Design

For a Concrete- Steel Arch

Luten' Method

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as a Thesis for the

DEGREE OF CIVIL ENGINEER.

Department of Engineering,

University of Alabama,

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Description of a concrete arch designed by Luten's method.

Span of arch	= 30' 0"
Rise	= 4' 6"
Thickness at crown	= 12"
Radius of intrados at crown	= 35'

Radius of extrados = I ÷c where I = radius of intrados at the crown and C = thickness at the crown in inches.

Hence radius of extrados = 35' ÷ 2.4 = 37.4'

Depth of earth filling over crown = 18"

To determine curve of intrados construct an ellipse whose major and minor axises are equal to the required span and rise. Pass a segment of a circle through the springings of the ellipse and crown. Bisect the vertical ordinates between the ellipse and circle. Approximate the resulting curve with arcs of circles adjusting curve at the springings to become tangent to the verticals.

The equation of the catenary is $y=\frac{m}{2}\left(e^{\frac{x}{m}} + e^{-\frac{x}{m}}\right)$ a catenarian rib 30' span and rise 3'6". It is required to determine a catenary which will be in equilibrium under loading represented by its ordinates to the directrix and which can be vertically projected into a transformed catenary of the required dimensions. If the ratio in which the ordinates are decreased in order to produce the transformed catenary be represented by r then ordinate of the latter curve $y'=\frac{rm}{2}$

$(e^{\frac{x}{m}} + e^{\frac{-x}{m}})$ But rm is equal to the ordinate at the crown of the transformed catenary calling rm,m' we have

$$y' = \frac{m'}{2}(e^{\frac{x}{m}} + e^{\frac{-x}{m}}) \text{ solving for m we have}$$

$$m = \frac{x}{\text{hyp,log}(\frac{y'}{m'}) + \sqrt{(\frac{y'}{m'})^2 - 1}}$$

m' = 2' 5"

x = 15' 0"

y' = 5' 6"

Hence $m = \frac{15}{\text{hyp log } 5.312}$

$\therefore$ m = 9'0"

The transformed catenary is derived by reducing each ordinate of the catenary in the ratio of 9 to 2. This is easily done graphically by assuming two points r and s on any convenient vertical line and at distances from the directrix in the ratio of 2 to 9. And then passing lines from s through points on the catenary to meet the directrix, and from these pints on the directrix back to r. It will be readily seen that such lines divide the vertical ordinates in the same ratio of two to nine, and will consequently locate points on the transformed catenary. The horrizontal thrust for the transformed catenary is the same as in the catenary, the load in each case being limited by the curve. By spacing the points of application of the normal loads at the same horizontal intervals as the vertical loads, the normal pressure will be equal, and the load line may then be drawn with elements of uniform length parallel to successive normal loads. The elements of the curve of equilibrium are drawn parallel to the successive rays of

the force polygon and limited in length by the points of intersection with the normal forces.

Assume the thickness of the rib originally to be 2a and after addition of material 2x. The distance between the central and neutral axis to be zf the intensity of stress in the extreme fiber and f the resultant force acting through the center of gravity of the original section the stress to be considered as increasing uniformly with its distance from the neutral axis. We have by taking moments about the center of gravity of the tensile stresses on the section

$$F\ (x-a+z)+2/3\ (x-z) = f\ \frac{x+z}{2}\ 2/3\ (x+z)+2/3\ (x-z)$$

And by summation of forces of the section

$$F = f\ \frac{x+z}{2} - f\ \frac{(x-z)}{(x+z)}\ \frac{(x-z)}{2}$$ and from these two equations

$$z = \frac{x^2}{3(x-a)}$$

Also $$\frac{fx+fz}{2} - \frac{f\ (x^2-2xz+z^2)}{2\ (x+z)} = F$$

$$fx^2+2fxz+fz^2-fx^2+2fxz-fz^2=2F(x+z)$$

$$4fxz = 2F(x+z)$$

$$f = \frac{F}{\frac{2}{zx}}\ (x+z)$$ for the compressive stress in the extreme fiber, and

$$f' = \frac{F}{2}\ \frac{z+x}{zx}\frac{(z-x)}{(z+x)}$$ for the tensile stress in the extreme fibre on the opposite side of the section. From which $f=\frac{F}{2}\frac{(4x-3a)}{x^2}$ and $f' = \frac{F}{2}\ \frac{(2x-3a)}{x^2}$ The maximum values of f and f' are determined by placing their first differential coefficient with respect to x equal to zero $$\frac{df}{dx} = \frac{8x^2F - F(4x-3a)4x}{4x^4} = \frac{8x^2F-16x^2F+12axF}{4x^4} =$$

$$= \frac{F}{2}\,\frac{(2x-3a)\ 2x-4x2}{x^4} = 0$$

$$df' = \frac{F}{2} - \frac{(2x-3a)\ 2x-2x^2}{x^4} - = 0$$

$$4x^2-6ax - 2x^2 = 0$$

$$2x^2 = 6ax$$

$$x = 3a \qquad \text{and } f' = 1/3\ F/2a$$

$\frac{F}{2a}$ is, however, the intensity of the uniform stress on the section before the addition of material at one side of the section. It follows, therefore, that the addition of material on one side of a rib under uniform compression, may increase the intensity of compressive stress by not more than one third, and may produce tensile stress not greater than one third of the uniform compressive stress.

