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Thermal leptogenesis in brane world cosmology

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The thermal leptogenesis in brane world cosmology is studied. In brane world cosmology, the expansion law is modified from the four-dimensional standard cosmological one at high temperature regime in the early universe. As a result, the well-known upper bound on the lightest light neutrino mass induced by the condition for the out-of-equilibrium decay of the lightest heavy neutrino, $\tilde{m}_1 \lesssim 10^{-3}$ eV, can be moderated to be $\tilde{m}_1 \lesssim 10^{-3}\ \mathrm{eV} \times (M_1/T_t)^2$ in the case of $T_t \leq M_1$ with the lightest heavy neutrino mass (M_1) and the "transition temperature" (T_t), at which the modified expansion law in brane world cosmology is smoothly connecting with the standard one. This implies that the degenerate mass spectrum of the light neutrinos can be consistent with the thermal leptogenesis scenario. Furthermore, as recently pointed out, the gravitino problem in supersymmetric case can be solved if the transition temperature is low enough $T_t \lesssim 10^{6-7}$ GeV. Therefore, even in the supersymmetric case, thermal leptogenesis scenario can be successfully realized in brane world cosmology.

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I. INTRODUCTION

The origin of the cosmological baryon asymmetry is one of the prime open questions in particle physics as well as in cosmology. The asymmetry must have been generated during the evolution of the universe. In fact, such a generation is possible if three conditions, (i) the existence of baryon number violating interactions, (ii) C and CP violations and (iii) the departure from thermal equilibrium, are satisfied [1].

Among various mechanisms of baryogenesis, leptogenesis [2] is attractive because of its simplicity and the connection to neutrino physics. Particularly, the simplest scenario, namely, thermal leptogenesis, requires nothing but the thermal excitation of heavy Majorana neutrinos which generate tiny neutrino masses via the seesaw mechanism [3] and provides several implications for the light neutrino mass spectrum [4]. In leptogenesis, the first condition is satisfied by the Majorana nature of heavy neutrinos and the sphaleron effect in the standard model (SM) at the high temperature [5], while the second condition is provided by their CP violating decay. The departure from thermal equilibrium is provided by the expansion of the universe.

The out-of-equilibrium decay is realized if the decay rate is smaller than the expansion rate of the universe,

$$\Gamma_{N_1} < H|_{T=M_1}, \tag{1}$$

where Γ_{N_1} and M_1 are the decay rate and the mass of the lightest heavy neutrino, respectively, and H denotes the Hubble parameter. Note that the expansion law is governed by the gravitational theory. Therefore, if general relativity is replaced by another theory at a high energy scale, the universe would undergo nonstandard expansion. One of such examples is the brane world cosmology [6]. The following discussion is based on the so-called "RS II" model first proposed by Randall and Sundrum [7]. In the model, the Friedmann equation for a spatially flat spacetime is given by

$$H^2 = \frac{8\pi G_N}{3}\rho\left(1 + \frac{\rho}{\rho_0}\right) \tag{2}$$

where

$$\rho = \frac{\pi^2}{30} g_* T^4 \tag{3}$$

is the energy density of the radiation with g_* being the effective degrees of freedom of relativistic particles,

$$\rho_0 = 96\pi G_N M_5^6, \tag{4}$$

with M_5 being the five-dimensional Planck mass [1], and the four dimensional cosmological constant has been tuned to be zero. Here we have omitted the term so-called "dark radiation", since this term is severely constrained by big bang nucleosynthesis [8] and does not affect the results in this paper. The second term proportional to ρ^2 is a new

[1]We define M_5 as the "reduced" five-dimensional Planck mass. In some papers, the normal five-dimensional Planck mass is used. The reduced five-dimensional Planck mass is defined as $M_5/(8\pi)^{1/3}$ by using the normal one (M_5).

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ingredient in the brane world cosmology and leads to a nonstandard expansion law.

Note that according to Eq. (2) the evolution of the early universe can be divided into two eras. At the era where $\rho/\rho_0 \gg 1$ the second term dominates and the expansion law is nonstandard (brane world cosmology era), while at the era $\rho/\rho_0 \ll 1$ the first term dominates and the expansion of the universe obeys the standard expansion law (standard cosmology era). In the following, we call a temperature defined as $\rho(T_t)/\rho_0 = 1$ "transition temperature", at which the evolution of the early universe changes from the brane world cosmology era into the standard one. The transition temperature T_t is determined as

$$T_t \simeq 1.6 \times 10^7 \left(\frac{100}{g_*}\right)^{1/4} \left(\frac{M_5}{10^{11}\ \mathrm{GeV}}\right)^{3/2} \mathrm{GeV}, \tag{5}$$

once M_5 is given. Using the transition temperature, we rewrite Eq. (2) into the form,

$$H^2 = \frac{8\pi G_N}{3}\rho\left[1 + \left(\frac{T}{T_t}\right)^4\right] = H_{st}^2\left[1 + \left(\frac{T}{T_t}\right)^4\right], \tag{6}$$

where H_{st} is the Hubble parameter in the standard cosmology.

