SUPERSYMMETRIC MINIMAL B-L
AND LEFT-RIGHT MODELS

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Abstract

We propose a simple gauged $\mathrm{U}(1)_{B-L}$ extension of the minimal supersymmetric Standard Model (MSSM), where R-parity is conserved as usual in the MSSM. The global $B-L$ (baryon minus lepton number) symmetry in the MSSM is gauged and three MSSM gauge-singlet chiral multiplets with a unit $B-L$ charge are introduced, ensuring the model free from gauge and gravitational anomalies. We assign an odd R-parity for two of the new chiral multiplets The scalar component of the R-parity even superfield plays the role of a Higgs field to beak the $U(1)_{BL}$ symmetry through its negative mass squared which is radiatively generated by the renormalization group running of soft supersymmetry (SUSY) breaking parameters. Because of our novel R-parity assignment, three light neutrinos are Dirac particles with one massless state. Since R-parity is conserved, the lightest superpartner (LSP) neutralino is a prime candidate of the cosmological dark matter. In particular, the $B-L$ gauge boson (Z'), once discovered at the Large Hadron Collider, will be a novel probe of the Dirac nature of the light neutrinos since its invisible decay processes include the final states with one massless (left-handed) neutrino and two Dirac neutrinos, in sharp contrast with the conventional $B-L$ extension of the SM or MSSM, where the right-handed neutrinos are heavy Majorana particles and decay to the SM leptons. We generalize a variation to the SUSY Left-Right symmetric model based on the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{BL}$. The charge conjugate $SU(2)_L$ singlets are put into $SU(2)_R$ doublets, mirroring the former. We only introduce a second Higgs bidoublet to produce realistic fermion mass matrices. We calculate renormalization group evolutions of soft SUSY parameters at the one-loop level down to low energy. We find that an $SU(2)_R$ slepton doublet acquires a negative mass squared at low energies, so that the breaking of $SU(2)_R \times U(1)_{BL} \to U(1)_Y$ is realized by a non-zero vacuum expectation value of a right-handed sneutrino. Small neutrino masses are produced through neutrino mixings with gauginos. Mass limits on the $SU(2)_R \times U(1)_{BL}$ sector are obtained by direct search results at the LHC as well as lepton-gaugino mixing bounds from the LEP precision data.

Dedication

In memory of David J. Miller

List of Abbreviations and Symbols

$SU(N)$	N-dimensional Special Unitary Group
$U(N)$	N-dimensional Unitary Group
EW	Electroweak
EWSB	Electroweak Symmetry Breaking
c	Speed of light
G_N	Gravitational Constant
α_{em}	Fine structure
$\hbar$	Reduced Planck constant
QED	Quantum Electrodynamics
QCD	Quantum Chromodynamics
h.c.	Hermitian conjugate
SM	Standard Model
VEV	Vacuum Expectation Value
CKM	Cabibbo-Kobayashi-Maskawa Matrix
PMNS	Pontercorvo-Maki-Nagawa-Sakata Matrix
M_{pl}	Planck Mass
SUSY	Supersymmetry
eV	Electronvolts
GeV	Giga-electronvolts

TeV	Tera-electronvolts
$\not{R}_p$	R-parity breaking/violating
LSP	Lightest Superpartner
LHC	Large Hadron Collider
LR	Left-Right
LRM	Left-Right Model
RGE	Renormalization Group Equation
GUT	Grand Unified Theory
LEP	Large Electron-Positron Collider
δ_{ij}	Kronecker delta
ϵ_{ij}	Levi-Civita Symbol

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1 Introduction

We use quantum field theory to describe interactions between several different species of fundamental particles. The Standard Model (SM) is the best description of this in that every free parameter has been experimentally measured. Though there is physics that cannot be explained solely with the SM. It suffers from a gauge hierarchy problem, no neutrino masses, and no dark matter candidates to say the least. We introduce an additional global symmetry between fermions and bosons to try and rectify these open problems. The Lie algebra of the Lorentz group is called Poincaré algebra. If we further extend this algebra with two Weyl spinors, it is a Super-Poincaré algebra[1] called supersymmetry (SUSY). By doing this we can extend the SM to the Minimal Supersymmetric Standard Model (MSSM). This adds a scalar partner to every fermion and fermion partners for the Higgs and gauge fields. These additional fields add new interactions that solve the gauge hierarchy and has a dark matter candidate. The MSSM must be further extended to explain neutrino oscillation, lack of right-handed neutrinos and absence of low energy superpartners. In this thesis, we introduce new gauge groups to explain neutrino observations as well adhere to the experimental constraints.

In any high energy model the strength of the couplings that govern particle interactions are on a sliding scale with respect to energy. The fine structure coupling, $\alpha_{EM} \approx 1/137$, becomes larger as the center of mass energy of the system increases through radiative corrections. By analyzing every coupling of the MSSM, the soft mass squared for the up-type Higgs becomes negative at low energies if ran backwards from very high energies. This generates a non-gauge invariant vacuum and breaks the Electroweak gauge symmetry. If we introduce new gauge groups, they must be broken at low energies.

[1] Also known as a Z_2 graded algebra.

In chapter 1 we briefly review the SM and Electroweak Symmetry Breaking (EWSB). Following that we discuss a very common extension based on gauging baryon minus lepton number $(B - L)$, which is an anomaly free global symmetry in the SM. In Chapter 3 we introduce the MSSM and the tools we will need for our work. We propose in Chapter 4 and 5 a minimal SUSY $B - L$ extension to the MSSM. Chapter 5 and 6 are original works where the former is under the review process and the later has been accepted into Physics Letters B[2, 3]. In chapter 6 we study a minimal SUSY Left-Right (LR) model and perform a radiative analysis on it. In both chapters 5 and 6 we show we can recover the SM after radiative breaking of all the additional gauge symmetries.

2 The Standard Model: A Review

2.1 Mathematical Formulation[1]

I shall take the Kenneth Wilson's approach by setting the fixed point for our discussion at the end of the 1960s. The current understanding of the nature of so called "weak" interactions was taking shape as the Glashow-Weinberg-Salam model. Today we call it the Electroweak interaction and it's built on the neutral and charged current interaction with the charged fermions. This is a a result of a broken $SU(2) \times U(1)$ symmetry[4, 5], which is broken by a scalar doublet (now known as the Higgs field)[6, 7, 8]. The understanding of the strong nuclear force was formulated by many, but its current understanding was formed somewhere by the end of the 1970s[9].

The Standard Model is based on the Yang-Mills theory of a non-Abelian gauge invariant Lagrangian. Quantum Electrodynamics (QED) has shown remarkable success using an Abelian gauge invariant Lagrangian to describe relativistic electromagnetic interactions[12]. The Standard Model (SM) is based on the Yang-Mills theory of a non-Abelian gauge invariant Lagrangian. The SM is composed of three gauge group; hypercharge (Y), the weak (only acts on left-handed particles), and the colored strong nuclear force. We write this as $\mathcal{G}_{SM} = U(1)_Y \times SU(2)_L \times SU(3)_c$, respectively. The gauge bosons, which mediate the force, are represented by the adjoint representation of their respective group. All fields are massless prior to the Electroweak Symmetry Breaking (EWSB). The matter fields are composed of three generations of fermions: six quarks, six leptons, and one scalar: the Higgs (H). The six quarks are named up (u), down (d), strange (s), charm (c), bottom (b) and top (t). For the leptons,the charged leptons are the electron (e), the muon (μ) the tau (τ) and three

[1]This is textbook material, I refer you to these texts[10, 11]

$SU(3)_c$	Gluons: G^i_μ, $i = 1, ..8$	$G^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - ig_c[G^j_\mu, G^k_\nu]$
$SU(2)_L$	W^i_μ, $i = 1, 2, 3$	$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - ig_L[W^j_\mu, W^k_\nu]$
$U(1)_Y$	B_μ	$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$

Table 2.1.: The number of gauge bosons for a $SU(N)$ is $N^2 - 1$. The W^i_μ and B_μ will mix after EWSB into the $W^\pm_\mu$, Z_μ, and A_μ, the photon. The lower case g's are the couplings for each group.

neutrinos (ν) associated with each charged lepton. In the SM there are left and right-handed copies of each particle, except for neutrinos, as there are no right-handed neutrinos. The matter interacting under $SU(2)_L$ is represented as a doublet in a chiral basis. In QED all matter is written in the Dirac basis, but in the SM all matter is chiral (Weyl basis). The left or right-handed chiral wavefunction can be extracted from the Dirac basis using projection operators

$$P_L\psi = \frac{1}{2}(1 - \gamma_5)\psi = \psi_L \quad \text{and} \quad P_R\psi = \frac{1}{2}(1 + \gamma_5)\psi = \psi_R\,.$$

The left-handed matter is written in their doublets as

$$Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \tag{2.1}$$

$$L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix} \tag{2.2}$$

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.3}$$

In literature, the $SU(2)_L$ doublets will just be called $Q_{L\,i}$ and L_i where i runs from left to right in equations (2.1) and (2.2). The entire particle context of particle models are expressed as tables with their representations as seen for the SM in Table 2.2. The kinetic sector of the Lagrangian can be now written down. In an effort to compactify our notation, we

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q	3	2	1/6
u_R	$\bar{3}$	1	2/3
d_R	$\bar{3}$	1	-1/3
L	1	2	-1/2
e_R	1	1	-1
H	1	2	+1/2

Table 2.2.: The particle content of the Standard Model. The generation index here is assumed and will be further suppressed to make our notation a bit easier.

define the covariant derivative of a field as $D_\mu = \partial_\mu - igA^i_\mu \lambda^i$ where λ^i are the generators for an $SU(N)$ group, g is the gauge coupling and A_μ's are the gauge bosons. For $SU(3)$ the generators are the Gell-Mann matrices, for $SU(2)$ the generators are $1/2\sigma^a$, and for $U(1)$ the generator is a charge. Here σ^a are the Pauli matrices. In the case of a $U(1)$ group a particle of charge q's covariant derivative is written as $D_\mu = \partial_\mu - igqA_\mu$. The gauge kinetic terms in the Lagrangian are

$$\begin{aligned}\mathcal{L} \supset\ & \overline{Q} i\gamma^\mu D_\mu Q + \overline{u}_R i\gamma^\mu D_\mu u_R + \overline{d}_R i\gamma^\mu D_\mu d_R + \overline{L} i\gamma^\mu D_\mu L + \overline{e}_R i\gamma^\mu D_\mu e_R \\ & + (D_\mu H)^\dagger (D^\mu H) - \frac{1}{2g_c^2}\mathrm{Tr}\,(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{2g_L^2}\mathrm{Tr}\,(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4g_Y^2}F_{\mu\nu}F^{\mu\nu}\,.\end{aligned} \tag{2.4}$$

The rest of the Lagrangian can be separated into the Yukawa interactions[2] and the scalar potential. The only scalar in the SM is the Higgs, so it may be called the Higgs potential. In the weak sector the quarks experience a flavor mixing interaction, so for the potential we shall explicitly write the flavor indicies for the Yukawa terms. The renormalizable potential is

$$\mathcal{L}_Y = -Y_u^{ij}\overline{Q}_{i\,a}\sigma^{ab}H_b^\dagger u_{R_j} - Y_d^{ij}\overline{Q}_i \cdot H d_{R_j} - Y_e^{ij}\overline{L}_i \cdot H e_{R_j} \tag{2.5}$$

$$V_H = \frac{\lambda}{4}\left(|H|^2 - \frac{v^2}{2}\right)^2, \tag{2.6}$$

[2]fermion-Higgs-fermion interactions

where the three Y's are 3×3 matrices, v is the Higgs VEV, and λ is the Higgs quartic coupling. Prior to Electroweak Symmetry Breaking (EWSB), all particles are massless. In total there are 19 free parameters in the SM.