The analysis for live load stress is similar to the above. The forces acting on a section are the dead load thrust of 13,500# acting at a distance 3 inches from the central axis and the live load thrust of 3200# acting at a distance of 30 inches. The thickness of arch rib at point og section is 18 inches. Taking moments as before about the center of gravity of the tensile stresses we have

$$\frac{13500}{12}\left[3+z+2/3\ (.9-z)\right] + \frac{3200}{12}\left[30+z+2/3\ (9-z)\right]$$

$$= f\,\frac{(9+z)}{2}\left[2/3\ (9+z)+2/3\ (9-z)\right]$$

and summing forces on the section there results

$$\frac{13500}{2} + \frac{3200}{12} = f\frac{(9+2)}{2} - f\frac{(9-z)}{(9+z)}\left(\frac{9-z}{2}\right)$$

from which f is found to be 287# per square inch. the compressive stress on the extreme fiber, and the tensile stress is found from proportion

of this value to be 132# per square inch. Assuming that 60# of this is carried by the concrete than there remains $72(\frac{9-2}{2})12=2450\#$ for each foot of rib to be carried by the rods which are consequently chosen as steel rods $\frac{3}{4}$ inch in diameter spaced 2 ft. apart. When the load is consentrated at a point at one side of the crown tension will be produced on the under side of the rib near the point of loading if near the crown. The same method of analysis will show that the rod is required near the introdos at the crown for such cases of loading, and as its location near the extrados at the crown is not essential for any condition of loading, the rods pass from their position near the extrados at the haunches across the rib at a point where the tensile stresses are a minimum, to follow the intrados for a short distance near the crown. Three transverse straps of steel 1 x $\frac{1}{4}$ inch were introduced, passing under the rods and weaving up into the concrete between the rods. The straps prevented the rods embedded near the intrados from tearing through the concrete and tending to straighten under tension if the arch should fail by spreading of the abutments. The thrust in thi bridge is resisted by tie rods from abutment to abutment beneath the bed of the stream. The ties are of steel which are embedded in a layer of concrete 8 inches thick which forms a pavement with its surface flush with the bed. This pavement not only saves in the amount of material which will be used in the abutments but it also makes the bridge flood-proof, it also provides a strong foundation for the forms supporting the arch ring during erectin eliminating a large part of the danger of distortion of the arch ring

before hardening. It also confines the earth of the foundations, thus helping to prevent settlement of the abutments in soft foundations. The use of the tie transforms the horrizontal thrust of the arch into a vertical one equal to one half the weight of the loaded arch. The pavement is depressed 6 inches at the middle of the stream and the rod is embedded to a depth of at least 3 inches. The ties are not necessarily straight, but may also be depressed at the middle of the stream provided they are not depressed so much that the pull in the rods would lift the pavement. The limitable amount of depression allowed in the ties is computed to be 8½ inches. The outer finish of the arch should be ornamented as shown on the general design and the spandrel walls of the arch ring are to be built at least twelve inches in height above the roadway. A simple finish as shown on the general design is to be put on the spandrel walls at the top. The facing of the arch-ring is to be composed of neat cement mortar. There is a 6-inch water pipe traversing the arch ring as shown in the method of erection. As soon as the pavement is laid and has hardened for two days the forms should be erected upon a raft of 2-inch planks and as soon as this sinks to a bearing on the drum drive wedges under each of the uprights. The centers are to consist of a single thickness of two-inch plank spaced 4 ft. apart. The forms should be [illegible] thoroughly saturated with water and kept so in order to prevent shrinkage. The steel rods should be smooth round, of medium steel with scarf welds and open hooks turned at the ends, which are to be upset to prevent pulling out from the concrete. The fill and roadway over

this arch shall be completed before the forms are removed for unless this is done there is danger of the arch sinking slightly at the crown.

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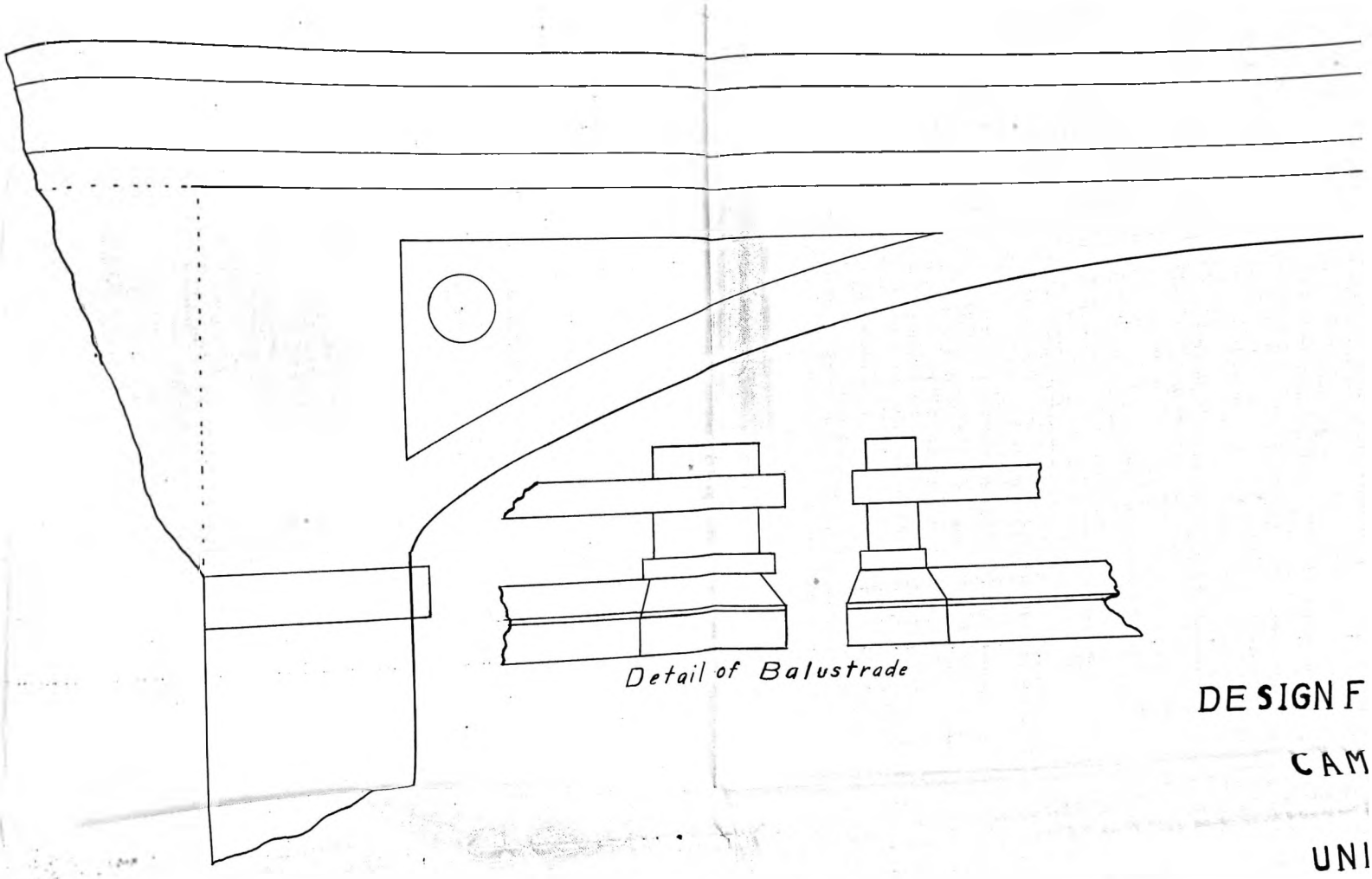

