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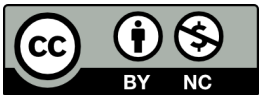

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The Life and Lasting Influence of Srinivasa Ramanujan

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INTRODUCTION

"The wonderful thing about Ramanujan is that he discovered so much, and yet he left so much more in his garden for other people to discover."— (Freeman, 263)

Many great men and women, like meteors, come, shine, leave a mark and disappear prematurely. Srinivasa Ramanujan (Figure 1) was one of these people. His birth, contributions to the field of mathematics, and early death all seem to have happened in a flash, in a short span of just 32 years in the early twentieth century (). His influence has since spread far from India to the entire world of mathematics and technology.

FIGURE 1 Bronze Bust of Ramanujan by Paul Garlund, Gustavus Adophus College, Minnesota. Photo Courtesy: Creative Commons: Google Image (accessed September 8, 2011).

Srinivasa Ramanujan was born to K. Srinivasa Iyengar and Komalatammal on December 22, 1887. They were a working-class Brahmin family in Erode, in southern India. Ramanujan was the first of three sons in his family. During his early school years, both Ramanujan's teachers and the upper classmates above him were astounded with his extraordinary proficiency in arithmetic, algebra, geometry, number theory, and trigonometry despite having had no formal training in these subjects. But it was in 1903 that G. S. Carr's *A Synopsis of Elementary Results in Pure Mathematics* laid a foundation for his advanced work in the subject for the next seventeen years (). Ramanujan's extreme love for numbers hindered his ability to concentrate on other subjects required to graduate from college. As a result, his formal education

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actually came to a standstill. However, his state of mathematical obsession was at least partially lifted as he was eventually obligated by the social customs of the times to care of his parents and provide financial support to his family after he married Janaki Ammal in 1909. He therefore took up a reliable, if only clerical, job at Madras Port Trust Office in 1912. Regardless of his pressing family concerns, Ramanujan continued his mathematics research writing. To save money on notebooks he would only enter his finished work after having worked out his conjectures, formulas, propositions, and results in great detail on an erasable slate. Despite his extreme preoccupation with mathematics, his supervisors at work, particularly Narayana Iyer, the chief administrator, who himself held a master's degree in mathematics, were highly supportive (). Ramanujan's self-taught profound mastery in mathematics, and the resulting notebooks, became a hot topic of conversation among the learned in Madras during 1913 and 1914 () and set the stage for a wider appreciation of his talent achievements at home and abroad (Srinivasa Rao 2011).

COLLABORATION AND FRIENDSHIP WITH HARDY

At the suggestion of his many well-wishers in India, particularly Seshu Iyer in January 1913, and despite a lack of interest in his work shown by two other British university mathematicians to whom he had sent earlier letters, Ramanujan sent yet another letter along with a set of mathematical results and 120 theorems to G. H. Hardy, the Cayley Lecturer in Mathematics at Cambridge University in England (). Upon receiving mail from a total stranger in a foreign land, Hardy glanced through the letter, only to be amazed by its contents. Hardy's first reaction was that the person purporting to be Ramanujan had to be either a well-established mathematician who was playing a trick on Hardy or a hitherto undiscovered genius mathematician (), whereupon Hardy called for his long-time friend and collaborator J. E. Littlewood to look over Ramanujan's work. It did not take long before the two were able to conclude that Ramanujan was indeed an as-yet undiscovered man of mathematical genius (). This famous letter along with many more written by Ramanujan to Hardy has since been published in the *Collected Papers of Srinivasa Ramanujan* () and in several other books and journals.

C. P. Snow wrote in the foreword to *A Mathematician's Apology* () “Once Hardy was determined [to bring Ramanujan to Cambridge University in England], no human agency could have stopped him, but Hardy needed a certain amount of help from a superhuman” (, 34) because Ramanujan had religious scruples about traveling across the ocean.