2.2 ElectroWeak Symmetry Breaking

In the Standard Model there is no dynamical source that creates a nonzero VEV for the Higgs boson. Several simple extensions have been proposed, one famous is known as the Coleman-Weinberg Mechanism which drives the Higgs quartic coupling negative through loop corrections[13]. The Higgs mechanism results in the breaking of the unified Electroweak to quantum electrodynamics (QED), $SU(2)_L \times U(1)_Y \to U(1)_{em}$. The neutral component of the Higgs doublet develops a nonzero VEV,

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h^0 \end{pmatrix}, \tag{2.7}$$

where h is the physical neutral Higgs. The measured value of $v = 246$ GeV. There is a change of basis associated with the EWSB. After breaking, there are two electrically charged gauge bosons and two neutral gauge bosons. In terms of the EW gauge basis we define the new EWSB gauge boson basis as

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}} \left(W^1_{\mu} \mp i W^2_{\mu} \right) \tag{2.8}$$

and

$$\begin{pmatrix} A_{\mu} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} B_{\mu} \\ W^3_{\mu} \end{pmatrix}, \tag{2.9}$$

where θ_W is the Weinberg angle defined by the EW couplings as

$$\sin\theta_W = \frac{g_Y}{\sqrt{g_L^2 + g_Y^2}}. \tag{2.10}$$

The $W^{\pm}$ and Z gauge bosons develop masses while the A is the photon in QED. Their masses are defined in terms of the Higgs VEV and gauge couplings,

$$
\begin{aligned}
m_Z &= \frac{v}{2}\sqrt{g_L^2 + g_Y^2}\,, \\
m_W &= \frac{v}{2} g_L\,, \\
m_A &= 0\,.
\end{aligned}
\tag{2.11}
$$

The charge relation for QED is $Q_{em} = \frac{1}{2}\sigma^3 + \frac{1}{2}Q_Y$ and the QED coupling by $e = g_L \cos\theta_W$. The fermions all get their masses from the Yukawa sector, except for neutrinos, which remain massless. The down quarks and charged lepton mass matrices are all measured in the diagonal basis, while the up quark mass matrix must be diagonalized. The matrix that does this is called the Cabibbo-Kobayashi-Maskawa (CKM) matrix, V_{CKM}. Each of the 3x3 matrices are expressed as

$$M_u = \frac{1}{\sqrt{2}} v Y_u V_{CKM}^{\dagger} \tag{2.12}$$

$$M_d = \frac{1}{\sqrt{2}} v Y_d \tag{2.13}$$

$$M_e = \frac{1}{\sqrt{2}} v Y_e\,. \tag{2.14}$$

This need for diagonalizing the up quark mass matrix comes from observed weak charged current having flavor mixing interactions with the EW bosons. If this were not observed all mass matrices would be diagonal.

2.3 Reasons for going Beyond the Standard Model (BSM)

With the Higgs boson detection at the Large Hadron Collider (LHC), all parameters of the SM have been measured[14]. However, there are still many open problems in physics that the SM does not answer.

2.3.1 Neutrino Oscillation

In weak interactions, the quarks will oscillate between flavors parameterized by the CKM matrix. It was found that the neutrinos, which were thought to be massless (and thus should not oscillate), they oscillate as they propagate. There are no right-handed neutrinos included in the SM either, neutrino oscillation shows us that there must be right-handed neutrino. Their mixings can be parameterized by the Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix) much akin to the CKM matrix, with different rotation angles. In fact, the CKM matrix is close to unity while the PMNS has much larger rotations. Because neutrinos are too light to measure their absolute mass, only the mass differences can be measured, which are sub-eV.

2.3.2 Gauge Hierarchy

Calculating the one-loop corrections to all the parameters of the SM yield logarithmic divergences (which is well behaved under renormalization) except, for the Higgs mass squared which diverges quadratically. A high energy cutoff scale is taken to keep the Higgs mass squared finite which is of the order 10^{18} GeV. This is the Planck scale, M_{Pl}, the highest energy scale in the SM where $M_{Pl} = \sqrt{\hbar c/8\pi G_N} = 2.4 \times 10^{18}$ GeV/c^2. We express the physical mass, m_H^2 as a sum of the bare Higgs mass, μ^2 in the Lagrangian plus all corrections, δm^2 as

$$m_H^2 = \mu^2 - \delta m^2 \,. \tag{2.15}$$

The problem arises that $m_H = 125$ GeV[15] and as previously stated $\delta m^2 \approx (10^{18}\,\text{GeV})^2$ then $\mu^2 \approx (10^{18}\,\text{GeV})^2$. This precise cancellation of two 10^{36} GeV^2 terms to produce a 10^4 GeV^2 term is known as the Gauge Hierarchy Problem.

2.3.3 Dark Matter

I we normalized the ratio of mass to luminosity of the Sun and compare this other observed galaxies, we find there is a large portion of non luminous matter. Further observations of curves of galaxies show there is massive amounts of mass that doesn't interact under the SM, but exerts a significant gravitational pull. The Planck experiment measures many cosmological parameters that are evidence of the presence of dark matter [16]. Currently, there is no viable dark matter candidate in the SM. The reader might read further in Ref. [17].

2.3.4 EWSB Mechanism and Higgs Potential Stability

At present, the SM potential (2.5) has no internal mechanism for the Higgs to develop a nonzero VEV. In addition, it is the only particle to have a mass term prior to the EWSB. At large energies, through radiative corrections to the quartic coupling, there is a possibility that the Higgs could develop a nonzero VEV[18]. If this is the case, the EW symmetry would be meta-stable. The RGEs for the quartic coupling, λ and the rest of the SM are found in the Appendix. Possible non-SUSY extensions using an additional $U(1)$ gauge group can keep the EW symmetry stable at larger energies [19].

3 Baryon minus Lepton Number

Baryon and lepton number are accidental global symmetries of the SM. If we gauge the $B-L$ symmetry with a gauge coupling, g_{BL}, the global number symmetries are automatic[1]. In this section, we will briefly talk about the model independent aspects of including a $U(1)_{BL}$ gauge group into the SM. One of the main motivations for introducing a $B-L$ symmetry is to include SM singlet right handed neutrinos, N^c, to generate neutrino mass. Three generations must be added to cancel out diagrams like Figure 3.1 to cancel out the SM matter running in the loop and make this model gauge anomaly free. This symmetry must be broken or there would be additional $B-L$ bosons interacting with our fermions. We include a $B-L$ Higgs called Φ. The particle contents of the simplest extension are seen in Table 3.1.

[1]A published review on this can be found at [20]

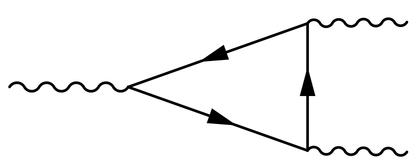

Fig. 3.1.: One-loop corrections like this must disappear for $U(1)$ to be anomaly free. This comes from the requirement that $U(1)$ gauge bosons do not self-interact.

	$U(1)_{BL}$
Q	$1/3$
U^c	$-1/3$
D^c	$-1/3$
L	-1
E^c	1
N^c	1
H	0
Φ	-2

Table 3.1.: Particle charges under $U(1)_{BL}$. Here we suppress the generation indicies for the fields here. The Φ is the $B-L$ Higgs.

In the Lagrangian we can write the neutrino Yukawa interaction as well as the neutrino $B-L$ Higgs interaction,

$$-\mathcal{L} \supset Y_{ij}^{D} H \cdot L_i N_j^c + Y_i^M \Phi N_i^c N_i^c, \tag{3.1}$$

where Y_ν^D is the Dirac Yukawa coupling and Y^M is the Majorana Yukawa coupling. The $B-L$ Higgs Φ develops a nonzero VEV of v_{BL} creating a Majorana mass term for the N^c and the Z' gauge boson,

$$\langle\Phi\rangle = \frac{v_{BL}}{\sqrt{2}} \tag{3.2}$$

$$m_{Z'} = 2 g_{BL} v_{BL} \tag{3.3}$$

$$M_{N_i^c} = \sqrt{2} v_{BL} Y_i N_i^c N_i^c. \tag{3.4}$$

Prior to EWSB the Feynman diagrams for the effective coupling at $B-L$ breaking are seen in Figure 3.2. As the energy of our system decreases the N^c does not propagate and the interaction becomes an effective 4-point interaction. After EWSB and the SM Higgs develops

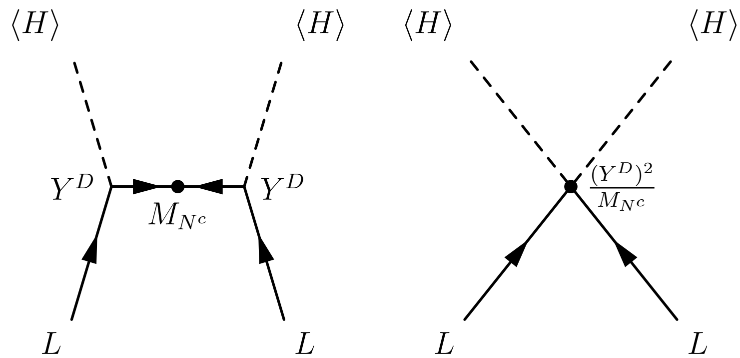

Fig. 3.2.: After the $B - L$ Higgs, Φ, gets its VEV, we move to integrate out the heavy neutrino mass, M_{N^c}. After we integrate out N^c, the diagram on the left becomes an effective 4-point interaction on the right.

a nonzero VEV, this diagram just becomes a mass term for the left-handed neutrinos, ν_L, of

$$m_\nu = \frac{v^2(Y^D)^2}{2M_{N^c}}. \tag{3.5}$$

This process is called the "see-saw mechanism" because the large M_{N^c} mass in the denominator with a small Dirac mass in the numerator produces very small neutrino masses. This was first proposed by Minkowski in reference [21].

4 Supersymmetry

4.1 Superfield Formalism

4.1.1 Superfields

Here we will give a truncated review of N=1 Supersymmetry. More information can by seen in a common text in ref [22]. Supersymmetry (SUSY) adds a spacetime symmetry between bosons and fermions. An electron would have a bosonic superpartner known as a selectron. In an exact supersymmetry they would have the same mass, but all superpartners have been ruled out below the TeV energy scale. This means supersymmetry has to be broken. SUSY breaking will be discussed later on.