Detail of Balustrade

DESIGN F

CAM

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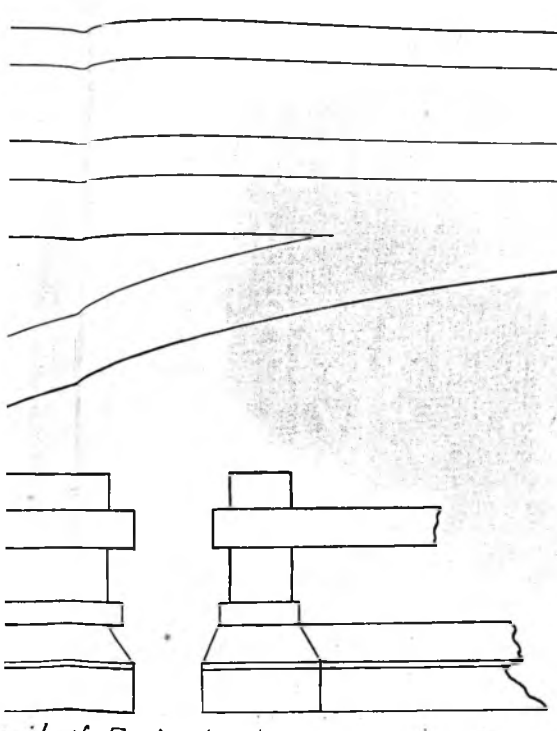
Scale lin to 1½ft



ail of Balustrade

DESIGN FOR A CONCRETE ARCH

OVER

CAMPUS CREEK

30 FT SPAN

UNIVERSITY OF ALA.

Scale 1in to 1½ft.

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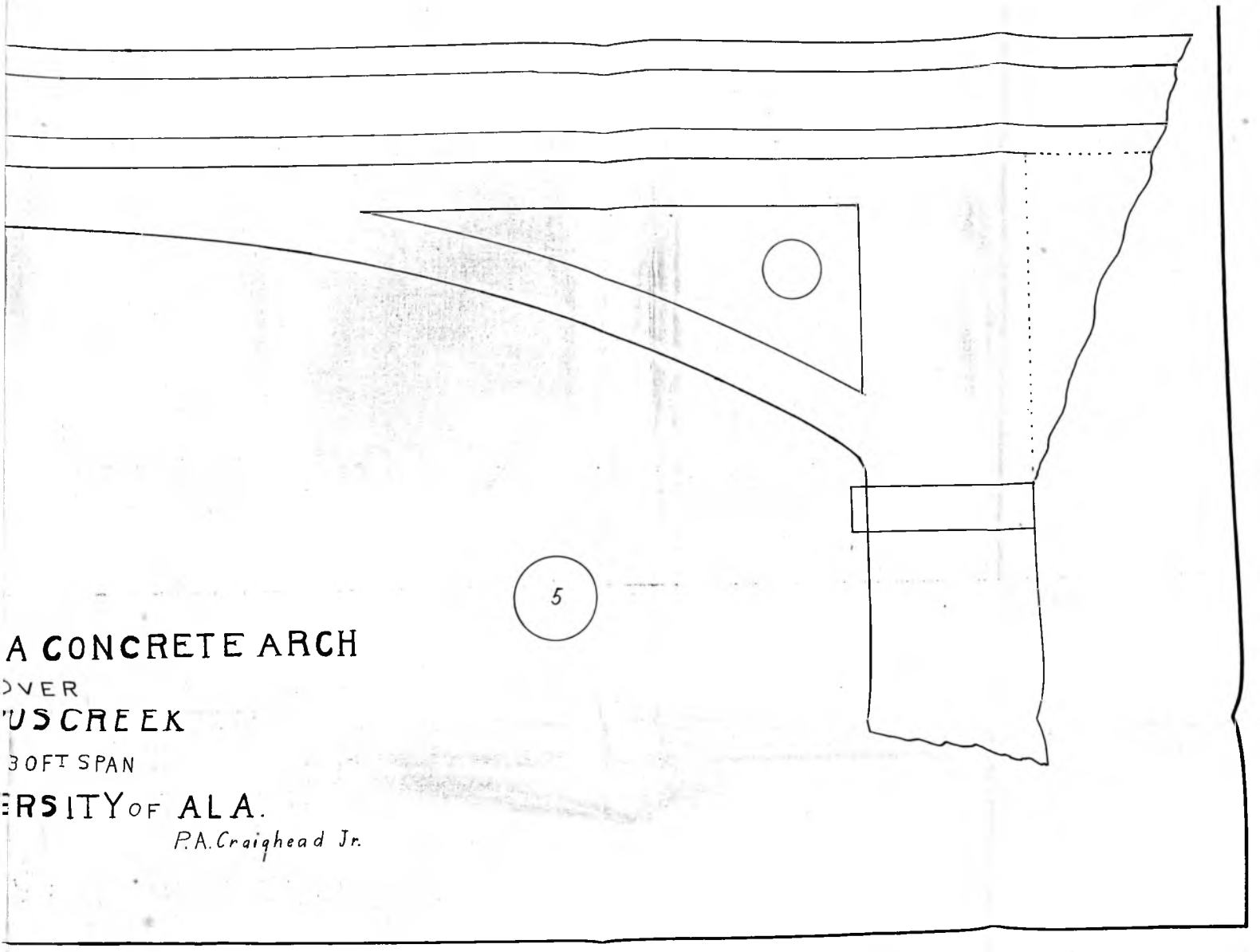

