possess the flexibility that enables it to be programmed on the computer problem board. The following analysis will consist of several general problems in industrial engineering, their manual conventional solution procedure, and the method of solving them on the analog computer.

Definition of Selected Experimental Problems

The experimental problems were selected from six of the main fields of the industrial engineering profession. These fields may be defined in the following manner:

Methods Engineering

Methods engineering is the use of those tools of industrial engineering that are concerned with the planning and perfecting of more effective work methods. It involves the introduction of improvements that will result in optimizing the overall job.[1]

Statistical Methods

Statistics are data or figures that enable the industrial engineer to analyze situations, to study trends, and to make predictions about future events. A statistical study involves the collection, analysis, and interpretation of information about operations. Some statistical methods include such tools as:

1. Work sampling.
2. Forecasting.
3. Statistical quality control.
4. Time series analysis.
5. Correlation analysis.[2]

Production Control

Production control is the task of co-ordinating manufacturing activities in accordance with manufacturing plans so that preconceived schedules can be attained with optimum economy and efficiency.[3]

Production control can be thought of as monitoring production activities or regulating production. The functions

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[1] H. B. Maynard (ed.), Industrial Engineering Handbook (1st ed.; New York: McGraw-Hill Book Company, 1956), section 2, p. 3.

[2] *Ibid.*, section 8, p. 284.

[3] William Voris, Production Control (Homewood, Illinois: Richard D. Irwin, Inc., 1956), p. 3.

of production control are:

1. Planning
2. Scheduling
3. Dispatching
4. Progress inspection
5. Correction.[4]

Job Evaluation

Job evaluation is the complete operation of determining the value of an individual job in an organization in relation to the other jobs in the organization. It begins with job analysis to obtain a job description and includes relating the descriptions by some systems designed to determine the relative value of the jobs or groups of jobs. It also involves the pricing of these values by establishing minimum and maximum salaries for each group of jobs based on their relative value. The operation ends with the final checking of the resulting salary system.[5]

Plant Layout

Plant layout is the physical arrangement of industrial facilities. The arrangement may be existing or proposed, and includes the space necessary for material movement, storage, indirect laborers, operating equipment and services. The layout objective is to arrange all the

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[4] Ibid., p. 319-20.

[5] "Industrial Job Evaluation Systems, Department of Labor, U. S. Employment Service, p. 19.

manufacturing or operating facilities in such a manner that it is easy to operate, yet safe and satisfying for the employees concerned.[6]

Operations Research

Operations research is the name given to a method or system intended to apply the systematic analysis of organization by:

1. The use of both schematic and mathematical models.
2. The use of shift and rework operations.
3. The use of the multi-discipline approach.
4. The effort to analyze the overall approach.

It is characterized by finding out what the problem really is and the art of separating the analytic functions from the authoritative decisions.[7]

The techniques which make available the quantitative data for the operations researcher to use in his analyses are quite voluminous. However, most of these techniques are a direct part of industrial engineering, and makes the industrial engineer a vital member of the operations research team which utilizes the multi-discipline approach. These tools which most frequently confront industrial engineers in operations research analyses are:

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[6] Maynard, op. cit., Section 7, pp. 26-27.

[7] A definition given by Professor Wyllys G. Stanton during the fall semester of 1958, in I. E. 129-(Operations Research).

1. Linear programming
    - a. Simplex method
    - b. Distribution method
    - c. Profit preference method
    - d. Dynamic programming method
2. Statistical methods
    - a. Analysis of variance
    - b. Correlation analysis
    - c. Chi-square analysis
    - d. Test of significance
3. Queueing theory
4. Monte Carlo technique

Experimental Analysis Presentation

As can be seen in the definition of both the broad scope of industrial engineering and the six fields which comprise a majority of its applications, the problems are essentially concerned with analyzing quantitive data. The numerical problems most frequently encountered are those involving economic evaluations and statistical analysis. For example, Table 1 shows a grouped distribution of data representing a pace rating made by forty time study engineers. There data must be analyzed by using statistical techniques so more information can be acquired. The mean, a measure of central tendency, was the first analysis made. The best method for finding the mean on the analog computer was the formula following the table.

TABLE 1

Pace Ratings

Range of Ratings	Frequency
75-85	5
85-95	8
95-105	14
105-115	8
115-125	5

$$\bar{X} = \sum_{i=1}^{N} \frac{F_i X_i}{N}$$

where

$\bar{X}$ = The mean

F = Frequency

X = Interval midpoints of the grouped data

N = Sample size.

An analytical solution of the data consists of the following:

$$\bar{X} = \frac{F_i}{N}(X_i) + \frac{F_2}{N}(X_2) + \frac{F_3}{N}(X_3) + \frac{F_4}{N}(X_4) + \frac{F_5}{N}(X_5)$$

$$= \frac{5}{40}(80) + \frac{8}{40}(90) + \frac{14}{40}(100) + \frac{8}{40}(110) + \frac{5}{40}(120)$$