This modification of the expansion law at a high temperature ($T > T_t$) leads some drastic changes for several cosmological issues. In fact, some interesting consequences in the brane world cosmology such as the enhancement of the dark matter relic density [9] and the suppression of the overproduction of gravitino [10] have been pointed out. In this paper, we investigate how the modified expansion law in the brane world cosmology affects the thermal leptogenesis. Clearly, if $T_t < M_1$, we can expect some effects according to the condition of Eq. (1).

II. A BRIEF OVERVIEW OF THERMAL LEPTOGENESIS

In the seesaw model, the smallness of the neutrino masses can be naturally explained by the small mixings between left-handed neutrinos and heavy right-handed Majorana neutrinos N_i. The basic part of the Lagrangian in the SM with right-handed neutrinos is described as

$$\mathcal{L}_N = -h_{ij}\overline{l_{L,i}}HN_j - \frac{1}{2}\sum_i M_i\overline{N_i^C}N_i + h.c., \tag{7}$$

where $i, j = 1, 2, 3$ denotes the generation indices, h is the Yukawa coupling, l_L and H are the lepton and the Higgs doublets, respectively, and M_i is the lepton-number-violating mass term of the right-handed neutrino N_i (we are working on the basis of the right-handed neutrino mass eigenstates). In this paper, we assume the hierarchical mass spectrum for the heavy neutrinos, $M_1 \ll M_2, M_3$, for simplicity as in many literature.

In the case of the hierarchical mass spectrum for the heavy neutrinos, the lepton asymmetry in the universe is generated dominantly by CP-violating out-of-equilibrium decay of the lightest heavy neutrino, $N_1 \to l_L H^*$ and $N_1 \to \overline{l_L}H$. The leading contribution is given by the interference between the tree level and the one-loop level decay amplitudes, and the CP-violating parameter is described as [11,12]

$$\varepsilon \equiv \frac{\Gamma(N_1 \to H + \bar{l}_j) - \Gamma(N_1 \to H^* + l_j)}{\Gamma(N_1 \to H + \bar{l}_j) + \Gamma(N_1 \to H^* + l_j)} \tag{8}$$

$$= \frac{1}{8\pi}\frac{1}{(hh^\dagger)_{11}}\sum_{i=2,3}\mathrm{Im}(hh^\dagger)_{1i}^2\left[f\left(\frac{M_i^2}{M_1^2}\right) + 2g\left(\frac{M_i^2}{M_1^2}\right)\right]. \tag{9}$$

Here $f(x)$ and $g(x)$ correspond to the vertex and the wave function corrections,

$$f(x) \equiv \sqrt{x}\left[1 - (1+x)\ln\left(\frac{1+x}{x}\right)\right],\qquad g(x) \equiv \frac{\sqrt{x}}{2(1-x)}, \tag{10}$$

respectively. These functions are slightly modified in supersymmetric models [12]. In our case, both functions are reduced to $\sim -\frac{1}{2\sqrt{x}}$ for $x \gg 1$, and ε can be simplified as

$$\varepsilon \simeq \frac{3}{16\pi}\frac{1}{(hh^\dagger)_{11}}\sum_{i=2,3}\mathrm{Im}(hh^\dagger)_{1i}^2\frac{M_1}{M_i}. \tag{11}$$

Through the relations of the seesaw mechanism, this formula can be roughly estimated as

$$\varepsilon \simeq \frac{3}{16\pi}\frac{M_1 m_3}{v^2}\sin\delta \simeq 10^{-6}\left(\frac{M_1}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{m_3}{0.05\ \mathrm{eV}}\right)\sin\delta, \tag{12}$$

where m_3 is the heaviest light neutrino mass, $v = 174$ GeV is the vacuum expectation value (VEV) of Higgs and $\sin\delta$ is an effective CP phase. Here we have normalized m_3 by 0.05 eV which is a preferable value in recent atmospheric neutrino oscillation data $\sqrt{\Delta m_\oplus^2} \simeq 0.05$ eV [13]. Using the above ε, the resultant baryon asymmetry generated via thermal leptogenesis is described as

$$\frac{n_b}{s} \simeq \frac{\varepsilon}{g_*}d, \tag{13}$$

where $g_* \sim 100$ is the effective degrees of freedom in the universe at $T \sim M_1$, and $d \leq 1$ is the so-called dilution factor. This factor parametrizes how the naively expected value $n_b/s \simeq \varepsilon/g_*$ is reduced due to washing-out processes. To evaluate the resultant baryon asymmetry precisely, numerical calculations are necessary, and the lower bound on the lightest heavy neutrino mass in order to obtain the realistic baryon asymmetry in the present universe $n_b/s \simeq 10^{-10}$ for fixed $\varepsilon/g_* \sin\delta \simeq 10^{-10}$ has been found to be $M_1 \gtrsim 10^9$ GeV [14].