A CONCRETE ARCH

OVER

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3 OFT SPAN

ERSITY OF ALA.

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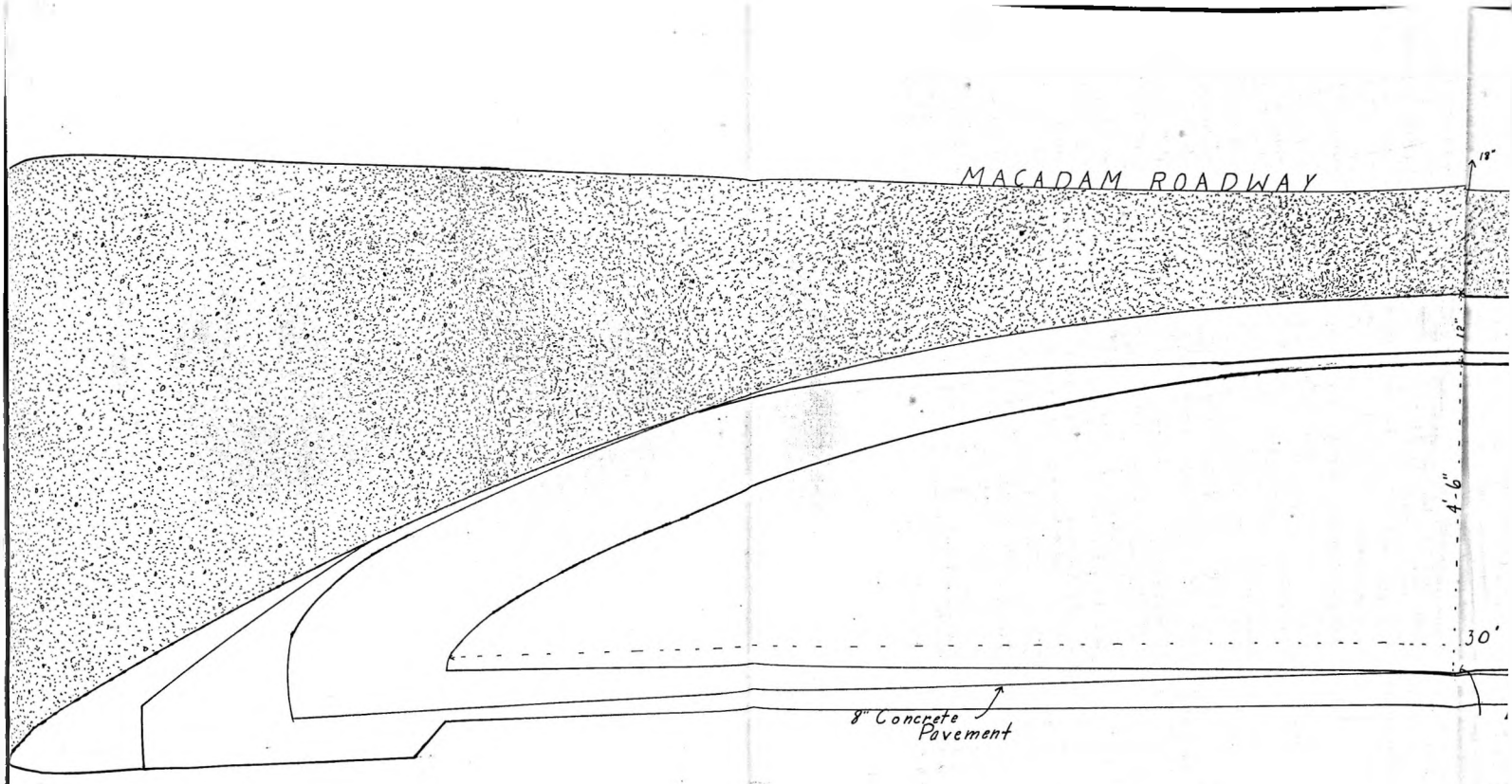
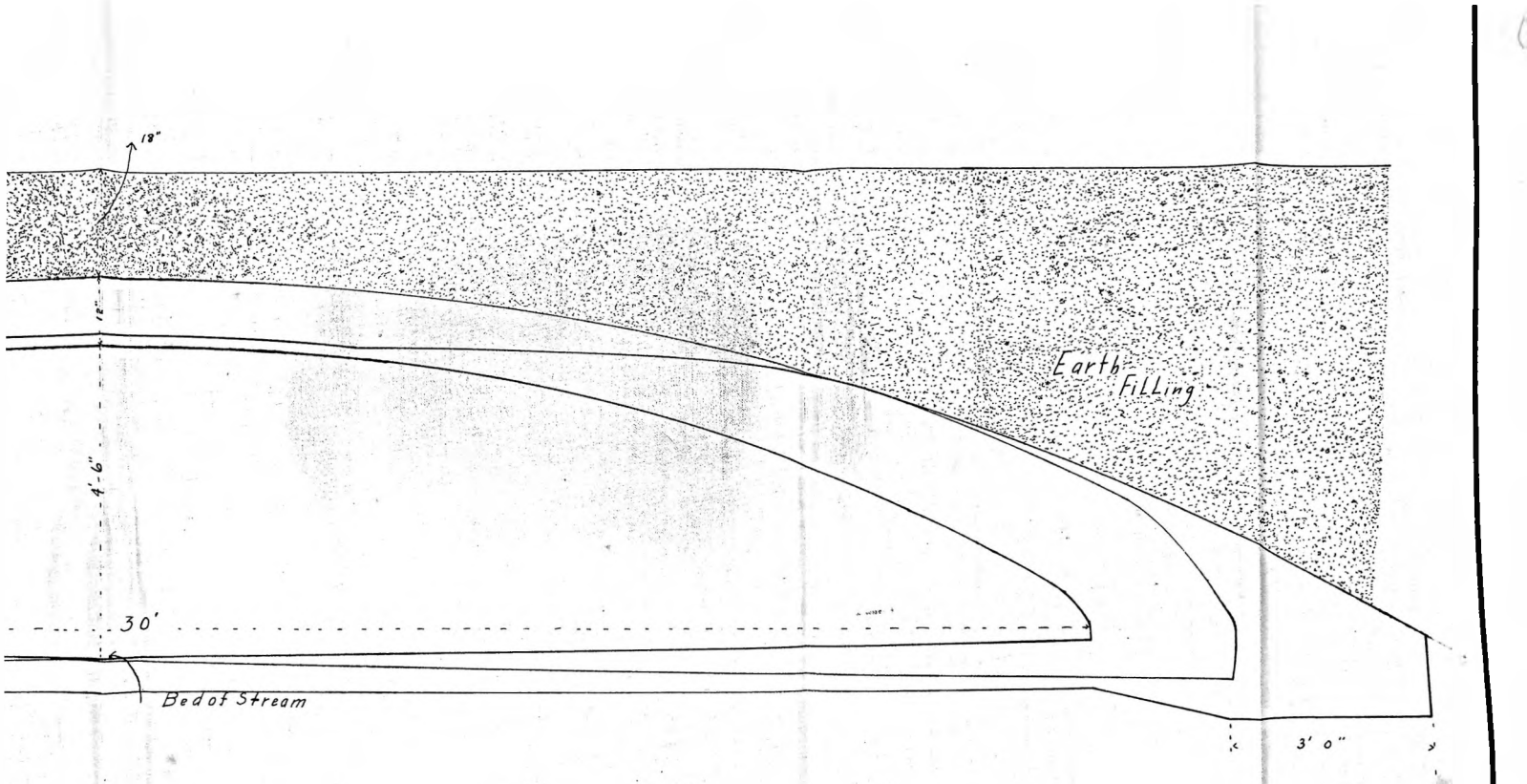


LONGITUDINAL SECT

Scale 1 in to 1½ ft.