$$= 100 .$$

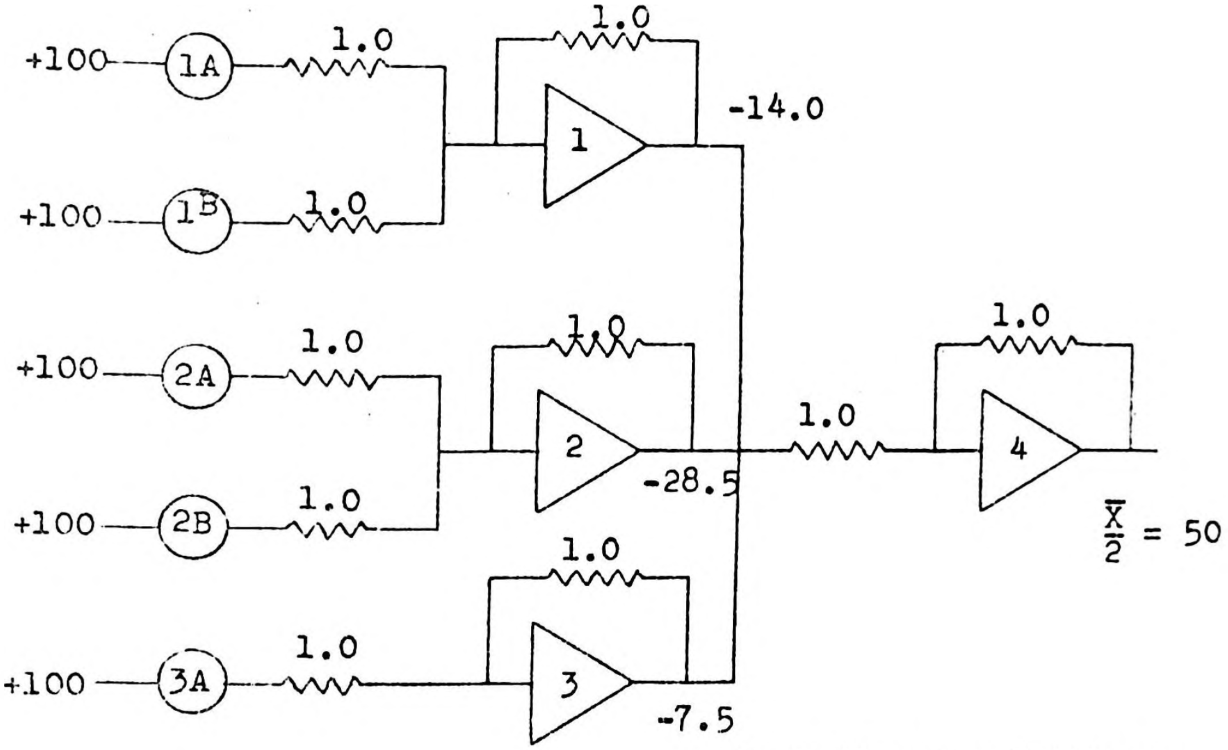
This procedure was programmed on the analog computer by using all summing amplifiers. The quotient of $\frac{F}{N}$ is the relative frequency, thus was always less than unity. This quotient was set on the potentiometer and connected to the summing amplifiers to give the product of the frequency divided by the sample size, times the midpoint of the interval of grouped data. Since the Heath Computer has only 100 volt sources available, the amplitude scale had to be adjusted. Dividing the equation by a factor of two gave a maximum amplitude equal to 60 volts and a minimum amplitude equal to 40 volts. The potentiometers volts also had to reduce the voltages from 100 volts to their desired quantity. This caused the potentiometer to have to be set on a value which was the product of the relative frequency multiplied by the voltage reduction factor.

The adjusted equation was

$$\overline{X}/2 = (5/40)(80/2) + (8/40)(90/2) + (14/40)(100/2) + (8/40)(110/2) + (5/40)(120/2)$$

which was programmed as shown in Figure 9, page 46.

Theoretically the mean of the distribution on the computer should have been 100. However, the computer's answer was approximately 3.5 per cent in error. The computer answer that was acquired after very close and precise calibration was 96.5. Apparently this error was caused by (1) extra loading on the potentiometer which was the effect of having the resistor in series with the potentiometers, (2) the error of setting the potentiometers on such low values, and (3) the use of only summing amplifiers. The latter problem was taken care of in another program, where a relationship of the feed-back resistance and the input resistance formed a division circuit, i.e. it was



Potentiometer Settings

1A (5/40)(40/100) = 0.05

1B (8/40)(45/100) = 0.09

2A (14/40)(50/100) = 0.175

2B (8/40)(55/100) = 0.11

3A (5/40)(60/100) = 0.075

Fig. 9 - Schematic Diagram for the Calculation of a Distribution Mean (Resistors are in megohms).

divided by a constant as shown in Figure 5, page 24. This latter program still required the potentiometers to be set on a product value; however, if the quantity and ratings of the patch resistors were not the limitation, the problem could be programmed in the manner described in Figure 10, page 48.

The programs presented in Figures 9 and 10 could have been modified by the use of components that were available. The available resistor ratios that could have been used with the Heath Computer can be represented by combinations of the following resistors as the input and feed-back impedancies for the amplifiers: 0.1, 0.2, 0.5, 1.0, 2.0, 5.0, and 10.0 megohms. By considering each input as a special problem, the input and feed-back components can be varied to give the potentiomenters settings less restraints.

Another characteristic of the distribution of pace ratings that needed to be determined was the measure of dispersion. This measure is called the standard deviation, which when squared is the variance. The formula for finding the standard deviation is:

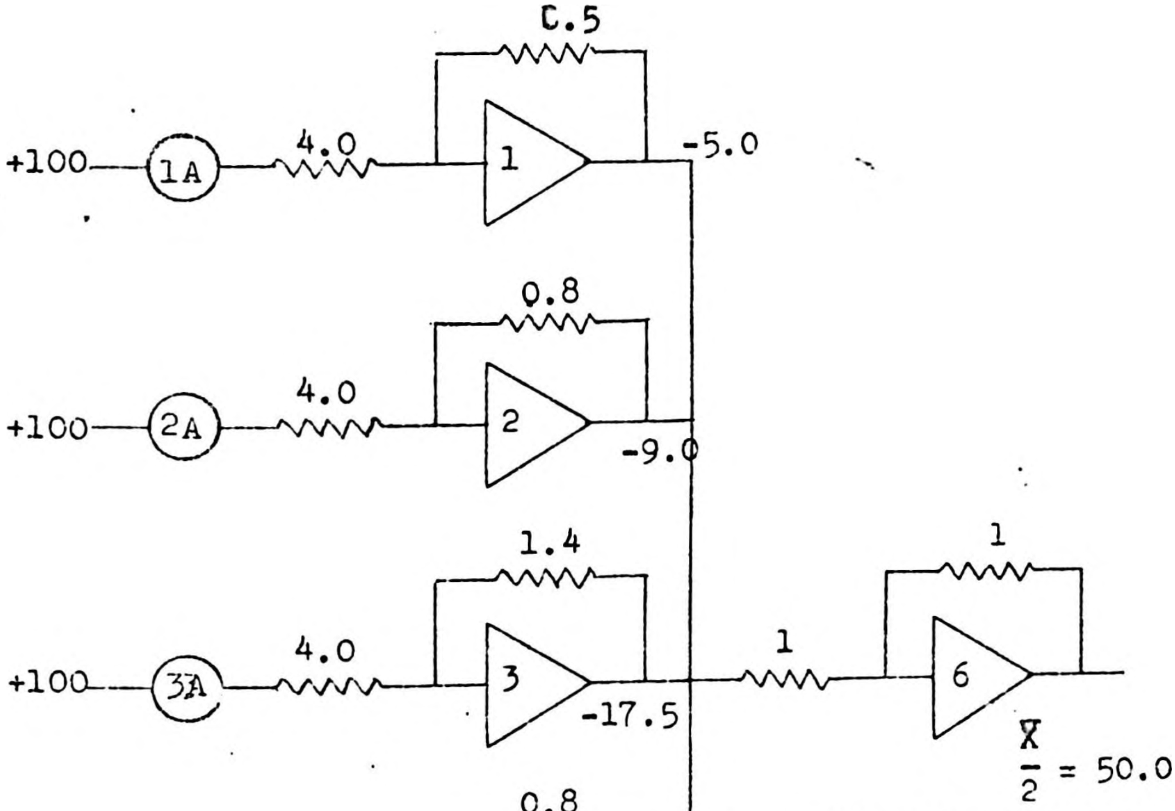
$$\sigma = \sqrt{\sum_{i=1}^{N} \frac{F_i X_i^2}{N} - \left(\sum_{i=1}^{N} \frac{F_i X_i}{N} \right)^2}$$

Where

σ = Standard deviation

F = Frequency

X = Interval midpoints of the grouped data.



Potentiometer Settings

1A	0.40
2A	0.45
3A	0.50
4A	0.55
5A	0.60

Fig. 10 - Schematic Diagram for the Calculation of a Distribution Mean without Component Limitations (Resistors are in megohms).

Both sides of the formula for finding the standard deviation was squared, which gave the variance of the distribution.

Then,

$$\sigma^2 = \sum_{i=1}^{N} \frac{F_i X_i^{\,2}}{N} - \left(\sum_{i=1}^{N} \frac{F_i X_i}{N} \right)^2$$

where,

σ^2 = Variance.

The second term in the formula for finding the variance was the same thing that was used for finding the mean, except that for the second measure.

Then,

$$\sigma^2 = \sum_{i=1}^{N} \frac{F_1 X_i^{\,2}}{N} - X^2$$

The mathematical procedure for numerically finding the variance was as follows:

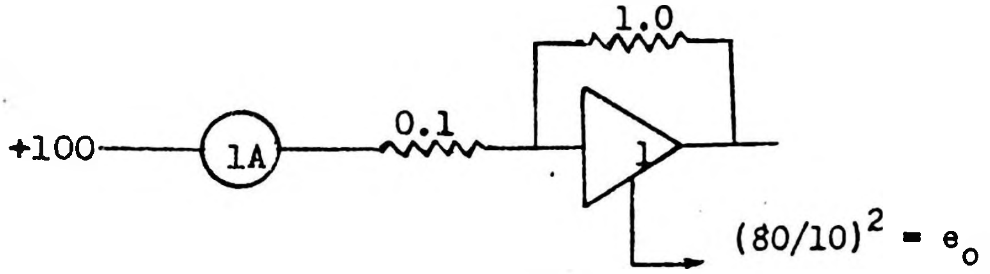
$$\sigma^2 = \frac{F_1}{N}(X_1)^2 + \frac{F_2}{N}(X_2)^2 + \frac{F_3}{N}(X_3)^2 + \frac{F_4}{N}(X_4)^2 + \frac{F_5}{N}(X_5)^2 - (\bar{X})^2$$

By substituting the data from Table 1, Page 44, into the above equation the following solution was obtained:

$$\sigma^2 = (5/40)(80)^2 + (8/40)(90)^2 + (14/40)(100)^2 + (8/40)(110)^2 + (5/40)(120)^2 - (100)^2 = 140$$

$\sigma = \sqrt{140}$ = Standard deviation

An analog computer program to solve such numerical operations was unrealistic, since each number had to be squared and multiplied by its relative frequency. The squaring of these numbers could only be accomplished by adjusting the input and feed-back resistors and the coefficient potentiometers so that their product would equal the magnitude of the number to be squared. For example, in finding the square of eighty, the following procedure was necessary. It was obvious that the answer would be greater than 100; therefore, a scale factor had to be used. Ten was selected because eight squared was less than 100, which is the maximum voltage limit of the computer. The program was as follows :



Potentiometer Setting

(1A) $(0.08)(0.8) = 0.064$

Fig. 11 - Schematic for Squaring Eighty

$$e_o = -K\,(R_f)/(R_1)e_i = -(0.064)(1.0/0.1)(100) = -64 \text{ volts}$$

Then, $(80)^2 = -(10)^2(64) = -6400$

For the case in Figure 11, the potentiometer could have been set at 0.64, and the ratio of the input resistor and the feed-back resistor would have been one.