We combine a scalar field $\phi(x^\mu)$ with its fermion superpartner, $\psi_A(x^\mu)$ into a single chiral superfield[1], Φ, with chiral fermionic superspace coordinates θ^A and $\bar{\theta}_{\dot{A}}$ and expand following the rules in Appendix A to get

$$\Phi\left(y,\theta,\bar{\theta}\right) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y), \tag{4.1}$$

$$\Phi^\dagger\left(y,\theta,\bar{\theta}\right) = \phi^\dagger(y) + \sqrt{2}\bar{\theta}\bar{\psi}(y) + \bar{\theta}\bar{\theta}F^\dagger(y). \tag{4.2}$$

The F-term is a product of a Taylor expansion with respect to the superspace coordinates. It has no degrees of freedom and using the Euler-Lagrange equations it can be shown to

[1]The subscript A is the chiral index that sums from $A = 1, 2$. If no index is present, the implicit contraction is taken.

contain additional scalar interactions. There exist chiral superderivatives,

$$\bar{\mathcal{D}}_{\dot{A}} = -\frac{\partial}{\partial\theta^{\dot{A}}} + 2i\theta^{B}\sigma^{\mu}_{B\dot{A}}\frac{\partial}{\partial y^{\mu}}, \tag{4.3}$$

$$\mathcal{D}_{A} = \frac{\partial}{\partial\theta^{A}} - 2i\sigma^{\mu}_{A\dot{B}}\theta^{\dot{B}}\frac{\partial}{\partial y^{\mu}}, \tag{4.4}$$

that allow us to describe a irreducible chiral superfields that obey $\bar{\mathcal{D}}_{\dot{A}}\Phi = 0 = \mathcal{D}_{A}\Phi^{\dagger}$ and where $y^{\mu} \equiv x^{\mu} - i\theta\sigma^{\mu}\bar{\theta}$. A general Taylor expansion, with substituting x^{μ} back in, on the superfield's component fields produce

$$\begin{aligned} \Phi\left(y,\theta\right) &= \phi(x) - i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\phi(x) - \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^{\mu}\partial_{\mu}\phi(x) \\ &\quad + \sqrt{2}\theta\psi(x) + \tfrac{i}{\sqrt{2}}\theta\theta\partial_{\mu}\psi(x)\sigma^{\mu}\bar{\theta} + \theta\theta F(x), \end{aligned} \tag{4.5}$$

$$\begin{aligned} \Phi^{\dagger}\left(\bar{y},\bar{\theta}\right) &= \phi^{\dagger}(x) + i\theta\sigma^{\mu}\bar{\theta}\partial_{\mu}\phi^{\dagger}(x) - \tfrac{1}{4}\theta\theta\bar{\theta}\bar{\theta}\partial^{\mu}\partial_{\mu}\phi^{\dagger}(x) \\ &\quad + \sqrt{2}\bar{\theta}\bar{\psi}(x) - \tfrac{i}{\sqrt{2}}\bar{\theta}\bar{\theta}\theta\sigma^{\mu}\partial_{\mu}\bar{\psi}(x) + \bar{\theta}\bar{\theta}F^{\dagger}(x). \end{aligned} \tag{4.6}$$

We introduce a real superfield known as the "vector superfield" that contains the gauge superfields. This contains the gauge bosons, A_{μ}, the gauginos, λ, and the D-term, D which shares the similar properties as the F-term. This vector superfield in terms of y^{μ} is

$$\begin{aligned} V(y,\theta,\bar{\theta}) &= \theta\sigma^{\mu}\bar{\theta}A_{\mu}(y) + \theta\theta\bar{\theta}\bar{\lambda}(y) + \bar{\theta}\bar{\theta}\theta\lambda(y) \\ &\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D(y) + i\frac{\partial}{\partial y^{\mu}}A^{\mu}(y)\right), \end{aligned} \tag{4.7}$$

$$\begin{aligned} V(\bar{y},\theta,\bar{\theta}) &= \theta\sigma^{\mu}\bar{\theta}A_{\mu}(\bar{y}) + \theta\theta\bar{\theta}\bar{\lambda}(\bar{y}) + \bar{\theta}\bar{\theta}\theta\lambda(\bar{y}) \\ &\quad + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\left(D(\bar{y}) - i\frac{\partial}{\partial \bar{y}^{\mu}}A^{\mu}(\bar{y})\right). \end{aligned} \tag{4.8}$$

With this we define one final field, the chiral spinorial field-strength superfields that hold the kinetic gauge terms using the chiral superderivatives,

$$W_A = -\frac{1}{4}\bar{\mathcal{D}}\bar{\mathcal{D}}\mathcal{D}_A V(y,\theta,\bar{\theta}), \tag{4.9}$$

$$\bar{W}_{\dot{A}} = -\frac{1}{4}\mathcal{D}\mathcal{D}\bar{\mathcal{D}}_{\dot{A}} V(\bar{y},\theta,\bar{\theta}). \tag{4.10}$$

These fields follow the same chiral properties as the superfields such that $\mathcal{D}_B \bar{W}_{\dot{A}} = 0 = \bar{\mathcal{D}}_{\dot{B}} W_A$. In this thesis we will omit the full expression for the chiral field strengths; their use will be apparent in the Lagrangian.

We can now write down a gauge and SUSY invariant Lagrangian. We write the kinetic terms for superfields, known as the Khaler potential. To include gauge interactions we insert the vector superfield and Taylor expand ($V^n = 0$ for $n \geq 3$) to achieve the component Lagrangian shown below,

$$\mathcal{L} \supset \int d^4\theta \Phi^\dagger e^{2ig_iQ_iV_i}\Phi \tag{4.11}$$

$$= |F|^2 + (D_\mu\phi)^\dagger(D^\mu\phi) + i\bar{\psi}\gamma^\mu D_\mu\psi + g_iQ_iD|\phi|^2 - \sqrt{2}g_iQ_i(\bar{\psi}\bar{\lambda}\phi + h.c.). \tag{4.12}$$

The g_i is the gauge coupling corresponding to a gauge group and Q_i is the charge of the superfield under this gauge group. It is summed over multiple gauge groups. For non-Abelian gauge groups the charge is replaced to the generators. The new SUSY additions to the kinetic terms are the F-term squared, D-term-scalar-scalar and gaugino-fermion-scalar interaction. The kinetic gauge fields are written as

$$\frac{1}{2}\int d^2\theta \mathrm{Tr}[W^A W_A] + \frac{1}{2}\int d^2\bar{\theta}\mathrm{Tr}[W_{\dot{A}} W^{\dot{A}}] \tag{4.13}$$

$$= \frac{1}{2}D^a D^a - \frac{1}{2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + \mathrm{Tr}[\bar{\lambda}\bar{\sigma}^\mu D_\mu \lambda],$$

where we assume a non-Abelian form. For an Abelian group the covariant derivative, $D_\mu \to \partial_\mu$. Combining (4.14) and (4.12) we obtain a gauge invariant Lagrangian. Notice there are no derivatives on the D-term, D. We can use Euler-Lagrange Equations to find

$$D^a = -g_i \phi^\dagger Q_i^a \phi. \tag{4.14}$$

Now we describe the interaction between superfields in a holomorphic superpotential,

$$\mathcal{L} = \int d^4\theta \Phi^\dagger e^{2ig_iQ_iV_i}\Phi + \left(\int d^2\theta W^A W_A + h.c\right) + \left(\int d^2\theta \mathcal{W}(\Phi_i) + h.c.\right), \tag{4.15}$$

$$\mathcal{W}(\Phi_i) = h_i\Phi_i + \tfrac{1}{2}m_{ij}\Phi_i\Phi_j + \tfrac{1}{3!}f_{ijk}\Phi_i\Phi_j\Phi_k. \tag{4.16}$$

If the F-term related couplings are computed from the superpotential and combined with $|F|^2$ from the Kahler potential in the Lagrangian we find

$$\mathcal{L}_F = F_iF_i^\dagger + F_i\frac{\partial \mathcal{W}}{\partial \Phi_i}\Big|_{\theta=\bar{\theta}=0} + F_i^\dagger \frac{\partial \mathcal{W}^\dagger}{\partial \Phi_i^\dagger}\Big|_{\theta=\bar{\theta}=0}. \tag{4.17}$$

From this we can use Euler-Lagrange Equations to find in general

$$F_i = -\frac{\partial \mathcal{W}^\dagger}{\partial \Phi_i^\dagger}\Big|_{\theta=\bar{\theta}=0}. \tag{4.18}$$

The fermion masses can be written down in terms of superpotential derivatives as

$$\mathcal{L}_{Yukawa} = -\frac{1}{2}\left(\psi_i\psi_j\frac{\partial^2\mathcal{W}}{\partial\Phi_i\partial\Phi_j}\Big|_{\theta=\bar{\theta}=0} + h.c.\right). \tag{4.19}$$

The F-term and D-terms are purely made of scalar fields. They combine to become the scalar potential,

$$V(\phi,\phi^\dagger) = |F|^2 + \frac{1}{2}D^aD^a. \tag{4.20}$$

The superscript on the D-term is the non-Abelian gauge index that sums over the generators ($a = 1, 2, 3$ for $SU(2)$ and $a = 1, 2..8$ for $SU(3)$ etc). Because SUSY is still exact, the scalar masses are described canonically as $m^2_{ij} = \frac{\partial^2 V(\phi_i,\phi_j)}{\partial\phi_i\partial\phi_j}$ and one can notice that scalars and fermions have the same mass.

4.1.2 Soft Supersymmetry Breaking

To have a phenomenologically viable SUSY model, the fermion and scalar fields in a superfield must have some mass splitting. We introduce "soft" SUSY breaking terms to obtain mass splitting while remaining small in compared to the supersymmetric part of the Lagrangian. The term "soft" means that SUSY is explicitly broken by the terms, but still free from quadratic divergences. The mechanism behind SUSY breaking is unknown to us in detail, so we assume there is a hidden sector of nature that communicates to the visible sector via a non-dynamical "spurion" superfield, $S = \theta^2 F_S$, to generate the soft masses. This an explicit breaking that obeys the correct mass dimensions and gauge invariance. The scalars obtain a soft mass m^2, the gauginos a Majorana mass, M and the bilinear scalar term of $\mathcal{B}$. We also introduce a trilinear scalar coupling, $\mathcal{A}$ that still obeys all the gauge symmetries. The soft SUSY breaking terms in the Lagrangian are

$$\mathcal{L}_{SOFT} = -m^2_{ij}\phi^\dagger_i\phi_j - \frac{1}{2}(M\lambda^a\lambda^a + h.c.) - \frac{1}{2}(\mathcal{B}_{ij}\phi^\dagger_i\phi_j + h.c.) + \frac{1}{3!}\left(\mathcal{A}_{ijk}\phi_i\phi_j\phi_k + h.c.\right). \quad (4.21)$$

The importance of SUSY breaking and its role in EWSB of the Minimal Supersymmetric Standard Model (MSSM) will be discussed in the next section.

Chiral Superfield	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$Q = \begin{pmatrix} u \\ d \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$1/6$
U^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-2/3$
D^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	$1/3$
$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$
E^c	$\mathbf{1}$	$\mathbf{1}$	1
$H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$1/2$
$H_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$-1/2$

Table 4.1.: Particle content of the MSSM. Here, we suppress the generation indices on the quark and lepton superfields.

4.2 The Minimal Supersymmetric Standard Model

4.2.1 Extending the Standard Model with SUSY

The MSSM is broken into three parts: the kinetic gauge and matter (Kähler Potential) terms, the superpotential, and the soft SUSY breaking terms. We promote all the matter fields from the SM to chiral superfields with the addition of a second Higgs superfield to maintain gauge invariance and holomorphy for the superpotential (recall the conjugate Higgs in the Yukawa potential). The particle content is listed in Table 4.1 for the MSSM with gauge groups $U(1)_Y \times SU(2)_L \times SU(3)_C$ with couplings g_Y, g_L, and g_3 respectively. In addition, to the usual particles of the SM all new matter superpartners will be noted with a tilde, using E^c for example as

$$E^c = \tilde{e}^c + \sqrt{2}\theta e^c + \theta\theta F_{E^c}.$$

We denote the gauginos for $U(1)_Y \times SU(2)_L \times SU(3)_C$ as λ_Y, λ_L^a, and λ_3^a respectively. We will not write out the complete kinetic sector, but rather state (4.12) and (4.14) are easily extendable to the particle contents in Table 4.1.