L SECTION

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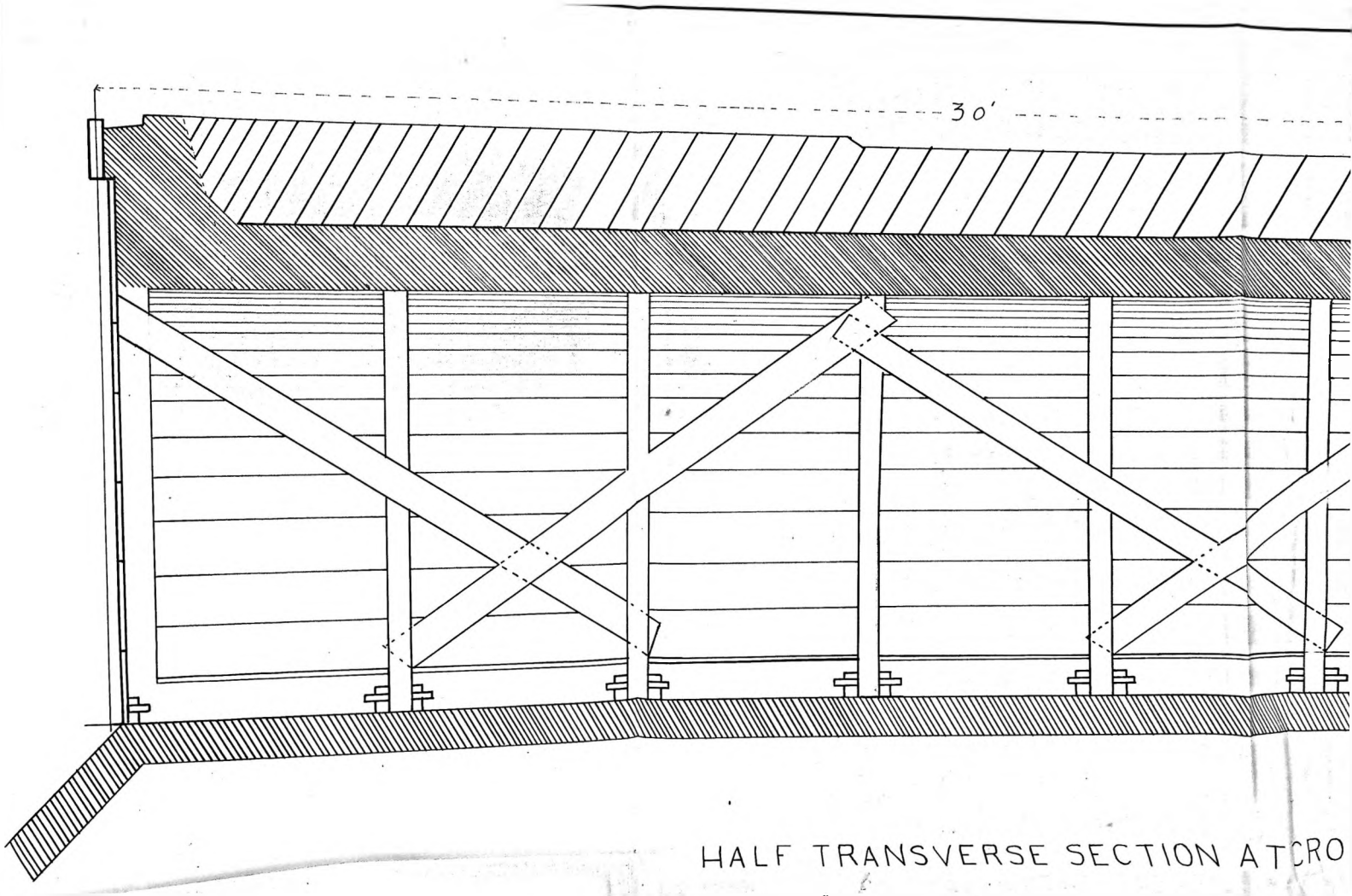
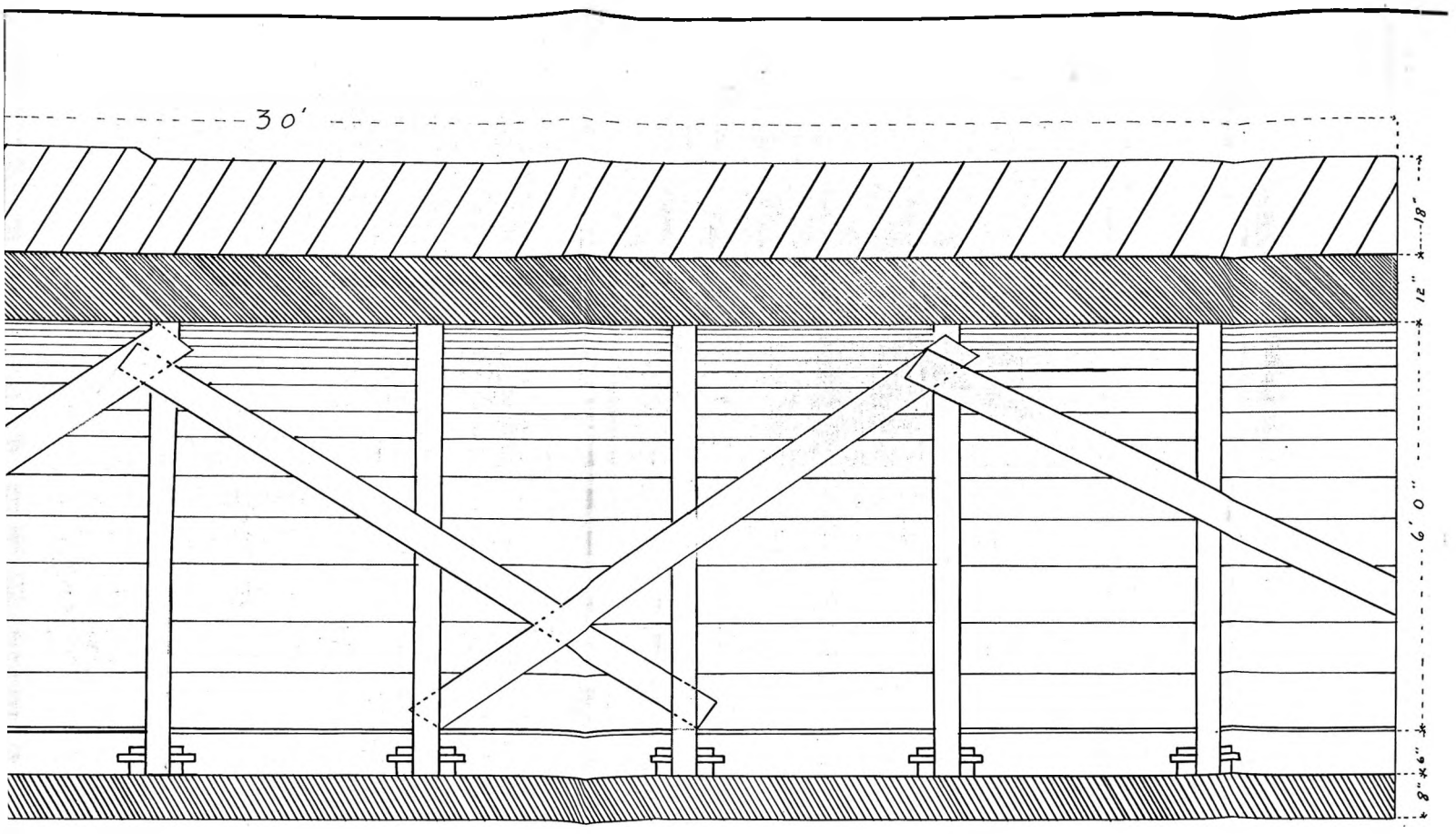