Then

$$e_o = -\,0.64\ (100) = -64$$

and $(80)^2 = 100(64) = 6400$

By the time that all the manipulations were figured for squaring the numbers, multiplying the squares by the frequencies and then dividing the product by the sample size, more numerical calculations had been performed by the manual methods than was left to do on the analog computer. Such an operational procedure was necessary for each group of data.

Such problems as have been presented in the foregoing analysis are typical of statistical analysis techniques. Other techniques such as generating distribution curves or setting up tests of significance entail essentiality the same operations. For example, in computing an analysis of variance test, the operations include finding the variances of two or more classifications of data, so that a comparison can be made. Also, in considering the binomial and Possion distributions there was a need for the computer to evaluate factorials of numbers. For example, in finding the number of arrangements that can be made with N things taken r at a time is,

$$P_r^N = \frac{N!}{(N-r)!} = \frac{N(N-1)(N-2)(N-r)!}{(N-r)!}$$

if

$N = 20$

and $r = 5$

Then

$$P_5^{20} = \frac{(20)(19)(18)(17)(20-5)!}{(20-5)!}$$

The computer could not make the cancellations nor the large multiplcations without a vast number of numerical calculations for the component settings and extensive computer programs. Even in these extreme cases, the answer could not be relied upon, because of the error in both computing and calibrating the values for the computer components.

From the experimental results of the foregoing statistical methods, it is concluded that theanalog computer is not a data analyzer, because it has (1) no storage, thus no memory (2) low accuracy, when the problem setup consists only of summing amplifiers with extensive use of potentiometers, (3) limited means of multiplying and dividing by a variety of constants, and (4) limited problem board space for complex problems. The data used was a simple case, far from the average run of problems encountered in actual practice.

As mentioned earlier, the industrial engineer is also confronted with engineering economics evaluations. Such problems are encountered in operations research and production control. Most of these problems are problems in accounting, or problems involving the evaluations of alternatives in terms of dollars per period. Such problems employ simple arithmetic and logic; however, some problems are concerned with minimum cost points of variable data. These problems can be worked into the form of an ordinary differential equation by differentiating the algebraic relationship. The following problem was one used as an experiment to determine what the small-scale analog computer can do in this area.

$$Y = NC + XD + \frac{NCI}{2X} + \frac{NT}{X}$$

Where

Y = All cost incident to the year's needs

N = Number of units of the material under consideration that will be needed during the year

C = Purchase price per unit

X = Number of lots ordered per year = N/n

N = Quantity per lot = N/x

D = Order cost per lot in per cent, per year

I = Carry charge in per cent,per year.

T = Cost of storage per unit, per year.

The normal method of finding the minimum cost point is to differentiate the polynomial one time, set the derivative equal to zero and solve for the value of X. This value of X would be that point on the abscissa where the slope of the line would equal to zero.

The problem was programed for the analog computer in the following manner.

$$Y = NC + XD + \frac{NCI}{2X} + \frac{NT}{X}$$

$$\frac{dY}{dX} = D - \frac{NCI}{2X^2} - \frac{NT}{X^2}$$

$$\frac{d^2Y}{DX^2} = \frac{2NCI}{X^3} + \frac{2NT}{X^3} = \frac{2N\ (CI + T)}{X^3}$$

The polynomial was differentiated twice to get it into simple form. The following data were used to make the problem practical:

N = 2000 units

C = \$1.00

D = \$10.00

I = 10%

T = \$ 0.10

Then, $\frac{d^2Y}{DX^2} = \frac{\$800.00}{X^3}$

Since the independent variable of the problem has to be expressed as a function of time, X had to be zero at the beginning of the computer solution. This, of course, caused the quotient of \$400.00 divided by X^3 to equal to infinity at this time zero. The problem was programmed in the following manner. By substituting given values into the equation

$$Y = 2000 + 10X + 400/X$$

When $X = 0$, $Y = \$2000$, then an amplitude scale change had to be made. Both sides of the equation were divided by 50. Then

$$0.02\ Y = 40 + 0.2\ X + 8/x$$

and $0.02\ \frac{DX}{DX} = 0.2 - \frac{8}{X^2}$

and $0.02\ \frac{d^2Y}{dX^2} = 16/X^3$

The schematic program is shown in Figure 12, page 55.

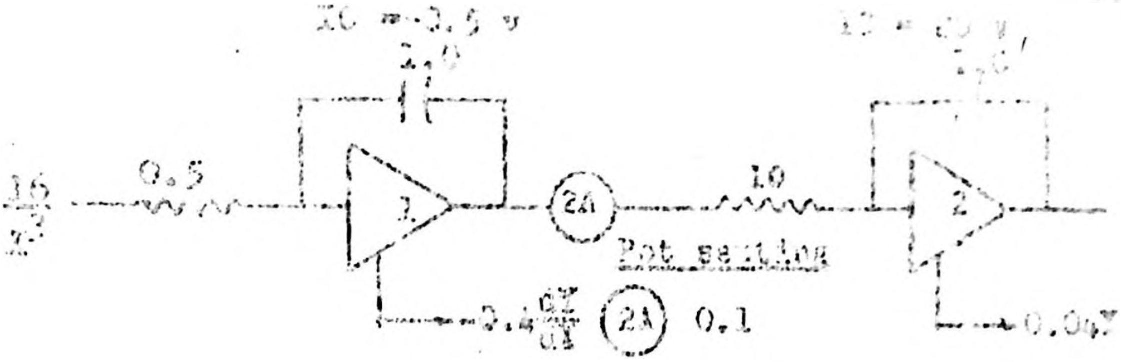


Fig. 12.-- A Schematic Diagram for Computing a Minimum Cost-Point

The computer never gave a solution because of the overload of the amplifiers when the relays were first engaged. There was a method to prevent this malfunction described in a text,[8] but it required the use of servomechanisms, which the author did not have available. What was meant to be accomplished was the plotting of the derivative of Y with respect to X. The point on the abscissa where the derivative crossed would have been the minimum cost-point.