The superpotential for the MSSM is

$$\mathcal{W} = -Y^u_{ij} Q_i \cdot H_u U^c_j - Y^d_{ij} H_d \cdot Q_i D^c_j - Y^e_{ij} H_d \cdot L_i E^c_j + \mu H_d \cdot H_u, \tag{4.22}$$

with the same Yukawa matrices from the SM and we introduce the Higgsino mass parameter μ. We use the $SU(2)$ spinor notation $F \cdot G = \epsilon_{AB} F^A G^B$. There are no quartic couplings in the superpotential because the mass dimension of $\mathcal{W}$ is 3. The SM has an accidental symmetry of baryon and lepton number conservation. In the MSSM we can write down some baryon and lepton number violating terms that still respect the gauge symmetries. We introduce a Z_2 parity known as R-parity to forbid the following lepton and baryon number violating terms. The R-Parity breaking, $\not{R}_p$, superpotential is

$$\mathcal{W}_{\not{R}_p} = -\epsilon_i L_i \cdot H_u + \frac{1}{2}\lambda_{ijk} L_i \cdot L_j E^c_k + \lambda'_{ijk} L_i \cdot Q_j D^c_k + \frac{1}{2}\lambda'_{ijk} U^c_i D^c_j D^c_k, \tag{4.23}$$

where i, j, k are flavor indices. The coupling ϵ_i has mass dimension of one while the others are dimensionless. The couplings ϵ_i, λ_{ijk}, and λ'_{ijk} violate lepton number while λ'_{ijk} violate baryon number. R-parity, R, for a field of baryon number B, lepton number L and spin S is defined as

$$R_p = (-1)^{3(B-L)+2S}. \tag{4.24}$$

If R-parity remains exact, then the lightest superpartner (LSP) remains stable. This allows for a cold dark matter candidate in the MSSM.

(S)Particle	Spin	R_p
Quark q	1/2	+
Squark $\tilde{q}$	0	−
Lepton l	1/2	+
Slepton $\tilde{l}$	0	−
Higgs $h_{u,d}$	0	+
Higgsino $\tilde{h}_{u,d}$	1/2	−
Gauge boson A^a_μ	1	+
Gaugino λ^a	1/2	−

Table 4.2.: The R-parity assignments. We explicitly write the gauge index on the gauginos to differ them from our three lambda R-parity violating couplings.

4.2.2 Soft SUSY Breaking in the MSSM

The absence of the superpartners at the LHC[23] means that SUSY is broken at low energies. We extend equation (4.21) for the MSSM and separating it into V_{SOFT} for the scalars and the gaugino mass terms

$$\begin{aligned} V_{SOFT} =&(m^2_{\tilde{q}})_{ij}\tilde{q}^\dagger_i\tilde{q}_j + (m^2_{\tilde{u}^c})_{ij}\tilde{u}^{c\dagger}_i\tilde{u}^c_j + (m^2_{\tilde{d}^c})_{ij}\tilde{d}^{c\dagger}_i\tilde{d}^c_j + \\ &(m^2_{\tilde{l}})_{ij}\tilde{l}^\dagger_i\tilde{l}_j + (m^2_{\tilde{e}^c})_{ij}\tilde{e}^{c\dagger}_i\tilde{e}^c_j + m^2_u|h_u|^2 + m^2_d|h_d|^2 + \\ &(A^u_{ij}\tilde{q}_i\cdot h_u\tilde{u}^c_j + A^d_{ij}h_d\cdot\tilde{q}_i\tilde{d}^c_j + A^e_{ij}h_d\cdot\tilde{l}_i\tilde{e}^c_j + h.c.) + \\ &(B\mu h_d\cdot h_u + h.c), \end{aligned} \tag{4.25}$$

$$\mathcal{L}_\lambda = \frac{1}{2}M_Y\lambda_Y\lambda_Y + \frac{1}{2}M_L\lambda^a_L\lambda^a_L + \frac{1}{2}M_c\lambda^a_c\lambda^a_c, \tag{4.26}$$

where, in general, the soft masses for scalars need not be diagonal. The introduction of the soft masses, specifically the Higgs soft masses, solves the hierarchy problem. The quadratic divergences for the Higgs by fermions runnning in a one-loop correction are canceled out by the sfermion running in a loop. Another effect the Higgs soft masses produce is a mechanism to generate EWSB which will be discussed later on. Now we move to discussing the details of EWSB in the MSSM.

4.2.3 EWSB in the MSSM

The two neutral scalar components of both Higgs doublets acquire a nonzero VEV that is related to the known SM VEV, $v = 246$ GeV, by

$$v_u^2 + v_d^2 = v^2 = (246\,\text{GeV})^2, \tag{4.27}$$

where we can parameterize their ratio by

$$\tan\beta = \frac{v_u}{v_d}. \tag{4.28}$$

The nondiagonalized fermion mass matrices are then described as

$$(M_u)_{ij} = \frac{v_u (Y_u)_{ij}}{\sqrt{2}}, \quad (M_d)_{ij} = \frac{v_d (Y_d)_{ij}}{\sqrt{2}}, \quad (M_e)_{ij} = \frac{v_d (Y_e)_{ij}}{\sqrt{2}}. \tag{4.29}$$

The relations for the EW gauge boson masses, m_Z^2 and m_W^2, remain unchanged. The neutral Higgs scalar potential is a combination of (4.20) and the Higgs soft masses is

$$V_H^0 = \frac{1}{8}(g_Y^2 + g_L^2)(|h_d^0|^2 - |h_u^0|^2)^2 + (m_u^2 + |\mu|^2)|h_u^0|^2 + (m_d^2 + |\mu|^2)|h_d^0|^2 - B\mu(h_d^0 h_u^0 + h.c.), \tag{4.30}$$

which has a minimum of

$$V_H^{min} = \frac{1}{32}(g_Y^2 + g_L^2)(v_d^2 - v_u^2)^2 + \frac{1}{2}(m_u^2 + |\mu|^2)v_u^2 + \frac{1}{2}(m_d^2 + |\mu|^2)v_d^2 - B\mu v_u v_d, \tag{4.31}$$

when $\langle h_u^0 \rangle = v_u/\sqrt{2}$ and $\langle h_d^0 \rangle = v_d/\sqrt{2}$. We can get the following relations from $\partial V_H^{min}/\partial v_d = 0 = \partial V_H^{min}/\partial v_u$

$$-2B\mu = (m_d^2 - m_u^2)\tan 2\beta + m_Z^2 \sin 2\beta, \tag{4.32}$$

$$|\mu|^2 = \frac{m_u^2 \sin^2\beta - m_d^2 \cos^2\beta}{\cos 2\beta} - \frac{1}{2}m_Z^2. \tag{4.33}$$

There are five physical Higgs bosons in the MSSM since two complex scalar doublets have eight degrees of freedom and three are eaten by the gauge bosons through the Higgs mechanism. In the neutral sector there are two CP even Higgs, h^0 and H^0, and one CP odd Higgs, A^0. The charged sector has two charged Higgs, $H^\pm$. There are two free parameters in the Higgs sector, $\tan\beta$ and the mass of A^0, m_A^2. Including the $W^\pm$ and Z boson masses, m_W and m_Z respectively, the other four masses are defined as

$$m_{H^\pm}^2 = m_A^2 + m_W^2, \tag{4.34}$$

$$m_{h^0,H^0}^2 = \tfrac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}\right). \tag{4.35}$$

If m_A is very large, the heavy neutral Higgs, H^0, decouples and the lighter neutral Higgs, h^0, acts like the SM Higgs with mass

$$m_{h^0}^2 \approx m_Z^2 \cos^2 2\beta, \tag{4.36}$$

This obviously presents a problem since $m_Z = 91.2$ GeV and $m_h = 125$ GeV. This can be resolved by including one-loop corrections from the top quark mass, m_t, and the top squark mass, $m_{\tilde{t}}$. At one-loop

$$m_{h^0}^2 \approx m_Z^2 + \frac{3m_t^4}{4\pi^2 v^2}\ln\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right). \tag{4.37}$$

where the mass of the top squark must be $\mathcal{O}$(1-10 TeV) to raise the tree level mass up to 125 GeV for the Higgs.

Through the new SUSY gauge couplings for the Higgs with the gauginos and higgsinos, EWSB will generate masses for new fermionic mass eigenstates. We first express $a = 1, 2$ of the $SU(2)_L$ gauginos, λ_L^a, as $\lambda_L^\pm = \frac{1}{\sqrt{2}}(\lambda_L^1 \mp \lambda_L^2)$. Gathering the charged terms we get the mass term

$$-\mathcal{L}_{\chi^\pm} = \begin{pmatrix} \lambda_L^- & \tilde{h}_d^- \end{pmatrix} \begin{pmatrix} M_L & \sqrt{2}m_w \sin\beta \\ \sqrt{2}m_W \cos\beta & \mu \end{pmatrix} \begin{pmatrix} \lambda_L^+ \\ \tilde{h}_u^+ \end{pmatrix} \tag{4.38}$$

$$= \tilde{M}_1^\pm \bar{\chi}_1^+ \chi_1^+ + \tilde{M}_2^\pm \bar{\chi}_2^+ \chi_2^+, \tag{4.39}$$

where $\chi_{1,2}^+$ are called charginos and written in the diagonal basis. The neutral higgsinos and neutral gauginos mix in a Majorana basis and are diagonalized. We show the mixing mass matrix as

$$\mathcal{M}_{\chi^0} = \begin{pmatrix} M_Y & 0 & -m_Z c_\beta s_W & m_Z s_\beta c_W \\ 0 & M_L & m_Z c_\beta c_W & m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta c_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \tag{4.40}$$

which can be diagonalized in the Lagrangian as

$$\mathcal{L}_{\chi^0_{1,2,3,4}} = -\frac{1}{2}(\psi^0)^T \mathcal{M}_{\chi^0} \psi^0 = -\frac{1}{2}\sum_i^4 \tilde{M}_i^0 \chi_i^0 \chi_i^0, \tag{4.41}$$

where

$$(\psi^0)^T = \begin{pmatrix} \lambda_Y & \lambda_L^3 & \tilde{h}_d^0 & \tilde{h}_u^0 \end{pmatrix}. \tag{4.42}$$

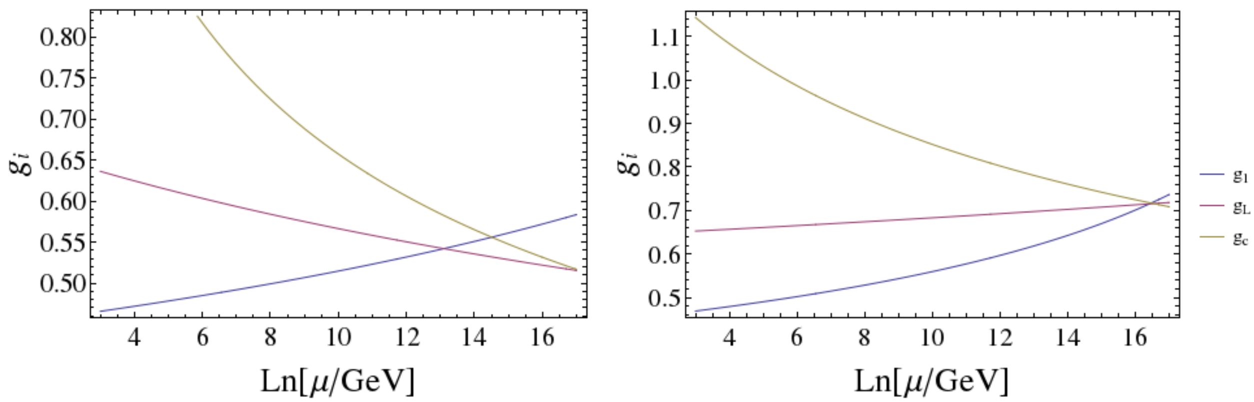

Fig. 4.1.: The plots above shows the gauge couplings versus the $\log \mu$. The left plot is the SM while the right plot is the MSSM.

The above diagonalized mass terms will not be given here because of their complicated nature; numerically solving them is much easier. The lightest neutralino, χ_1^0, we identify as the LSP a dark matter candidate.