> Fortunately his mother had the highest respect for the Goddess of Namakkal. One morning Ramanujan's mother made a startling announcement. She had had a dream on the previous night, in which she saw her son seated in a big hall among a group of Europeans, and the Goddess of Namkkal had commanded her not to stand in the way of her son fulfilling his life's purpose. This, say Ramanujan's Indian biographers, was a very agreeable surprise to all concerned. (, 34–35)

Ramanujan arrived in England in 1914, and the Hardy–Ramanujan association and collaboration began. One of the earliest joint theorems of Ramanujan and Hardy involved the partitions of a number *n* that determines the number of ways in which *n* can be expressed as a sum of whole numbers or, more formally:

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> A partition of integer “n” is a division of n into any number of positive integral parts, where the number of partitions of “n” is denoted by $p(n)$. For example: 4 = 3 + 1 = 2 + 2 = 2 + 1 + 1 = 1 + 1 + 1 + 1. In effect, the number 4 has 5 partitions, or p (4) = 5. (, 111)

Littlewood said, “We owe the [partition] theorem to the happy collaboration of two men of quite unlike gifts, in which each contributed the best, most characteristic and most fortunate work that was in him” (Turnbull 1993, 140).

Far in advance of the invention of the modern computer, Ramanujan also devised his infinite series 1/π, which is now incorporated in computer algorithms to enable one to calculate $\mathrm{pi}\ \ (\pi)$ to millions of decimal places ().

When often asked what made Ramanujan stand out in his field, Hardy would answer an extraordinary memory. Ramanujan could remember numbers in a mysterious way, and he has quoted what Littlewood once said: “Every integer was one of the personal friends of Ramanujan” (, 12). The following spontaneous exchange is instructive. According to an account by (, 113), it happened during a visit by Hardy to an ailing Ramanujan.

Hardy: I came in the taxi-cab 1729. It is rather a dull number. I hope it is not an unfavorable omen.
Ramanujan: No, it is a very interesting number.
Hardy: How?
Ramanujan: It is the smallest number expressible as the sum of two cubes in two ways: (1729 = $1^3 + 12^3 = 9^3 + 10^3$).

In 1917, Ramanujan contracted tuberculosis and went into a nursing home in Cambridge. He later moved to sanatoria at several locations in and around London. In the last letter that he wrote to Hardy in January 1920 he discussed his mock-*theta* functions, the work he began before left Cambridge on sick leave. In this letter Ramanujan explained why he named his latest research the mock-*theta* function: “I am extremely sorry for not writing you a single letter up to now. … I discovered very interesting functions recently which I call ‘mock’ *theta*-functions. Unlike the ‘false’ *theta*-functions (studied partially by Prof. Rogers in his interesting papers) they enter into mathematics as beautifully as the ordinary *theta*-functions. I am sending you with this letter some examples … ” (, xxxi). This mock-*theta* function is now known as Ramanujan's *theta* function.

In his presidential address to the London Mathematical Society in 1935, G. N. Watson stated:

> Ramanujan's discovery of the mock *theta* functions makes it obvious that his skill and ingenuity did not desert him at the oncoming of his untimely end. As much as any of his earlier work, the mock *theta* functions are an achievement sufficient to cause his name to be held in lasting remembrance. (, 80)

The collaboration of Ramanujan–Hardy is summed up well in Part II of *Ramanujan's Notebooks*, by editor Bruce C. Berndt:

> The relation between Hardy and Ramanujan is unparalleled in scientific history. Each had enormous respect for the abilities of the other. … Although Ramanujan returned from England

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> with a terminal illness, [according to his wife] he never regretted accepting Hardy's invitation to visit Cambridge. (, on the back of the dedication page)

Hardy noted: “I still say to myself when I am depressed and find myself forced to listen to pompous and tiresome people: Well, I have done one thing you could never have done, and that is to have collaborated with Littlewood and Ramanujan on something like equal terms” (, 148).

HOW THE FOUNDER OF INDIAN LIBRARY SCIENCE BECAME THE RESCUER OF THE THIRD OF RAMANUJAN'S NOTEBOOKS

Shiyali Ramamrita Ranganathan (1892–1972), widely considered the father of Indian library science, played a vital role in preserving the work and memory of Ramanujan. By a fortunate happenstance, Ranganathan was trained as a mathematician and, in fact, had published a number of short papers on the history of mathematics and the lives of mathematicians, before he was ultimately named the first librarian at the University of Madras. He recalled that he had become acquainted with Ramanujan's work, which was even then being serialized in some Indian publications while Ranganathan was still a university student:

> It was a happy idea of the Founder of the Society (Indian Mathematical Society), V. Ramaswamy Ayyar, to have instituted the pages “Questions and Solutions.” I remember with what avidity some of us young men used to look forward to these pages from issue to issue. (, 119)

In 1924, Ranganathan went to England for one year to study library science and observe UK libraries at work. Ranganathan met Hardy in Oxford and discussed the archiving of Ramanujan's two notebooks back in India and mentioned his search for a reported “third book” that Ramanujan might have left behind in Cambridge that Ranganathan wished to consult so that he might write the biography of Ramanujan with the knowledge of any new mathematics within those notes. To his surprise, Hardy went on to find the notebook for him and insisted that it be returned to a place of honor in India.