The computer did prove to be good for plotting the values of pdynomials. The method is identical to the one just described for the minimum cost problem, i. e. the polynomial is differentiated manually, so that the computer can integrate to find the value. The following polynomial was used as an experimental problem.

$$Y = 5 + 0.5X + 0.3X^2 + 0.02X^3$$

where $X = 10$

Then $dY/dX = 0.5 + 0.6X + 0.06X^2$

$$d^2Y/dX^2 = 0.6 + 0.12X$$

$$d^3Y/dX^3 = 0.12$$

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[8] Johnson, op. cit., pp. 73-74.

When $X=10$, $Y=60$, so an amplitude change was unnecessary.

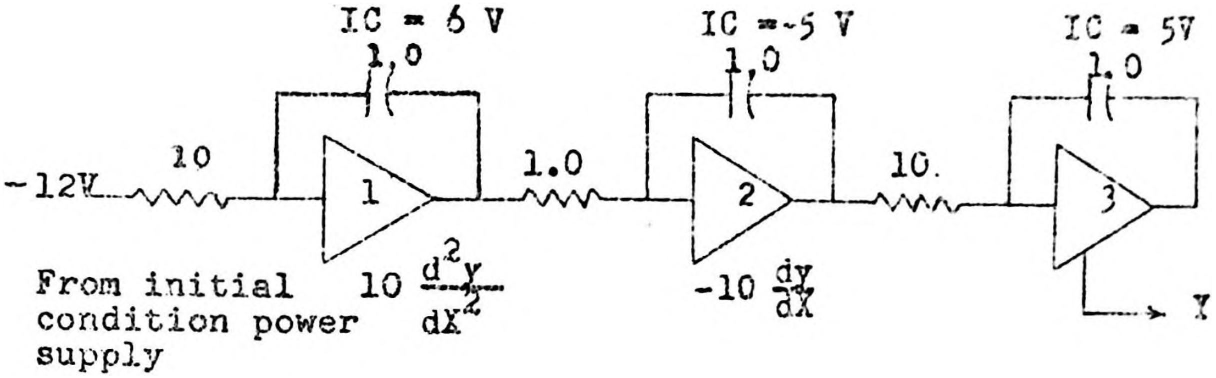

Fig. 13 - Schematic for the Evaluation of a Polynomial (capicators in micro farads and resistors in megohms).

The evaluation of this polynomial is shown in Figure 14, page 57. The output of the amplifier number three of the computer was put into an X-Y Recorder, with a horizontal speed generated from the computer, equal to one inch per second. The results are accurate to approximately one per cent. The answer is presented in problem time where X was considered to be a function of time. The ordinate and abscissa of the solution are calibrated for actual problem values.

The computer's limitations prohibited it from being useful for computing the equation by the method of least squares. This was primarily due to its inability to sum and square masses of data. The author attempted a method of fitting an equation to a polynomial by varying the initial conditions on the integrators for a schematic similar to Figure 13. This is a way, by trial and error, that an approximation can be attained after the data are plotted on the scaled recorder paper. Such curve fitting techniques are not accurate enough for industrial engineering