4.2.4 Radiative Corrections

If radiative corrections to one-loop are taken into account in the MSSM several new features are produced. First in Figure 4.1 there are the three gauge couplings plotted with respect to energy. We solve the RGEs (A.1)-(A.3) for the SM and (A.10)-(A.12) for the MSSM using EW data. The introduction of the superpartners changes the running of the gauge couplings to higher energies to unify at the GUT scale, motivating unification theories. We take this notion of high energy unification to minimize the number of free parameters in the model. At the GUT scale we use m_0 for all scalar masses, $m_{1/2}$ for all the gaugino masses, and A_0 for all trilinear couplings. In addition to these three parameters we have $\tan \beta$ and sign(μ). This is the minimal amount of freedom we have in the MSSM. Taking

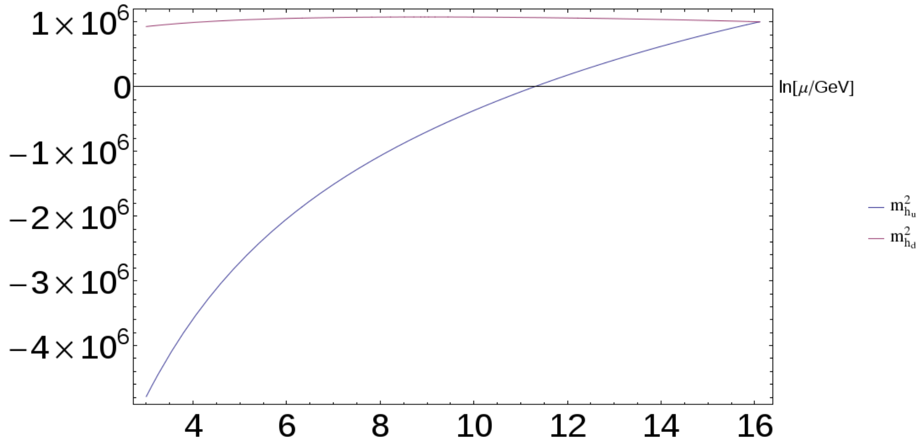

Fig. 4.2.: The plot of $m^2_{h_u}$ and $m^2_{h_d}$ with energy. While $m^2_{h_d}$ remains positive while $m^2_{h_u}$ runs negative at low energies, the $B\mu$ term causes former to becomes negative at the EWBS scale.

into account all the one-loop corrections to $m^2_{h_u}$ we can approximate (A.19) as

$$\frac{dm^2_{h_u}}{dt} \approx \frac{3|Y_t|^2 m^2_{h_u}}{8\pi^2} \tag{4.43}$$

because the $Y_u^{33} = Y_t$ is much larger than all other Yukawa terms and $Y_{33} > g_L^2 > g_Y^2$ at low energies. If this is run from the GUT scale down to the EW scale we see in Figure 4.2 that $m^2_{h_u} < 0$.

4.2.5 Beyond the MSSM

At the end of chapter one there were several shortcomings listed for the SM. By introducing SUSY and extending the SM to the MSSM we removed the hierarchy problem with sparticle cancellations, introduce a natural mechanism for EWSB through radiative corrections to the Higgs soft mass, and propose the LSP neutralino as a dark matter candidate. In addition, the MSSM gives motivation for a grand unified theory at very high energies through gauge coupling unification.

The MSSM still has no right handed neutrinos and does not account for neutrino oscillation and currently there are no hints of SUSY at the LHC. The recently measured Higgs mass is still within the acceptable parameter region for the MSSM phenomenologically. There is still no origin of R-parity in the MSSM, there are possibilities in extending the MSSM.

5 SUSY $B-L$

We supersymmeterize the $B-L$ model as an extension to the MSSM. To maintain gauge invariance and holomorphy we introduce a second $B-L$ Higgs superfield Φ^c. The particle contents are in Table 5.1. We can write down the terms relevant for the neutrino physics

	$U(1)_{BL}$
Q	$1/3$
U^c	$-1/3$
D^c	$-1/3$
L	-1
E^c	1
N^c	1
H_u	0
H_d	0
Φ	-2
Φ^c	2

Table 5.1.: We list the supersymmetric version of Table 3.1.

and the Higgs sector,

$$\mathcal{W}_{N^c} = Y_{ij}^D H_u \cdot L_i N_j^c + Y_i^M \Phi N_i^c N_i^c, \tag{5.1}$$

$$\mathcal{W}_{Higgs} = \mu H_u \cdot H_d + \mu_\Phi \Phi \Phi^c. \tag{5.2}$$

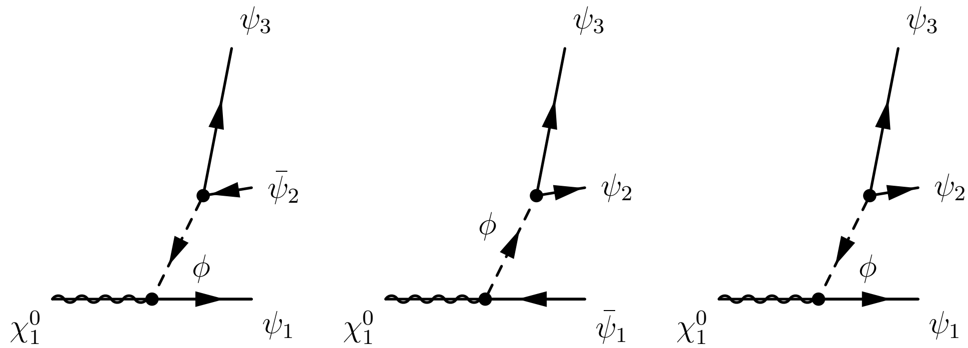

Fig. 5.1.: Here we have three diagrams for the R-parity violating decay of the MSSM neutralino LSP, the neutralino. We call Diagram 1 on the left and Diagram 2 in the middle and Diagram 3 on the right

When Φ and Φ^c develops nonzero VEVs, the $B - L$ symmetry is broken. In most models at least one generation of N^c gets a nonzero VEV[24, 25]. In the upcoming chapters this reason will be further elaborated on, but for now we will just analyze the vacuum potential. The Z' mass is described by

$$m_{Z'} = g_{BL} v_{BL}, \tag{5.3}$$

where

$$v_{BL}^2 = \langle N_3^c \rangle^2 + 4\langle \Phi \rangle^2 + 4\langle \Phi^c \rangle^2. \tag{5.4}$$

Since $\tilde{N}_3^c$ develops a nonzero VEV, R-parity is broken. The Dirac Yukawa coupling for the right-handed neutrinos after the sneutrino develops a VEV is

$$\mathcal{W} \supset \frac{v_{BL}(Y^D)_{3i}}{\sqrt{2}} L_i \cdot H_u = \epsilon_i L_i \cdot H_u. \tag{5.5}$$

Once this bilinear R-parity violating term is generated, the other lepton number violating terms from (4.23) are generated through mixings with either L or H_u. In Figure 5.1 are the Feynman diagrams for the LSP neutralino decays[1]. Here we consider four end main decays

[1]Neutralinos are no longer stable, gravitinos can be a dark matter candidate.

χ_1^0	ϕ	$\psi_1\psi_2\psi_3$
Diagram 1	$\tilde{e}_j$	$e_j\bar{e}_i\nu_k$
	$\tilde{d}_j$	$d_j\bar{d}_i\nu_k$
	$\tilde{\nu}_j$	$\nu_j\bar{e}_i e_k$
	$\tilde{\nu}_j$	$\nu_j\bar{d}_i d_k$
	$\tilde{u}_j$	$u_j\bar{d}_i e_k$
	$\tilde{e}_j$	$e_j\bar{d}_i u_k$
Diagram 2	$\tilde{e}_j$	$\bar{e}_i e_j\nu_k$
	$\tilde{d}_j$	$\bar{d}_i e_j\nu_k$
	$\tilde{d}_j$	$\bar{d}_i u_j e_k$
Diagram 3	$\tilde{u}_j$	$u_i d_k d_j$
	$\tilde{d}_j$	$d_i u_k d_j$

Table 5.2.: Neutralino decays corresponding to Figure 5.1

with different mediating sfermions: $\chi_1^0 \to \bar{e}_i e_j\nu_k$, $\bar{d}_i d_j\nu_k$, $\bar{d}_i u_j e_k$, $u_i d_j d_k$. In Table 5.2 we list all permutations of the decay.

6 R-parity Conserving Minimal Supersymmetric B-L Model

6.1 Introduction

The $B-L$ (baryon number minus lepton number) is the unique anomaly-free global $\mathrm{U}(1)_{BL}$ symmetry in the Standard Model (SM). This symmetry is easily gauged, and the so-called minimal $B-L$ model is a simple gauged $B-L$ extension of the SM, where three right-handed neutrinos and an SM gauge singlet Higgs field with two units of the $B-L$ charge are introduced. The three right-handed neutrinos are necessarily introduced to make the model free from all gauge and gravitational anomalies. Associated with a $B-L$ symmetry breaking by a Vacuum Expectation Value (VEV) of the $B-L$ Higgs field, the $B-L$ gauge field (Z' boson) and the right-handed neutrinos acquire their masses. After the electroweak symmetry breaking, tiny SM neutrino masses are generated via the seesaw mechanism [21].

Although the scale of the $B-L$ gauge symmetry breaking is arbitrary as long as phenomenological constraints are satisfied, a breaking at the TeV scale is probably the most interesting possibility in the view point of the Large Hadron Collider (LHC) experiments. However, mass squared corrections of the $B-L$ Higgs (any Higgs fields in 4-dimensional models, in general) are quadratically sensitive to the scale of a possible ultraviolet theory, and as a result the $B-L$ symmetry breaking scale is unstable against quantum corrections. As is well-known, supersymmetric (SUSY) extension is the most promising way to solve this vacuum instability. Very interestingly, SUSY extension of the minimal $B-L$ model offers a way to naturally realize the $B-L$ symmetry breaking at the TeV scale. With suitable inputs of soft SUSY breaking parameters at a high energy, their renormalization group (RG) evolutions drive the $B - L$ Higgs mass squared negative and therefore the $B - L$ gauge symmetry is radiatively broken [26, 24, 25]. Since the scale of the negative mass squared is controlled by the soft SUSY breaking parameters, the $B - L$ breaking scale lies at the TeV from naturalness.

SUSY extension opens a further possibility. As has been proposed in Ref. [24], it is not necessary to introduce the $B - L$ Higgs field, since the scalar partner of a right-handed neutrino can play the same role as the $B - L$ Higgs field in breaking the $B - L$ gauge symmetry. Hence, we can define the minimal SUSY $B - L$ model by a particle content, where only three right-handed neutrino chiral superfields are added to the particle content of the minimal SUSY SM (MSSM). It is interesting that such a particle content can be derived from heterotic strings [27, 28]. In Ref. [29], a negative soft mass squared of a right-handed sneutrino is assumed to break the $B - L$ gauge symmetry, so that the $B - L$ symmetry breaking occurs at the TeV scale. Associated with this symmetry breaking, R-parity is also spontaneously broken, and many interesting phenomenologies with the R-parity violation have been discussed [30, 31, 32, 33]. Through the non-zero VEV of the right-handed sneutrino, mixings between neutrinos, MSSM Higgsinos, MSSM neutralinos and $B-L$ gaugino are generated. Although the neutrino mass matrix becomes very complicated, it has enough number of degrees of freedom to reproduce the neutrino oscillation data with a characteristic pattern of the mass spectrum [34, 35].