HALF TRANSVERSE SECTION AT CRO

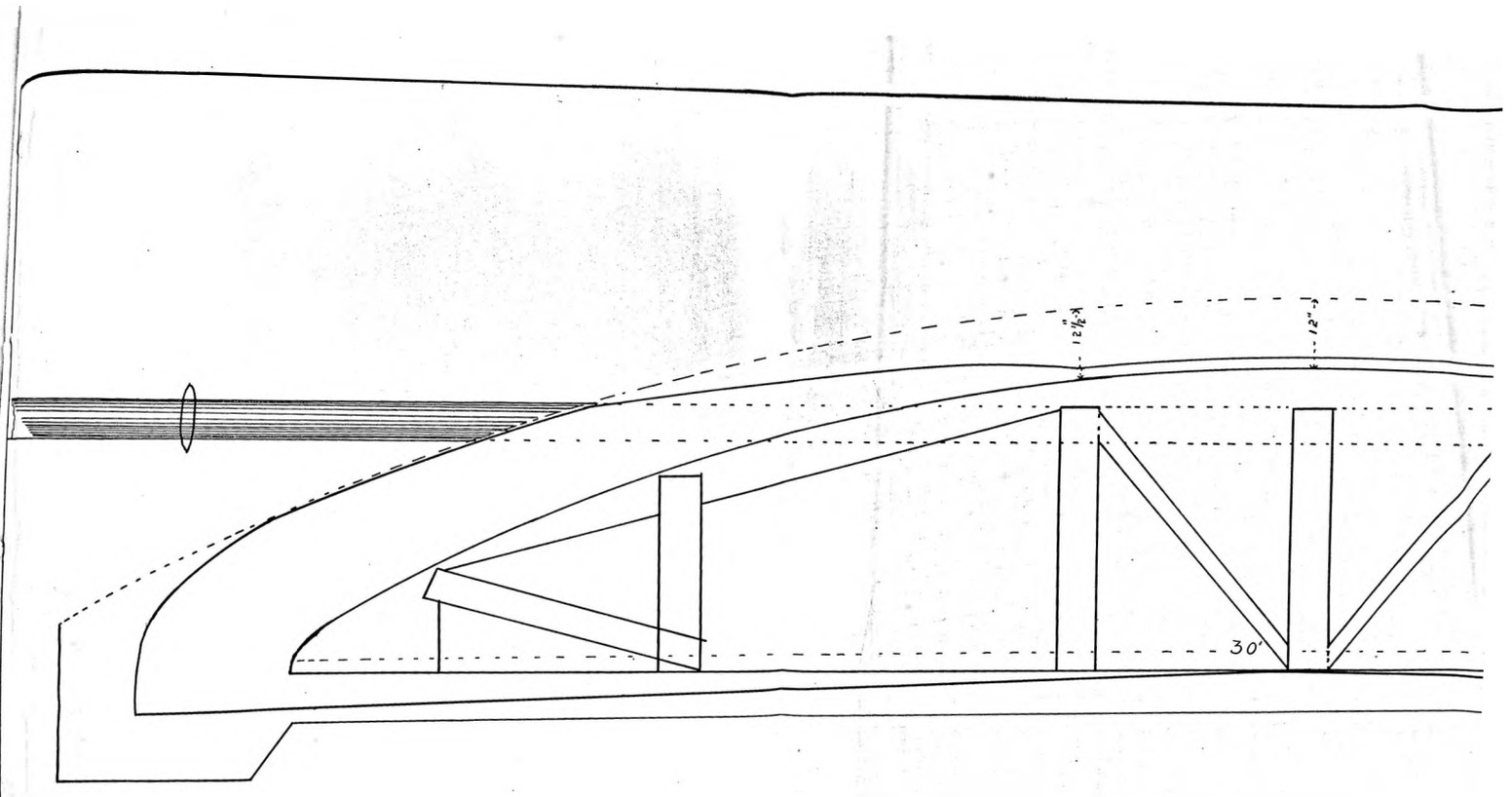
Scale 1 in to 1½ ft.



