Thesis on the design of a circular reinforced concrete arch for C. E. degree

by

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The following data is assumed. (1) Span of intrados curve 30 feet. (2) Rise 12 feet. Thickness of arch at crown 2 feet.

Having this data , proceed as follows:- Divide the arch span into equal spaces, beginning at the center of the span, and erect ordinates. Draw a semicircle through these points called intrados curve. From the center of the intrados curve draw two lines making angles of 60° with the vertical. The thickness of the ring at these points can be computed from the relation $s_1 : \frac{bh^2}{6} = s_2 : \frac{bh^2}{6}$ when

s_1 = length of arc at crown intercepted by two ordinates.

s_2 = length of arc at 60° intercepted by two ordinates.

$\frac{bh^2}{6}$ = section modulus at corresponding points.

Now draw the extrados curve through these three points, and draw the center of gravity line between these two curves.

A blanket of earth four feet deep at center of arch span is assumed as part of the dead load. This earth absorbs impact, and thereby lessens the liability of concrete to crack.

Compute the weight of earth and concrete in a section of the arch one foot wide, assuming earth to weigh 100# and concrete 150# per cubic foot. All further discussion will relate to a section of the arch one foot in width. The live load is taken as 3000# per lineal foot of span.

The stresses are determined with the live load extending over one-half the bridge, as this is the position of loading which produces the greatest strains.

Draw a force polygon with any pole distance, and then construct trial equilibrium polygon. Notice carefully the system of numbering the ordinates. For instance , ray 2 , in the lower part of the force diagram , crosses ordinate two on the lefthand side of the arch. Each load is designated by the numbers between which it occurs.

Now the problem is to draw a line m m , so that the sum of the ordinates , called M ordinates, from this line to the trial equilibrium polygon, shall be zero. Ordinates measured above are plus and below minus. Table No. 1 , Column No. 1,contains the numbers of these ordinates. Columns Nos. 2 and 3 contain those parts of the ordinates included in the trial polygon measured to the scale used for the arch ring. Columns No. 4 contains the respective differences of symmetric ordinates about the center.

Beginning at zero ordinate , measure the lengths of ordinates in trial equilibrium polygon and put them in Columns Nos. 2 and 3 of table, - half of zero ordinate being put in left column, and half in right. In the fourth column is out the numerical differences of these ordinates In the fifth column these differences are multiplied by the numbers in Column No. 1. From this data compute V W from the relation $V W = \frac{12(d+2d+3d+.....)}{(n+1) (n+2)}$ where n = the number of spaces or number of ordinates, less one. V W always occurs on that side of the polygon which has the longest ordinates. The middle ordinate to m m is found by dividing R , the sum of the ordinates within the polygon, by the number of ordinates. Through the top of this ordinate draw m m , parallel to V W .

Likewise in the arch, measure the ordinates between O Y and the gravity axis. These values are found in column eight. Divide the sum of the ordinates by the number of spaces, plus one. Draw a line K K , parallel to O Y, and at a distance above it equal to the quotient

just found. Ordinates called K ordinates, in value measured from the line K K to the gravity axis of the arch, have a similar relation to the ordinates within the arch, as the m ordinates to the line m m have to the ordinates within the trial polygon.

The line m m cuts the trial equilibrium polygon in points E and E. Project these points upwards to e and e in line K K . Points e and e are points in the required pressure curve. Now measure the m and k ordinates which are found in Columns six and seven, nine and ten. These respective algebraic sums should be zero.

Next multiply each m and k ordinate by its corrssponding y ordinate, due regard being paid to signs. The products with their algebraic sums , are found in Columns eleven to fourteen inclusive.

Referring to the force diagram, draw P r parallel to m m . From r draw an indefinite horizontal line. The true pole for the pressure curve lies on this line. A pole taken anywhere on this line will give an equilibrium curve passing through the points e and e. The pole distance for the pressure curve = $\frac{\text{summation my}}{\text{summation ky}}$ X trial P.D.

Having the true pole distance draw the true force polygon. Then, beginning at c or e , with the proper ray, draw the true pressure curve. Beginning at either point, the curve should pass through the other point.

Now if the pressure curve falls outside of the arch, the thickness of arch assumed at center, was not great enough, and a new determination will have to be made with a greater thickness of arch.

Whereever the pressure curve falls without the middle third of the arch section, tension occurs in that section. Steel is required only at such sections.

Referring to Table two.

bh = area of concrete

a = " of steel

f = Stress per in steel

c = " " on concrete

p = $\frac{a}{bh}$ percentage of steel in concrete

M = external moment = product of force taken from pressure line multiplied by distance of pressure line , at section under consideration , from outside edge of middle third.

Es = Modulus of elas in steel = 30,000,000 per

Ec = " of elas in concrete ; varies according to proportion of mixture

$\frac{Es}{Ec}$ = e/ = rates of Moduli assumed at 10,15, 20 and 30, according to mixture

$$c = \frac{2pt}{x} \qquad (1)$$

$$f = \frac{M}{n\left(1-\frac{x}{3}\right)a} \qquad (2)$$

$$\frac{Es}{Ec} = (1 - x) = \frac{x^2}{2p} \qquad (3)$$

In Column 1, Table 2, assume saome value of p , and from relation the $p = \frac{a}{bh}$ find area of steel a.

Then from Formula (1) see what the stress in steel actually is, and vary p until stress is about 12,000 lbs. From equation (2) find c , which must be 500 lbs , or less.

The cost for concrete and false work will be $12.00 a cubic yard , according to figures from the L. S. & M. S. R. R., which will make a single track bridge of this type 10 feet wide cost about $12,500 exclusive of foundations.

, The steel will weight 4500 lbs and cost $56.25 , neglecting freight. Total cost $12,ſ56.25.

In this work frequent reference was made to the ELASTIC ARCH by Burton R. Lefler and REINFORCED CONCRETE by Taylor and Thompson. Many thanks are also due to Prof. F. H. Sibley of the University of Alabama for assistance in the drafting room.

TABLE II

p	x			
	$E_s \div E_c$			
	10	15	20	30
0.001	0.138	0.158	0.181	0.217
2	181	217	246	292
3	217	258	292	344
4	246	292	328	384
5	270	320	358	418
6	292	344	384	446
7	311	365	407	471
8	328	384	428	493
9	344	402	446	513
10	358	418	463	531
12	384	446	493	562
14	407	471	519	588
16	428	493	542	611
18	446	513	562	631
0.020	0.465	0.531	0.580	0.649