In this chapter, we propose the minimal SUSY $B-L$ model with R-parity conservation. The particle content is the same as the one of the minimal SUSY $B - L$ model discussed above, while we assign an even R-parity to one right-handed neutrino chiral superfield (Φ) and an odd R-parity to the other two right-handed neutrino chiral superfields. The R-parity assignment for the MSSM fields is as usual. Because of this parity assignment and the gauge symmetry, the chiral superfield Φ has no Dirac Yukawa coupling with the lepton doublet fields. In fact, it does not appear in the renormalizable superpotential. We consider the case that the $B - L$ symmetry breaking is driven by a VEV of the R-parity even right-handed sneutrino. Phenomenological consequences in this model are very different from those of the conventional minimal SUSY $B-L$ model. As usual in the MSSM, R-parity is conserved and hence the lightest neutralino is a candidate of the dark matter. In addition to the lightest neutralino in the MSSM, the model offers a new candidate for the dark matter, namely, a linear combination of the fermion component of Φ and the $B-L$ gaugino. Since Φ has no Dirac Yukawa coupling, no Majorana mass term is generated in the SM neutrino sector, and as a result, the SM neutrinos are Dirac particles. With only the two right-handed neutrinos involved in the Dirac Yukawa couplings, the Dirac neutrino mass matrix leads to three mass eigenstates, one massless chiral neutrino and two Dirac neutrinos. A general 2-by-3 Dirac mass matrix includes a number of free parameters enough to reproduce the neutrino oscillation data. This Dirac nature of the SM neutrinos are quite distinctive from those in the usual $B-L$ model, where the right-handed neutrinos are heavy Majorana particles and the mass eigenstates are different from the light SM neutrinos. If the Z' boson is discovered at the LHC, this difference could be tested through its decay products and the decay width measurements.

This chapter is organized as follows. In the next section, we define our minimal SUSY $B-L$ model with a novel R-parity assignment. Then, we introduce superpotential and soft SUSY breaking terms relevant for our discussion. In Sec. 5.3, we discuss a way to radiatively break the $B-L$ gauge symmetry, while keeping R-parity manifest. Focussing on the $B-L$ sector, for simplicity, we perform a numerical analysis for the RG evolutions of the soft SUSY breaking masses of the right-handed sneutrinos, and show that the $B-L$ gauge symmetry is radiatively broken at the TeV scale by a VEV of the scalar component of Φ. In Sec. 5.4, we consider a new dark matter candidate which is a linear combination of the scalar component of Φ and the $B-L$ gaugino. We show a parameter set which can reproduce the observed dark matter relic density. We also briefly discuss an implication of the Dirac neutrinos to the LHC phenomenology through the Z' boson production in Sec. 5.5.

Chiral Superfield	SU(3)$_c$	SU(2)$_L$	U(1)$_Y$	U(1)$_{BL}$	R-parity
Q^i	3	2	+1/6	+1/3	−
U_i^c	$\bar{3}$	1	−2/3	−1/3	−
D_i^c	$\bar{3}$	1	+1/3	−1/3	−
L_i	1	2	−1/2	−1	−
Φ	1	1	0	+1	+
$N_{1,2}^c$	1	1	0	+1	−
E_i^c	1	1	−1	+1	−
H_u	1	2	+1/2	0	+
H_d	1	2	−1/2	0	+

Table 6.1.: Particle content of the minimal SUSY $B-L$ model with a conserved R-parity. In addition to the MSSM particles, three right-handed neutrino superfields (Φ and $N_{1,2}^c$) are introduced. We assign an even R-parity for Φ. $i = 1, 2, 3$ is the generation index.

6.2 Minimal SUSY $B-L$ model with a conserved R-parity

The minimal SUSY $B-L$ model is based on the gauge group of $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{BL}$. In addition to the MSSM particle content, we introduce three chiral superfields which are singlet under the SM gauge groups and have a unit $B-L$ charge. The new fields are identified as the right-handed neutrino chiral superfields, and their existence is essential to make the model free from all gauge and gravitational anomalies. Unlike direct supersymmetrization of the minimal $B-L$ model, the $B-L$ Higgs superfields are not included in the particle content. The key of our proposal is that we assign an even R-parity to one right-handed neutrino chiral superfield, in contrast with the minimal SUSY $B-L$

model proposed in Ref. [29], where all the right-handed neutrino superfields are R-parity odd as usual. The particle content is listed in Table 6.1.

The gauge and parity invariant superpotential which is added to the MSSM one is only the neutrino Dirac Yukawa coupling

$$W_{BL} = \sum_{i=1}^{2} \sum_{j=1}^{3} y_D^{ij} N_i^c L_j H_u. \tag{6.1}$$

Note that the Yukawa coupling for Φ is forbidden by the parity, and Φ has no direct coupling with the MSSM fields. After the electroweak symmetry breaking, the neutrino Dirac mass matrix is generated. Since this is a 2-by-3 matrix, one neutrino remains massless. Therefore, we have one massless neutrino and two Dirac neutrinos in the model. The 2-by-3 Dirac mass matrix has a sufficient number of free parameters to reproduce the neutrino oscillation data. Although we have introduced the special parity assignment, this may be unnecessary in the practical point of view. Without the parity assignment, the superpotential in Eq. (6.1) can include

$$W_{BL} \supset \sum_{j=1}^{3} y_D^{j} \Phi L_j H_u, \tag{6.2}$$

which are unique direct couplings between Φ and the MSSM fields. Let us now take a limit $y_D^j \to 0$, which switch off the direct communication of Φ with the lepton and Higgs doublets. In this sense, our parity assignment can be regarded as a result of symmetry enhancement caused by this limit. Since the neutrinos are Dirac particles, the Dirac Yukawa coupling constants must be extremely small in order to reproduce the observed neutrino mass scale. We will discuss a possibility to naturally realize such small parameters in the last section.

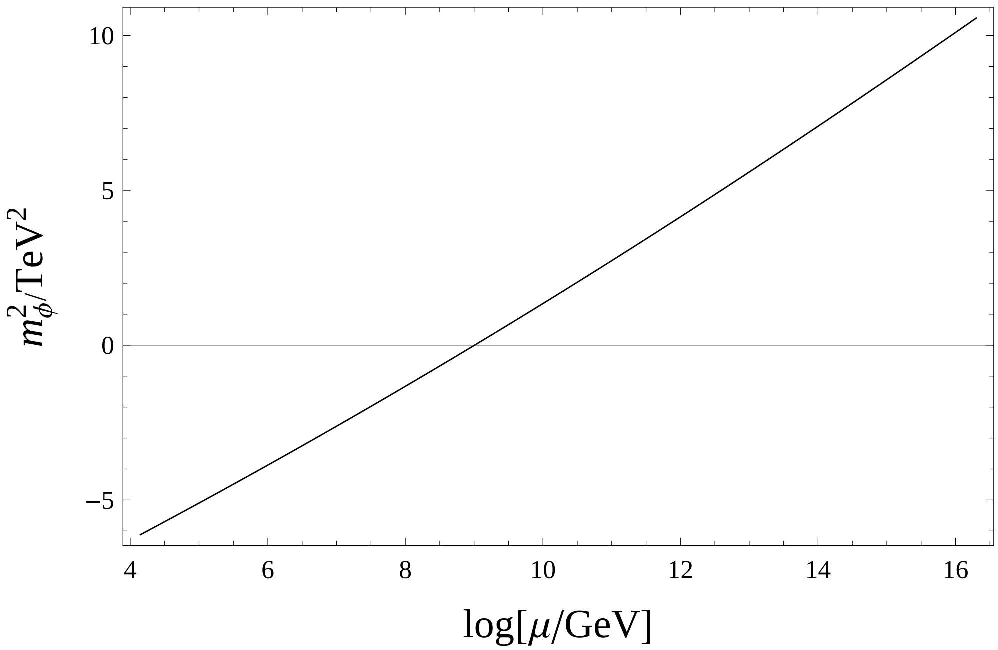

Fig. 6.1.: The RG evolution of the soft SUSY breaking mass m_ϕ^2 from M_U to low energies.

Next, we introduce soft SUSY breaking terms for the fields in the $B-L$ sector:

$$\mathcal{L}_{soft} \;=\; -\left(\frac{1}{2}M_{BL}\lambda_{BL}\lambda_{BL} + h.c.\right) - \left(\sum_{i=1}^{2} m^2_{\tilde{N}^c_i}|\tilde{N}^c_i|^2 + m_\phi^2|\phi|^2\right), \tag{6.3}$$

where λ_{BL} is the $B-L$ gaugino and $\tilde{N}^c_i$ and ϕ are scalar components of N^c_i and Φ, respectively. Since the Dirac Yukawa couplings are very small, we omit terms relevant to the couplings. In the next section, we analyze the RG evolutions of the soft SUSY breaking masses and find that m_ϕ^2 is driven to be negative and the $\mathrm{U}(1)_{BL}$ symmetry is radiatively broken. Although we do not assume the grand unification of our model, we take $M_U = 2 \times 10^{16}$ GeV as a reference scale at which the boundary conditions for the soft masses are given.

6.3 Radiative $B-L$ symmetry breaking

It is well-known that the electroweak symmetry breaking in the MSSM is triggered by radiative corrections which drives the soft mass squared of the up-type Higgs doublet negative. Because of this radiative symmetry breaking, the electroweak scale is controlled by the soft SUSY breaking mass scale and the SUSY breaking scale at the TeV naturally results in the right electroweak scale of $\mathcal{O}(100\text{ GeV})$. Similarly to the MSSM, a radiative $B-L$ symmetry breaking occurs by the RG evolution of soft SUSY breaking parameters from a high energy to low energies. However, the mechanism that drives m_ϕ^2 negative is different from the one in the MSSM where the large top Yukawa coupling plays a crucial role.

To make our discussion simple, we consider the RG equations only for the $B-L$ sector.[1] RG equations relevant for our discussion are

$$16\pi^2\mu\frac{dM_{BL}}{d\mu} = 32g_{BL}^2 M_{BL}, \tag{6.4}$$

$$16\pi^2\mu\frac{dm^2_{\tilde{N}_i^c}}{d\mu} = -8g_{BL}^2 M_{BL}^2 + 2g_{BL}^2\left(\sum_{j=1}^{2} m^2_{\tilde{N}_j^c} + m_\phi^2\right), \tag{6.5}$$

$$16\pi^2\mu\frac{dm^2_\phi}{d\mu} = -8g_{BL}^2 M_{BL}^2 + 2g_{BL}^2\left(\sum_{j=1}^{2} m^2_{\tilde{N}_j^c} + m_\phi^2\right), \tag{6.6}$$

where the $B-L$ gauge coupling obeys

$$16\pi^2\mu\frac{dg_{BL}}{d\mu} = 16g_{BL}^3. \tag{6.7}$$

In Eq. (6.5) the contributions from very small Dirac Yukawa couplings are omitted. In fact, the second term in the right-hand side of Eq. (6.6), which originates from the D-term interaction, plays an essential role to drive m_ϕ^2 negative. Since squarks and leptons have

[1] See Refs. [36, 37] for more elaborate analysis and parameter scans to identify parameter regions which are consistent with current experimental results.

$B-L$ charges, their soft squared masses also appear in the RG equations, but we have omitted them, for simplicity, by assuming they are much smaller than $m^2_{\tilde{N}^c_i}$ and m^2_ϕ.