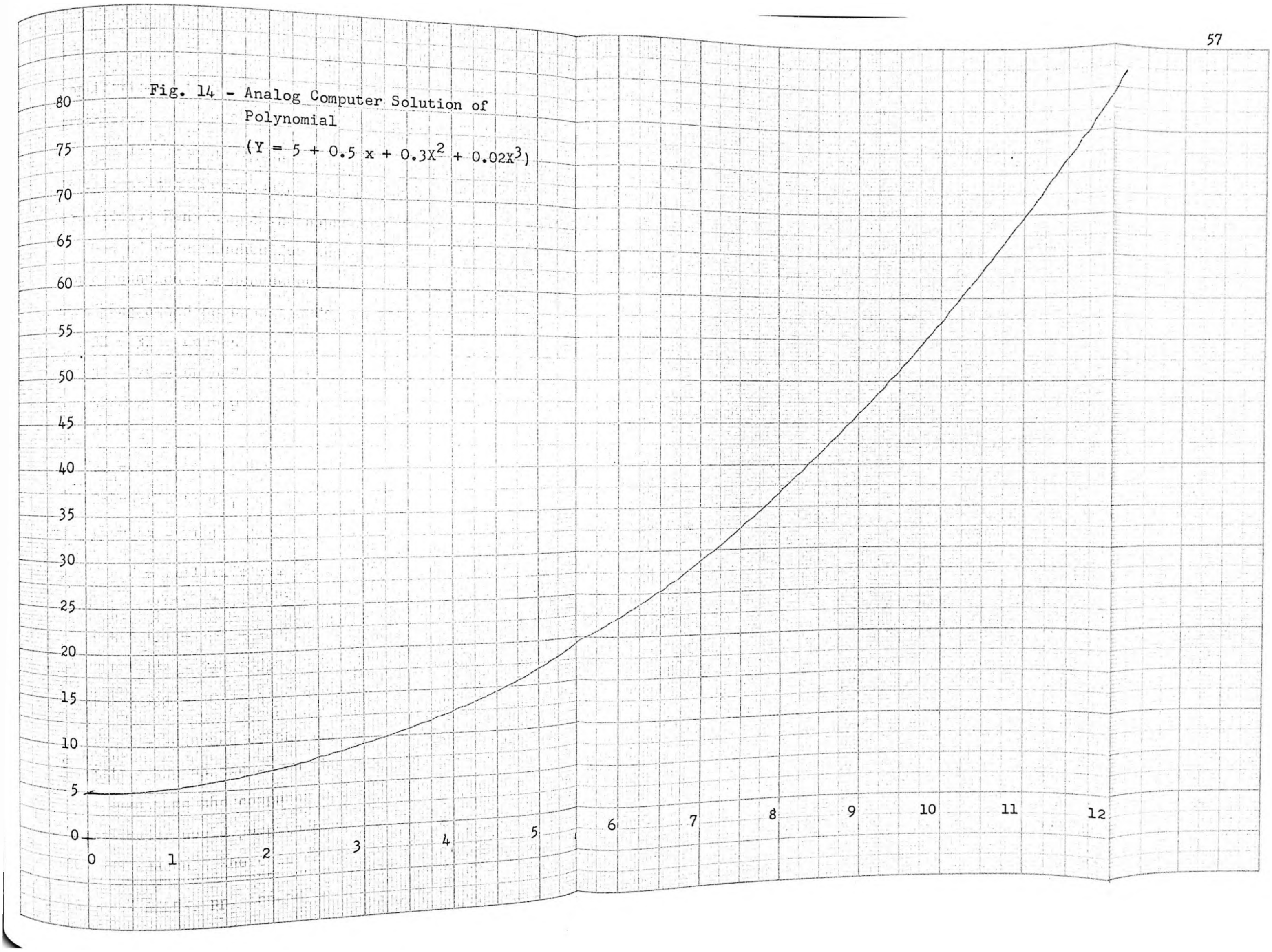

Fig. 14 - Analog Computer Solution of Polynomial

$(Y = 5 + 0.5\,x + 0.3X^2 + 0.02X^3)$

applications, thus the detailed technique will not be presented.

The last of the experimental problems encountered in this thesis was the solution of simultaneous algebraic equations. Many times in industrial engineering analysis the use of determinents for the solution of linear algebraic equations is necessary. The following equations were programmed as shown in Figure 15, page 59:

$$2X + 3Y + 4Z = 5$$
$$3X + 7Y + 5Z = 7$$
$$5X + 2Y + 2Z = 9$$

or,

$$2X = 3Y - 4Z + 5$$
$$7Y = 3X - 5Z + 7$$
$$2Z = 5X - 2Y + 9$$

The amplitude was adjusted by multiplying the equations through by five. Then

$$10X = 15Y - 20Z + 25$$
$$35Y = 15X - 25Z + 35$$
$$10Z = 25X - 10Y + 45$$

The solution of the algebraic equations never proved reliable. The voltmeter on the computer gave different answers each time the computer was turned on. In many cases the voltmeter would jump to plus or minus 100 volts without apparent reason. There is an alternate method[10] for solving

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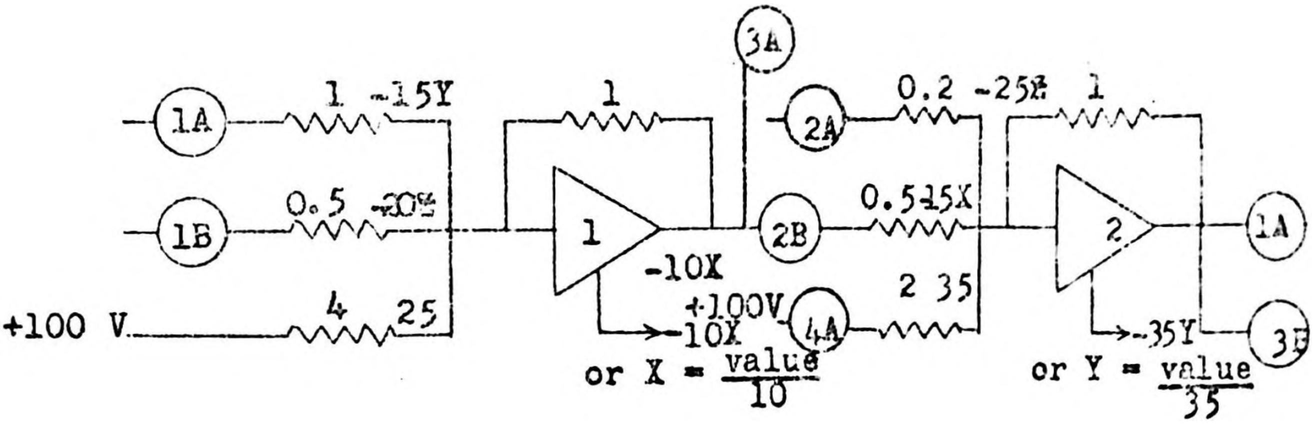
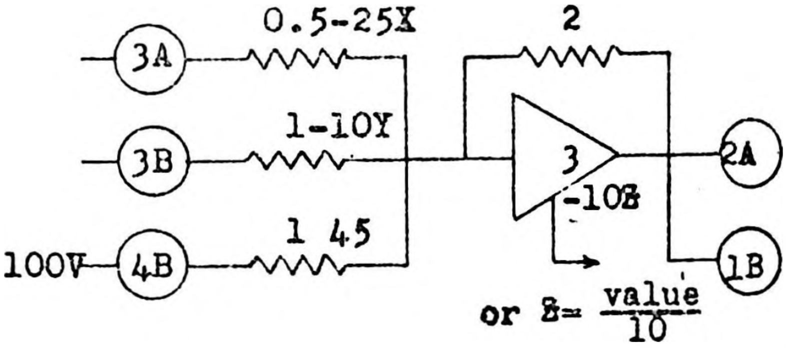
[10] Ibid., pp. 166-68.