For successful thermal leptogenesis, the reheating temperature after inflation should be higher than the lightest heavy neutrino mass. This fact causes a problem, when we consider the thermal leptogenesis in supersymmetric models. In supersymmetric models, if gravitinos produced from thermal plasma decay after big bang nucleosynthesis (BBN), high energy particles originating from the gravitino decay would destroy light nuclei successfully synthesized by the BBN. In order to maintain the success of the BBN, the number density of the produced gravitino is severely constrained to be small. The resultant number density of the produced gravitino is proportional to the reheating temperature, and then the upper bound on the reheating temperature has been found to be $T_R \lesssim 10^{6-7}$ GeV [15] for the gravitino mass being around the electroweak scale. The reheating temperature should be far below the lightest heavy neutrino mass. Therefore, in order to realize the successful thermal leptogenesis in supersymmetric models, new ideas are necessary, such as the gravitino as the lightest supersymmetric particle (LSP) [16].

In the standard cosmology, the expansion of the universe is governed by

$$H^2 = \frac{8\pi G_N}{3}\rho. \tag{14}$$

The condition of the out-of-equilibrium decay in Eq. (1) to provide sufficient lepton asymmetry without dilution, $d \sim 1$, is rewritten as

$$\tilde{m}_1 \equiv \sum_j (h_{1j}h^\dagger_{1j})\frac{v^2}{M_1} < \frac{8\pi v^2}{M_1^2} H|_{T=M_1} \equiv m_* \simeq 1\times 10^{-3}\ \mathrm{eV}, \tag{15}$$

with the decay width of the lightest heavy neutrinos

$$\Gamma_{N_1} = \sum_j \frac{h_{1j}h^\dagger_{1j}}{8\pi} M_1. \tag{16}$$

This condition can be regarded as the upper bound on the lightest neutrino mass m_1, since the inequality $m_1 \le \tilde{m}_1$ can be shown [17,18]. Considering Eq. (12), this upper bound is normally interpreted as an implication that thermal leptogenesis cannot generate sufficient baryon asymmetry in the case of the degenerate mass spectrum of light neutrinos [18,19].

III. LEPTOGENESIS IN BRANE WORLD COSMOLOGY

Let us consider the case where the lightest heavy neutrinos decays in the brane world cosmology era, namely $M_1 > T_t$. In the era, the expansion law of the universe is nonstandard such as

$$H = H_{st}\sqrt{1+\left(\frac{T}{T_t}\right)^4} \simeq H_{st}\left(\frac{T}{T_t}\right)^2. \tag{17}$$

Accordingly, the condition for the out-of-equilibrium decay of the heavy neutrino is modified as [2]

$$\sum_j \frac{h_{1j}h^\dagger_{1j}}{8\pi} M_1 < H(T=M_1) \simeq H_{st}(T=M_1)\left(\frac{M_1}{T_t}\right)^2. \tag{18}$$

Now we obtain the upper bound on the lightest neutrino mass in the brane world cosmology such that

$$\tilde{m}_1 < m_*\left(\frac{M_1}{T_t}\right)^2 \simeq 1\times 10^{-3}\ \mathrm{eV}\left(\frac{M_1}{T_t}\right)^2. \tag{19}$$

Note that the upper bound has been moderated due to the enhancement factor $(M_1/T_t)^2$ for $M_1 > T_t$. This result implies that, if T_t is low enough, the thermal leptogenesis scenario is successful even in the case of the degenerate light neutrino mass spectrum.