To illustrate the radiative $B-L$ symmetry breaking, we numerically solve the above RG equations from $M_U = 2 \times 10^{16}$ GeV to low energy, choosing the following boundary conditions.

$$g_{BL} = 0.311, \quad M_{BL} = 8.13 \text{ TeV}, \quad m_{\tilde{N}^c_1} = m_{\tilde{N}^c_2} = 20.0 \text{ TeV}, \quad m_\phi = 3.25 \text{ TeV}. \tag{6.8}$$

Fig. 6.1 shows the RG evolution of m^2_ϕ. The mass squared of ϕ becomes negative at low energies as shown in this figure, while the other squared masses remain positive. The mass squared hierarchy $m_{\tilde{N}^c_i} \gg m^2_\phi$ is crucial to drive $m^2_\phi < 0$. We now analyze the scalar potential with the soft SUSY breaking parameters obtained from the RG evolutions. We choose the VEV of ϕ as $v_{BL} = \sqrt{2}\langle\phi\rangle = 14$ TeV as a reference, at which the solutions of the RG equations are evaluated as follows:

$$g_{BL} = 0.250, \quad M_{BL} = 5.25 \text{ TeV}, \quad m_{\tilde{N}^c_1} = m_{\tilde{N}^c_2} = 19.6 \text{ TeV}, \quad |m_\phi| = 2.47 \text{ TeV}. \tag{6.9}$$

The scalar potential is given by

$$V = m^2_{\tilde{N}^c_1}|\tilde{N}^c_1|^2 + m^2_{\tilde{N}^c_2}|\tilde{N}^c_2|^2 + m^2_\phi|\phi|^2 + \frac{g^2_{BL}}{2}\left(|\tilde{N}^c_1|^2 + |\tilde{N}^c_2|^2 + |\phi|^2\right)^2 . \tag{6.10}$$

Solving the stationary conditions, we find (in units of TeV)

$$\langle\tilde{N}^c_1\rangle = \langle\tilde{N}^c_2\rangle = 0, \quad \langle\phi\rangle = \frac{\sqrt{-2m^2_\phi}}{g_{BL}} \simeq \frac{14}{\sqrt{2}}. \tag{6.11}$$

This result is consistent with our choice of $v_{BL} = 14$ TeV in evaluating the running soft masses.

In our parameter choice, the Z' boson mass is given by

$$m_{Z'} = g_{BL} v_{BL} = 3.5 \text{ TeV}. \tag{6.12}$$

The ATLAS and CMS collaborations at the LHC Run-2 have been searching for the Z' boson resonance with the dilepton final state and have recently reported their results which are consistent with the SM expectaions [38, 39]. In Ref. [40], the ATLAS and CMS search results are interpreted to a constraint on the Z' boson in the minimal $B-L$ model, where an upper bound of the the $B-L$ gauge coupling as a function of Z' boson mass has been obtained. We refer the results in Ref. [40] where it is shown that $g_{BL} \leq 0.328$ and 0.350 for $m_{Z'} = 3.5$ TeV from the ATLAS and CMS results, respectively.[2] Our parameter choice of $g_{BL} = 0.250$ for $m_{Z'} = 3.5$ TeV is consistent with the recent LHC Run-2 results.

6.4 Right-handed neutrino dark matter

As we here show in the previous section, the $B-L$ gauge symmetry is radiatively broken by the RG effects on the soft SUSY breaking masses. Since the breaking occurs by the VEV of R-parity even scalar field ϕ, R-parity is still manifest, by which the stability of the lightest R-parity odd particle is ensured. Thus, as usual in the MSSM, the lightest neutralino is a candidate of the dark matter. In addition to the MSSM neutralinos, a new dark matter candidate arises in our model, namely, the fermion component of Φ (ψ). We can call ψ R-parity odd right-handed neutrino. In this section, we study phenomenology of the right-handed neutrino dark matter.

A scenario of the right-handed Majorana neutrino dark matter was first proposed in [42] in the context of the non-SUSY minimal $B-L$ model, where a Z_2 parity is introduced and

[2] It is also shown in Ref. [40] that the ATLAS bound at the LHC Run-2 is more severe than the bound obtained from the LEP2 data [41] for $m_{Z'} \leq 4.3$ TeV.

an odd parity is assigned to one right-handed neutrino while the other fields are all parity-even. Because of the Z_2-parity conservation, the parity-odd right-handed neutrino becomes stable and hence the dark matter candidate. Phenomenology of this dark matter has been investigated [42, 43, 44]. Recently, in terms of the complementarity to the LHC physics, the right-handed neutrino dark matter has been investigated in detail in [40]. Supersymmetric version of the minimal $B-L$ model with the right-handed neutrino dark matter has been proposed in [25].

Our dark matter scenario that we will investigate in this section shares similar properties with the scenario discussed in [25]. However, there is a crucial difference that ψ has no Majorana mass by its own, but it acquires a Majorana mass through a mixing with the $B-L$ gaugino (λ_{BL}). After the $U(1)_{BL}$ symmetry breaking, a mass matrix for ψ and λ_{BL} is generated to be

$$M_\chi = \begin{pmatrix} 0 & m_{Z'} \\ m_{Z'} & M_{BL} \end{pmatrix}. \tag{6.13}$$

The mass matrix is diagonalized as

$$\begin{pmatrix} \psi \\ \lambda_{BL} \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \chi_\ell \\ \chi_h \end{pmatrix} \tag{6.14}$$

with $\tan 2\theta = 2m_{Z'}/M_{BL}$. Let us assume that the lighter mass eigenstate (χ_ℓ) is the lightest neutralino. Since ψ and λ_{BL} are the SM gauge singlets, possible annihilation processes of the dark matter are very limited. Furthermore, given a small $B-L$ coupling and the Majorana nature of the dark matter particle, the annihilation process via sfermion exchanges is not efficient. We find that a pair of dark matter particles can annihilate efficiently only if the dark matter mass is close to half of the Z' boson mass and the Z' boson resonance in the s-channel annihilation process enhances the cross section. Let us set $M_{BL} \simeq (3/2) m_{Z'}$, so that the lightest mass eigenvalue is found to be $m_{DM} \simeq m_{Z'}/2$ and $\cos^2\theta \simeq 0.8$. Our parameter choice in the previous section is suitable for this setup, $M_{BL} = (3/2) m_{Z'} = 5.25$ TeV for $m_{Z'} = 3.5$ TeV.

Let us now calculate the dark matter relic abundance by integrating the Boltzmann equation given by

$$\frac{dY}{dx} = -\frac{s\langle\sigma v\rangle}{x H(m_{DM})} \left(Y^2 - Y_{EQ}^2\right), \tag{6.15}$$

where temperature of the universe is normalized by the mass of the right-handed neutrino $x = m_{DM}/T$, $H(m_{DM})$ is the Hubble parameter at $T = m_{DM}$, Y is the yield (the ratio of the dark matter number density to the entropy density s) of the dark matter particle, Y_{EQ} is the yield of the dark matter particle in thermal equilibrium, and $\langle\sigma v\rangle$ is the thermal average of the dark matter annihilation cross section times relative velocity. Explicit formulas of the quantities involved in the Boltzmann equation are as follows:

$$\begin{aligned} s &= \frac{2\pi^2}{45} g_\star \frac{m_{DM}^3}{x^3}, \\ H(m_{DM}) &= \sqrt{\frac{4\pi^3}{45} g_\star} \frac{m_{DM}^2}{M_{Pl}}, \\ sY_{EQ} &= \frac{g_{DM}}{2\pi^2} \frac{m_{DM}^3}{x} K_2(x), \end{aligned} \tag{6.16}$$

where $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, $g_{DM} = 2$ is the number of degrees of freedom for the Majorana dark matter particle, $g_\star$ is the effective total number of degrees of freedom for particles in thermal equilibrium (in the following analysis, we use $g_\star = 106.75$ for the SM particles), and K_2 is the modified Bessel function of the second kind. In our scenario, a pair of dark matter annihilates into the SM particles dominantly through the Z' boson exchange in the s-channel. The thermal average of the annihilation cross section is given by

$$\langle\sigma v\rangle = (sY_{EQ})^{-2}\,\frac{m_{DM}}{64\pi^4 x}\int_{4m_{DM}^2}^{\infty} ds\;\hat{\sigma}(s)\sqrt{s}K_1\left(\frac{x\sqrt{s}}{m_{DM}}\right), \tag{6.17}$$

where the reduced cross section is defined as $\hat{\sigma}(s) = 2(s - 4m_{DM}^2)\sigma(s)$ with the total annihilation cross section $\sigma(s)$, and K_1 is the modified Bessel function of the first kind. The total cross section of the dark matter annihilation process $\chi_\ell\chi_\ell \to Z' \to f\bar{f}$ (f denotes the SM fermions plus two right-handed neutrinos) is calculated as

$$\sigma(s) = \frac{5}{4\pi} g_{BL}^4 \cos^2\theta \frac{\sqrt{s(s-4m_{DM}^2)}}{(s-m_{Z'}^2)^2 + m_{Z'}^2\Gamma_{Z'}^2}, \tag{6.18}$$

where all final state fermion masses have been neglected. The total decay width of Z' boson is given by

$$\Gamma_{Z'} = \frac{g_{BL}^2}{24\pi} m_{Z'}\left[15 + \cos^2\theta\left(1 - \frac{4m_{DM}^2}{m_{Z'}^2}\right)^{\frac{3}{2}}\theta\left(\frac{m_{Z'}^2}{m_{DM}^2} - 4\right)\right]. \tag{6.19}$$

Here, we have assumed that all sparticles have mass larger than $m_{Z'}/2$.

Now we solve the Boltzmann equation numerically, and find the asymptotic value of the yield $Y(\infty)$. Then, the dark matter relic density is evaluated as

$$\Omega_{DM}h^2 = \frac{m_{DM}s_0 Y(\infty)}{\rho_c/h^2}, \tag{6.20}$$

where $s_0 = 2890\ \mathrm{cm}^{-3}$ is the entropy density of the present universe, and $\rho_c/h^2 = 1.05\times10^{-5}$ GeV/cm^3 is the critical density. In our analysis, only three parameters, namely g_{BL}, $m_{Z'}$ and m_{DM}, are involved.[3] As mentioned above, a sufficiently large annihilation cross section is achieved only if $m_{DM} \simeq m_{Z'}/2$. Thus, we focus on the dark matter mass in this region and in this case $\cos^2\theta \simeq 0.8$. For $g_{BL} = 0.250$, $m_{Z'} = 3.5$ TeV and $\cos^2\theta = 0.8$, Fig. 6.2 shows the resultant dark matter relic abundance as a function of the dark matter mass m_{DM}, along with the bound $0.1183 \le \Omega_{DM}h^2 \le 0.1213$ (65) from the Planck satellite experiment [1] (two

[3] The mixing angle θ is determined once $m_{Z'}$ and m_{DM} are fixed.

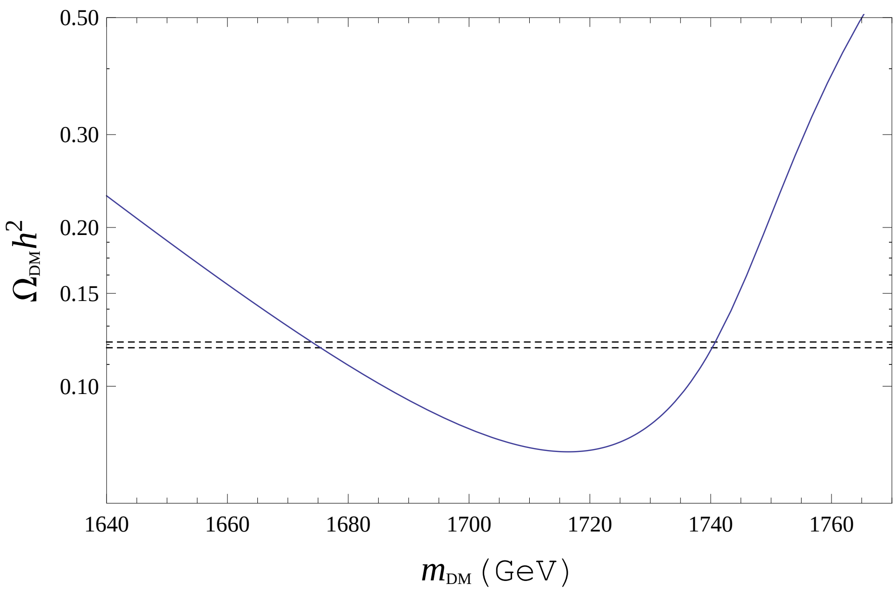

Fig. 6.2.: The relic abundance of the dark matter particle as a function of the dark matter mass (m_{DM}) for $g_{BL} = 0.250$, $m_{Z'} = 3.5$ TeV and $\cos^2\theta = 0.8$. The two horizontal lines denote the range of the observed dark matter relic density, $0.1183 \leq \Omega_{DM}h^2 \leq 0.1213$ [1].

horizontal dashed lines). We have confirmed that only if the dark matter mass is close to half of the Z' boson mass, the observed relic abundance can be reproduced.