THE DISCOVERY OF YET MORE OF RAMANUJAN'S “LOST NOTEBOOKS”

But Ramanujan's influence on mathematics was scarcely ended even then. In 1975, a mathematician from Penn State University, George Andrews, was asked to sort through the remaining papers of the late George N. Watson, a Cambridge University mathematics professor, who happened to work in the same field as Ramanujan and Andrews himself and who had from time to time consulted the archives of Ramanujan in the Wren Library at that university. Watson's papers were otherwise scheduled to be incinerated in the following week. To his astonishment, Andrews came across over one hundred loose pages filled with equations in Ramanujan's handwriting. Dr. Andrews realized that he was looking at the legendary mock *theta* functions that Ramanujan had hinted at in his last letter to Hardy after his return to India. Andrews has since dedicated his career to studying this “marvelous collection” and bringing out “Ramanujan's Lost Notebooks” in many volumes (4664).

Once again, it was found that Ramanujan left behind a wealth of knowledge in areas such as applied mathematics and combinatorics—the science of counting—and computing (). Because many of Ramanujan's ideas and the work he did during the last year of his life were not found for over fifty-five years, other mathematicians unknowingly claimed them as their own discoveries those that were subsequently shown to have been first discovered by Ramanujan in his hitherto unpublished work. Dr. Bruce Berndt, another Ramanujan scholar at University of Illinois at Urbana-Champaign, discussed this

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situation in depth in his article in the *Asia Pacific Mathematics Newsletter* entitled “Ramanujan Reaches His Hand from His Grave to Snatch Your Theorems from You”.

THE EPONYMOUS RAMANUJAN

Ramanujan's legacy can be measured by some of the mathematical entities that bear Ramanjuan's name; Ken , 649–650) listed them as follows:

- The Dougall–Ramanujan identity
- The Landau–Ramanujan constant
- Ramanujan's theta-function
- Ramanujan's class invariants gn and Gn
- Ramanujan's identity
- Ramanujan's T-function
- Ramanujan's continued fraction
- Ramanujan graphs
- Ramanujan's mock theta functions
- The Ramanujan–Nagell equation
- The Ramanujan–Petersson Conjectures
- Ramanujan sums
- Ramanujan's theta-operator
- The Rogers–Ramanujan identities

In addition, the *Ramanujan Journal*, a Springer publication, is devoted to the areas of mathematics influenced by Ramanujan and edited by Krishnaswami Alladi of the Department of Mathematics, University of Florida. Some of the key areas in which the articles are published in this journal are mock-theta functions, divisor functions, modular equations, partitions, Roger–Ramanujan identities, and real quadratic fields.

THE RAMANUJAN RECORD IN CITATIONS

Using the Web of Science's Cited Reference feature, a cited author search (generated in September 2011) of “Ramanujan S*” and cited years as “1900–1921” yielded 992 articles citing Ramanujan from 1965 to 2011 as shown in Table 1.

TABLE 1 Citation Report. Cited Author = (Ramanujan S) AND Cited Year = (1900–1920)

Results found	992
Number of times cited	12,607
Number of times cited without self-citations	11,268
Citing articles	97,642
Average citation per item	12.71
h-Index	46

Source: Thomson Reuters Web of Knowledge

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Figure 2 shows how many citations were made each year to any items in the set. It shows which years produced the largest number of citing articles and which years produced the smallest number.