In the above discussion, we have implicitly assumed that the lightest heavy neutrino is in thermal equilibrium at a high temperature $T > M_1$. In the following, let us verify whether this situation can be realized in the brane world cosmology. Since the right-handed neutrinos are singlet under the SM gauge group, the only interaction through which the right-handed neutrino can be in thermal equilibrium is the Yukawa coupling in Eq. (7).[3] Consider a pair annihilation process of the lightest heavy neutrino through the Yukawa couplings. The thermal averaged annihilation rate is roughly estimated as

$$\Gamma_{\mathrm{int}}(T) = n\langle\sigma v\rangle \simeq \frac{3\zeta(3)}{2\pi^2}T^3\frac{Ng_Y^4}{100}T^{-2}, \tag{20}$$

where N is the number of annihilation channels, g_Y stands for dominant Yukawa couplings, and the factor $1/100$ denotes the kinematical phase factor. The freeze-in temperature, T_{FI}, in the brane world cosmology era can be defined from the condition $\Gamma_{\mathrm{int}}(T_{FI}) = H(T_{FI}) \simeq H_{st}(T_{FI})(T_{FI}/T_t)^2$, and we obtain

$$T_{FI} \simeq 1.1\times 10^{10}\ \mathrm{GeV}\left(\frac{Ng_Y^4}{10}\right)^{1/3}\left(\frac{g_*}{100}\right)^{-1/6}\left(\frac{T_t}{10^7\ \mathrm{GeV}}\right)^{2/3}. \tag{21}$$

Here we have used a large value for the normalization of the Yukawa coupling constant. It is necessary to fine tune

[2] It is nontrivial whether the same condition of the out-of-equilibrium decay is applicable for the brane world cosmology. After this work is finished, a related preprint has appeared [20], where the same subject is addressed and Boltzmann equations are numerically solved. Their results justify our rough estimation here in the same manner as the one in the standard cosmology, though there are some discrepancies between resultant numerical values given by their numerical calculations and the ones we estimate in this paper.

[3] If we extend our model into a model such as the left-right symmetric model, we can consider the case where the right-handed neutrinos can be in thermal equilibrium through new gauge interactions [21].

each element of large Yukawa couplings, in order to obtain masses of the light neutrinos much smaller than the scale naively obtained through the seesaw mechanism. Large Yukawa couplings might be reasonable because the thermal leptogenesis with the degenerate light neutrino mass spectrum is possible in the brane world cosmology as discussed above. Consistency of our discussion, namely $T_t \leq T_{FI}$, leads to the upper bound on the transition temperature such as

$$T_t \lesssim 1.3 \times 10^{16}\ \mathrm{GeV}\left(\frac{N g_Y^4}{10}\right)\left(\frac{g_*}{100}\right)^{-1/2}. \tag{22}$$

The thermal leptogenesis in the brane cosmology era can take place if the lightest heavy neutrino mass is in the range

$$T_t < M_1 < T_{FI}. \tag{23}$$

Recall that $M_1 \simeq 10^{9-10}$ GeV is required in order to generate the sufficient baryon asymmetry. This implies 10^{9-10} GeV $> T_t \gtrsim 10^{6-7}$ GeV (10^{12-13} GeV $> M_5 \gtrsim 10^{10-11}$ GeV) from Eqs. (21) and (23).

Finally, there is an additional interesting possibility for the thermal leptogenesis in the supersymmetric case. In the standard cosmology, the thermal leptogenesis in supersymmetric models is hard to be successful, since the reheating temperature after inflation is severely constrained to be $T_R \lesssim 10^{6-7}$ GeV due to the gravitino problem [15]. However, as pointed out in Ref. [10], the constraint on the reheating temperature is replaced with the one on the transition temperature in the brane world cosmology. Therefore, when the transition temperature is low enough $T_t \lesssim 10^{6-7}$ GeV, the gravitino problem can be solved even if the reheating temperature is much higher. Interestingly, the transition temperature $T_t \simeq 10^{6-7}$ GeV we found above can realize the thermal leptogenesis scenario successfully and also solve the gravitino problem. However, for this scenario, one should notice that inflation needs to have a very efficient reheating with $T_R \simeq 10^{9-10}$ GeV, because the potential energy during inflation V_{inf} must be less than $M_5^4 \simeq (10^{10-11}\ \mathrm{GeV})^4$ in order for inflation models to be consistently treated in the context of the five-dimensional theory. In fact, such inflation models are possible but limited [22]. Further studies on inflation would provide informations for this possibility.

IV. CONCLUSION

We have studied the thermal leptogenesis in the brane world cosmology. The nonstandard expansion law of the brane world cosmology affects the condition of the out-of-equilibrium decay of the lightest heavy neutrino, and moderates the upper bound on the lightest light neutrino mass. As a result, the degenerate mass spectrum for the light neutrinos can be consistent with the successful thermal leptogenesis scenario, if the transition temperature is lower than the lightest heavy neutrino mass. We have verified that the lightest heavy neutrino can be in thermal equilibrium through the Yukawa couplings among light neutrinos and Higgs bosons and found the region of the transition temperature consistent with the successful thermal leptogenesis in the brane world era. Then, we obtain the constraint on T_t (M_5). Furthermore, in supersymmetric case, we have noticed that the transition temperature required for the successful thermal leptogenesis can solve the gravitino problem simultaneously.

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