Potentiometer settings

1A	0.429
1B	1.000
2A	0.500
2B	0.750
3A	0.625
3B	0.286
4A	0.750
4B	0.900

Fig. 15 - Schematic Diagram for the Solution of Simultaneous Algebraic Equations.

simultaneous algebraic equations; however, the author who presents it indicates that the desk calculator or some type of digital computer is far better for such solutions.

The aim of the foregoing analyses was to report on the research of the usefulness of a small-scale electronic analog computer to industrial engineers. Conclusions have been presented in part during the analyses, and Chapters V and VI give a more complete presentation of the conclusions and recommendations made as a result of this research.

CHAPTER V

CONCLUSIONS

It is concluded from this study that a small scale-electronic analog computer is not a good analysis tool for industrial engineering problems. Even though some of the problems could be solved on the analog computer, the limitations of both the mathematical form of the problems and the computer presented a logical solution sequence. Conclusions on each analysis was presented in Chapter IV, for each experimental problem. Since these problems have been determined as representative of the type problems encountered in industrial engineering from both an academic viewpoint and a practical viewpoint, similar problems would entail essentially the same procedure for solution. Industrial engineering problems are primarily concerned with analyzing quantative information about operations, while the analog computer was found to be an equation solver, not a data analyzer. The inherent limitations of the small-scale analog computer which made it generally not applicable are:

(1) Limited means of accurately putting masses of data in several summing amplifiers.

(2) No memory or storage.

(3) No means of squaring numbers without extensive numerical calculations for potentiometer settings.

(4) No square root circuit.

(5) Vast amount of time required to program problems requiring several additions and divisions.

(6) Limited problem board space for average complex problems.

(7) Machine circuit instability.

These are some of the most prevalent limitations encountered as a result of the research of this thesis. The machine was found to be very useful as an equation solver for systems of ordinary differential equations, which are usually found in other fields of engineering.

Since most of the problems that are encountered in a small company are concerned with analyzing and weighing information, it is concluded that the small company could not profit by relying on a small analog computer as an analysis tool. A small engineering consultant might justify such a computer if he could sell its services to clients who have a sufficient quantity of applicable problems.

The biggest accomplishments of this thesis was finding out what the computer, under study, would do as an analysis tool for industrial engineering. Since the industrial engineer is concerned with methods and organization, he must be aware of new technology, such as analog computer applications. In essence, he must know what will do a job for both himself and the other departments in the organization, if he is to have adequate functional relationships and efficiency in the organization.

CHAPTER VI

RECOMMENDATIONS

As a result of this study, the author has found that by acquiring additional auxiliary equipment some of the limitations could be partially or in some cases completely overcome. Such equipment would include (1) Servo or functional multipliers, (2) additional arbitrary function generating equipment, (3) an X-Y Recorder, (4) some differential relays, (5) electronic switches, (6) a cathode-ray oscillograph with a polaroid camera attachment. The quantity of each of these would depend on the complexity of the research. A detailed discussion on the operation and application of this equipment can be found in the references of this bibliography.

Such problems as a study of the use of an analog computer for optimizing production operations would be a contribution to the analog art. A good grasp of many practical situations and a thorough knowledge of mathematics beyond calculus would be essential for such a study.

APPENDIX I

QUESTIONS AND ANSWERS ON HEATH ELECTRONIC ANALOG COMPUTER

Questions Most Asked About The HEATH Electronic Analog Computer[1]

1. Q. - What is the drift of an operational amplifier?

   A. - 0.5 millivolts referred to the summing junction with the amplifier connected as an integrator with an RC time constant of one.

2. Q. - What is the noise level of an operational amplifier?

   A. - The noise level is below 2 millivolts referred to the summing junction.

3. Q. - What is the phase shift and frequency response of an operational amplifier?

   A. - The phase shift would equal one degree at 1200 cycles per second. The amplifier is flat to 2000 cycles per second.

4. Q. - What is the quiescent power required by the operational amplifier?

   A. - The quiescent power required for the amplifier is 3 watts.

5. Q. - What is the power output of the operational amplifier?

   A. - The power output for the amplifier is one watt at 100 volts.

6. Q. - What is the input and output impedance of the amplifier?

   A. - The input impedance is direct to the grid. The output is less than 50 ohms.

7. Q. - What is the open loop gain?

   A. - The open loop gain is 30,000 to 50,000.

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[1] Letter from The Heath Company, March 21, 1958.

8. Q. - What maximum closed loop gain would you suggest?

A. - The maximum recommended closed loop gain around an amplifier would be 10.

9. Q. - Would an operational amplifier drive a pen recorder?

A. - Yes, the amplifier will drive a pen recorder directly if the recorder pen does not require more than 10 mills of driving current.

10. Q. - What is the linear output of the amplifier?

A. - The amplifier is linear between plus and minus 100 volts.

11. Q. - What is the maximum load which might be placed on the amplifier?

A. - Since the amplifier is rate at 1 watt at 100 volts, the highest recommended load is 10,000 ohms.

12. Q. - May this computer be paralleled or enlarged?

A. - Yes. A large capacity interconnection panel is provided to the rear of the computer. This unit may be expanded with other HEATH computers or any make that is similar.