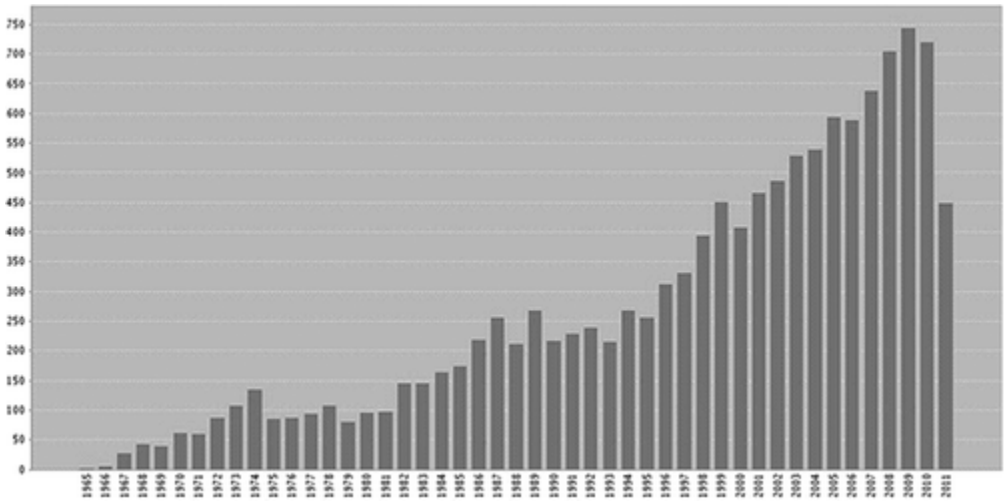

FIGURE 2 Graph of Citations in Each Year (1965–2011) Generated in September 2011.

Table 2 shows the subject areas in which Ramanujan's work was cited during 2011 as of October. There were forty-five articles, some of which were funded by agencies such as the National Science Foundation (NSF) and, most notably, the National Security Agency (NSA), essentially America's principle agency for communications intelligence and code-cracking.

TABLE 2 Citations to Ramanujan in Mathematics and Other Disciplines in 2011. Cited Author = (Ramanujan S*)

Mathematics (35)	Dermatology (1)	Marine freshwater biology (1)
Computer science (4)	Education educational research (1)	Ophthalmology (1)
Physics (3)	Environmental sciences ecology (1)	Pediatrics (1)
Engineering (2)	Forestry (1)	Water resources (1)
Biochemistry molecular biology (1)		

Source: Thomson Reuters Web of Knowledge

Table 3 indicates Ramanujan's most often cited articles according to the Web of Knowledge.

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TABLE 3 Most Often Cited References. Cited Author Search Ramanujan S, Cited Work 1900–1920

Authors	Journal title	Year of publication	Vol. no.	Number of citing articles
With Hardy	*Proceedings of the London Mathematical Society*	1918	17	316
Ramanujan	*Transactions of the Cambridge Philosophical Society*	1916	22	158
With Hardy	*Quarterly Journal of Mathematics*	1917	48	116
Ramanujan	*Quarterly Journal of Mathematics*	1914	45	111
Ramanujan	*Proceedings of the London Mathematical Society*	1915	14	55
Ramanujan	*Proceedings of the Cambridge Philosophical Society*	1919	19	54
Ramanujan	*Transactions of the Cambridge Philosophical Society*	1918	22	52
With Hardy	*Proceedings of the London Mathematical Society*	1917	16	44
Ramanujan	*Messenger of Mathematics*	1916	45	40
Ramanujan	*Messenger of Mathematics*	1915	44	28
Ramanujan	*Proceedings of the Cambridge Philosophical Society*	1919	19	28

Source: Thomson Reuters Web of Knowledge

CITATIONS TO RAMANUJAN AND RAMANUJAN-RELATED MATHEMATICS

To get a more mathematically focused idea of Ramanujan's influence, one can use the Author Citations feature in MathSci MR (Mathematical Reviews). Using this approach, as of September 2011, Ramanujan had been cited 443 times by 227 authors in the literature of pure and applied mathematics, statistics, and computing. These citations to Ramanujan are mostly from papers published between 2000 through the present, although in some cases, due to papers that were indexed slowly in MathSci MR, some citations to him made as early as 1997 are included (Table 4).

TABLE 4 Citations to Ramanujan and Ramanujan-Related Publications 1997–Present

MR author ID	Earliest index publication	Total publications	Total author/author-related publications	Total citations
144415	1957	44	199	443

Source: American Mathematical Society MathSciNet Mathematical Reviews on the web. (accessed September2011).

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