ILF TRANSVERSE SECTION AT CROWN

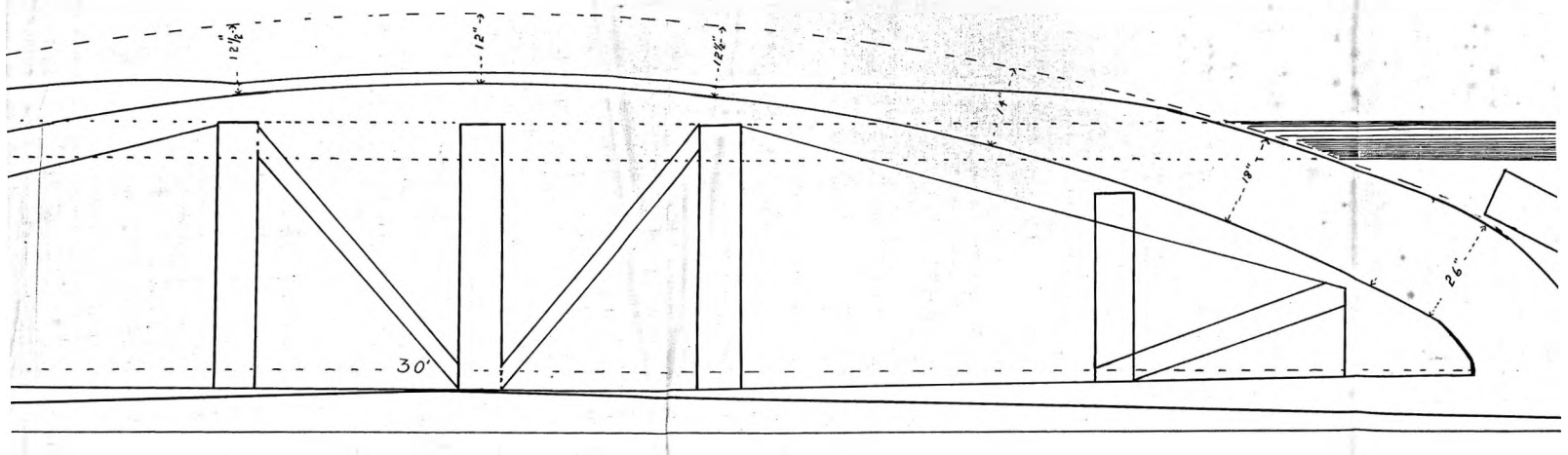
3

1 in to 1½ ft.



METHOD OF ERECTION

Scale 1 in to 1½ ft



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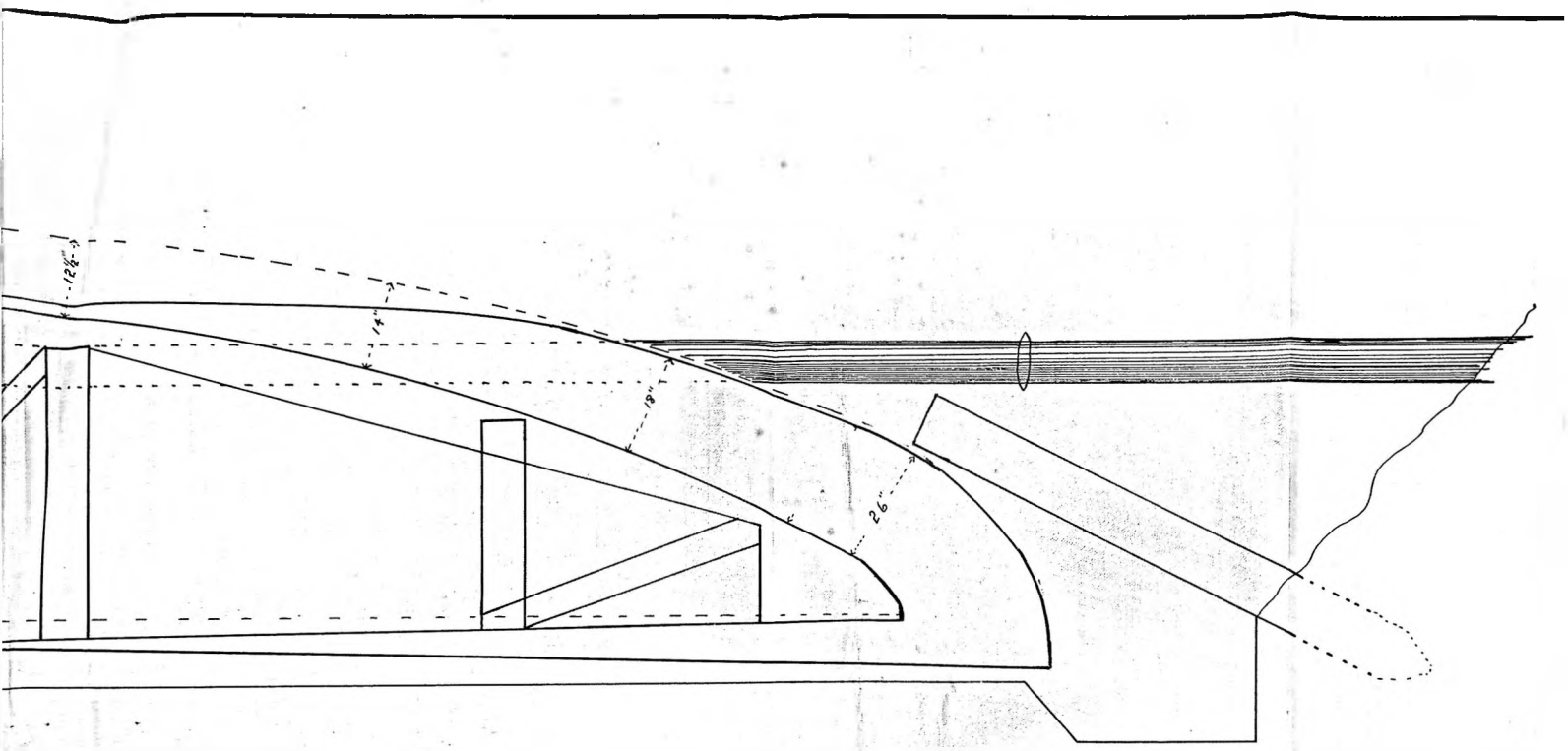
METHOD OF ERECTION

Scale 1in to 1⅛ ft.



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