13. Q. - What is the noise level of the power supply?

A. - The noise level of the power supply is less than one millivolt.

14. Q. - How is the computer affected by line changes and transients?

A. - The computer is subject to error from line changes. The error is greatly reduced if the voltage regulator transformer is used on the filaments.

15. Q. - What type of diodes are used?

A. - 6AL5 diodes are used.

16. Q. - How long will it take to assemble?

A. - Approximately 250 man hours.

17. Q. - How long does it take to learn to operate?

A. - The average engineer familiar with his problem may learn to simulate it on a computer within one to two days. Special applications may take longer.

18. Q. - Is it repetitive and how does one make it repetitive?

A. - This unit may be made repetitive by changing the RC constants and switch to the repetitive oscillator.

19. Q. - What is the principle of setting up the computer?

A. - The computer is in reality an analog for simulation of the dynamic characteristics of the problem under study. An electronic model is built and manipulated to find the desired results.

20. Q. _ Why are the 30 coefficient potentiometers used?

A. - The large number of potentiometers per amplifier permits a great deal of flexibility and lowers the cost, in that the sole requirement is high stability. Since the potentiometers are nulled against a standard, accuracy is maintained.

21. Q. - How does one start and stop the computer?

A. - The computer is started and stopped by the introduction of the initial conditions by the use of relays.

22. Q. - How does one read the computer?

A. - An indication of the answer can be read from the meter included in the computer, but a more accurate and complete solution will be obtained with the use of an oscilloscope or pen recorder.

23. Q. - What are the reference voltages and how are initial conditions started?

A. - The reference voltages are plus or minus 100 volts. The initial conditions are introduced by means of patch in relays.

24. Q. - Does one have to know calculus to operate the computer?

A. - Calculus is not absolute necessary. Since this is an analog unit certain configurations of "patching" may be used to directly simulate the dynamic systems.

25. Q. - What is the weight of the computer?

A. - The weight of the full computer is 168 lbs., shipping weight is 200 lbs.

26. Q. - What power requirements are necessary for the computer?

A. - 450 watts.

APPENDIX II

TECHNICAL PURPOSES, FEATURES, AND SPECIFICATIONS OF HEATH ELECTRONIC ANALOG COMPUTER COMPONENTS

Technical Purposes, Features, and Specifications of HEATH Electronic Analog Computer Components[2]

MODEL ES-400

Cabinet and Front Panel

Purpose: To house all the power supplies, amplifiers, and computing components, and to bring to a composite surface by means of jacks, all of the refined voltages for computing purposes.

Features: The panel contains 30 coefficient potentiometers, a nulling meter for computer voltage setting, a patch board panel which shows the computer block layout, a dividing network and meter which may be connected to any one of the 15 amplifiers and which may show full scale deflection of plus or minus 2.20 and 100 volts, and a plus and minus 100-volt standard.

Specifications: Power requirements are 105-125 volts, AC, 60 cycles, 420 watts.

Dimensions: 26 1/2 inches long, 15 inches high, 32 1/2 inches wide.

Weight: 70 pounds

MODEL ES-2

Amplifier Power Supply

Purpose: To supply power to the operational

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[2] Ibid.

amplifier and function generators.

Features: The plus and minus voltages are referenced from one standard and are interconnected so as to null or cancel power supply drift to amplifiers.

Specifications: Plus 250 volts at 250 mills, minus 250 volts at 250 mills, minus 450 volts at 50 mills, 6.3 volts A. C. at 12 amperes, and 6.3 volts A. C. at 2.5 amperes.

MODEL ES-100

Initial Condition Power Supply

Purpose: To supply initial condition voltages to the integrators.

Features: Low drift rate, ungrounded, floating supply, highly shielded.

Specifications: Uses two OB2 tubes, contains two separate supplies which can be varied from zero to 100 volts.

MODEL ES-201

Amplifier

Purpose: To provide an amplifier for integration, sign changing, addition, and multiplication by a constant.

Features: Highly stable with low drift, linear from plus 100 volts to minus 100 volts, will deliver 10 milliamperes without overloading, and has an open loop gain of 50,000.

Specifications: Uses on 12AX7, one 6BQ7A, one 6BH6 tubes. Power requirement - plus 250 volts and minus 450 volts. Quiescent power is less than 5 watts.

MODEL ES-151

Relay Power Supply

Purpose: To supply power to operate the functional relays.

Features: Has built in voltage surge network to insure simultaneous operation of the relays.

Specifications: Designed to supply 50 volts across four 10,000 ohms relays.

MODEL ES-50

Reference Power Supply

Purpose: To supply highly stable and accurate reference voltages.

Features: The positive and negative voltages are slaved together and referenced from a single 5651 tube. Has negligible noise.

Specifications: Output - Plus 100 volts, minus 100 volts, contains two 6X4, two 6U8, one 5651, tubes.

MODEL ES-505

Repetitive Oscillator

Purpose: To provide repetitive operation for the functional relays.

Features: Has an adjustable repetition rate of 0.6 to 6.0 times per second.

Specifications: Contains one 6J6 tube.

MODEL ES-600

Function Generator

Purpose: To provide a function of "X" for any input of "X".

Features: Variable break voltages, high static accuracy (.5%), ten straight line segments - five in the plus "X" direction and five in the minux "X" direction, break voltages may be varied from plus 100 volts to minus 100 volts.

Specifications: Power requirements - plus 250 volts at 16 milliamperes, minus 250 volts at 16 milliamperes, 117 volts A. C. at 100 milliamperes.

Input requirements - A voltage which varies with respect to time.

Output characteristics - approximation of functions by straight line segments. Contains five 6AL5 tubes.

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