6.5 Implication of Dirac neutrinos to LHC physics

Because of our R-parity assignment, the SM neutrinos are Dirac particles in our model. This is quite distinct from usual $B - L$ extension of the SM, where right-handed neutrinos are heavy Majorana states. Since the right-handed neutrinos are singlet under the SM gauge groups and the Dirac Yukawa coupling constants are very small in both Dirac and Majorana cases, the right-handed neutrinos can communicate with the SM particles only through Z' boson exchange.

As we mentioned above, the search for Z' boson resonance is underway at the LHC Run-2. Once discovered at the LHC, the Z' boson will allows us to investigate physics of the right-handed neutrinos through precise measurements of Z' boson properties. In this section we consider an implication of the Dirac neutrinos to LHC physics.

When the right-handed neutrinos are heavy Majorana particles as in the minimal $B-L$ model, a pair of right-handed Majorana neutrinos, if kinematically allowed, can be produced through Z' boson decays at the LHC. The right-handed neutrino subsequently decays to weak gauge bosons/Higgs boson plus leptons. Because of the Majorana nature of the right-handed neutrino, the final states include same-sign leptons. This is a characteristic signature from the lepton number violation, and we expect a high possibility to detect such final states with less SM background. For a detailed studies, see, for example, [45].

The Majorana neutrinos are heavy and can be produced only if they are kinematically allowed, while the Dirac neutrinos in our model are always included in the Z' boson decay products. However, they cannot be detected just like the usual SM neutrinos produced at colliders. This process may remind us of the neutrino production at the LEP through the resonant production of the Z boson. It was a great success of the LEP experiment that the precise measurement of the Z boson decay width and the production cross section at energies around the Z boson peak has determined the number of the SM neutrinos to be three [41]. We notice that the Z' production is quite analogous to the Z production at the LEP. Although the right-handed neutrinos produced by the Z' boson are completely undetectable, the total Z' boson decay width carries the information of the invisible decay width. A precise measurement of the Z' boson cross section at the LHC may reveal the existence of the right-handed Dirac neutrinos. To illustrate this idea, we calculate in the following the differential cross section for the process with the dilepton final states, $pp \to \ell^+\ell^-$ with $\ell = e, \mu$ mediated by photon, Z boson and Z' boson at the LHC with a collider energy $\sqrt{s} = 14$ TeV.

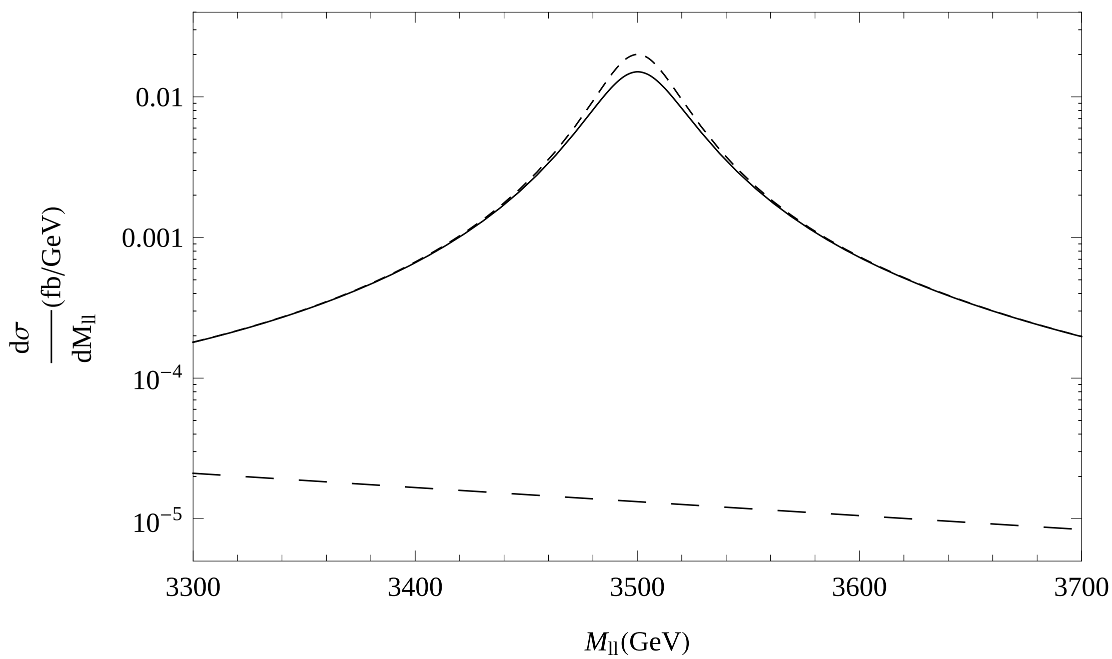

Fig. 6.3.: The differential cross section for $pp \to e^+e^-X + \mu^+\mu^-X$ at the 14 TeV LHC for $m_{Z'} = 3.5$ TeV and $g_{BL} = 0.250$. The solid and dashed curves correspond to the results for $N(\nu_R) = 2$ and 0, respectively. The horizontal long-dashed line represents the SM cross section, which is negligible compared with the Z' boson mediated process.

The differential cross section with respect to the final state dilepton invariant mass M_{ll} is described as

$$\frac{d\sigma(pp \to \ell^+\ell^- X)}{dM_{ll}} = \sum_{a,b} \int_{-1}^{1} d\cos\theta \int_{\frac{M_{ll}^2}{E_{\rm CMS}^2}}^{1} dx_1 \frac{2M_{ll}}{x_1 E_{\rm CMS}^2} \times f_a(x_1, Q^2) f_b\left(\frac{M_{ll}^2}{x_1 E_{\rm CMS}^2}, Q^2\right) \frac{d\sigma(\bar{q}q \to \ell^+\ell^-)}{d\cos\theta}, \tag{6.21}$$

where $E_{\rm CMS} = 14$ TeV is the center-of-mass energy of the LHC. In our numerical analysis, we employ CTEQ5M [46] for the parton distribution functions (f_a) with the factorization scale $Q = m_{Z'}$. The reader may refer to the Appendix in Ref. [47] for the helicity amplitudes to calculate $d\sigma(\bar{q}q \to \ell^+\ell^-)/d\cos\theta$. For the Z' boson mediated process, we consider two cases,

$N(\nu_R) = 0$ and $N(\nu_R) = 2$, where $N(\nu_R)$ is the number of right-handed (Dirac) neutrinos. For our case with $N(\nu_R) = 2$, the total Z' boson decay width is given in Eq. (6.19), while the number 15 in the bracket must be replaced to 12 for $N(\nu_R) = 0$.

Fig. 6.3 shows the differential cross section for $pp \to e^+e^-X + \mu^+\mu^-$ for $m_{Z'} = 3.5$ TeV and $g_{BL} = 0.250$, along with the SM cross section mediated by the Z-boson and photon (horizontal long-dashed line). The solid and dashed curves correspond to the results for $N(\nu_R) = 2$ and 0, respectively. The dependence of the total decay width on the number of right-handed neutrinos reflects the resultant cross sections. When we choose a kinematical region for the invariant mass in the range, $M_{Z'} - 100 \leq M_{ll}(\text{GeV}) \leq M_{Z'} + 100$, for example, the signal events of 892 and 1049 for $N(\nu_R) = 2$ and 0, respectively, would be observed with the prospective integrated luminosity of 1000/fb at the High-Luminosity LHC. The difference between $N(\nu_R) = 2$ and 0 are distinguishable with a $4 - 5\sigma$ significance.

7 Radiative Breaking of the Minimal Supersymmetric Left-Right Model

7.1 Introduction

Nature at low energies can be described by a vector-like model known as Quantum Electrodynamics (QED). Adding the strong interactions into the mix, nature retains its indifference to a fields' handedness. At higher energies, we encounter the Standard Model (SM) which is a chiral theory that is broken down into QED via Electroweak Symmetry Breaking (EWSB). Among the fermions in the SM only left-handed fields interact under $SU(2)_L$. This question of why does such a parity violation exist as well many others are not cannot be answered by the SM alone. Motivation for nature returning to vector-like at TeV scales and higher has led to Left-Right symmetric Models (LRMs) being introduced. The first LRM was a broken Pati-Salam model [48] introduced in [49] with the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{BL}$. The LR symmetry must be broken at low energies, TeV scale LRMs are being once again considered from the view point of the Large Hadron Collider (LHC) experiments. The current lower bound on the $SU(2)_R$ charged gauge boson (W_R) is found to be around 3 TeV [50] (see also [51] on the lower bound from rare decay processes).

Historically the first type of LR symmetry breaking was done by a $SU(2)_R$ doublet Higgs field[52, 53]. After the introduction of the seesaw mechanism [21], breaking LR symmetry by $SU(2)_L$ and $SU(2)_R$ triplets was considered. This case has new sets of unnaturalness problems with keeping the $SU(2)_L$ triplet vacuum expectation value (VEV) at the neutrino mass scale [54]. Its minimal superymmetric (SUSY) extensions have been suggested before, however broken by triplet superfields [55, 56, 49]. Triplet Higgs superfields lead to a $U(1)_{em}$ violating vacuum [57, 58]. To keep a $U(1)_{em}$ invariant vacuum, at least one generation of right-handed scalar neutrino $\tilde{N}^c$ must acquire a nonzero VEV. If we consider a supersymmetric LRM with the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{BL}$, the the right-handed slepton doublet plays a role of the $SU(2)_R$ doublet Higgs field and a VEV of right-handed scalar neutrino $\tilde{N}^c$ can break the LR symmetry down to the SM one [59]. It has been shown [29] that in the B-L extension of the minimal suspersymmetric Standard Model (MSSM), the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{BL}$ is successfully broken down to to the SM one by $\langle \tilde{N}^c \rangle$. In this context of the $U(1)_{BL}$ extension of the MSSM, radiative symmetry breaking can occur when $\tilde{N}^c$'s mass squared becomes negative at low energies [60, 61]. Generally the seesaw mechanism comes about from a triplet scalar VEV inducing a Majorana mass term for the right-handed neutrino. However in this model, the seesaw is induced by the mixing between gaugino and neutrino [34, 35].

The main focus of this section is to propose a class of supersymmetric LRMs, where only a second Higgs bidoublet superfield is newly introduced, and the LR symmetry is radiatively broken into the MSSM purely by the VEV of the neutral component of the right-handed slepton doublet. The LR symmetry breaking without any additional Higgs fields has been considered before [59], where a negative mass squared for the right-handed slepton doublet is assumed. Here we calculate the renormalization group equations (RGEs) at the one-loop level and evolve them from some intermediate scale down to the TeV scale. We find that the mass squared of the right-handed slepton becomes negative and hence the LR symmetry is radiatively broken. After the breaking, a charged lepton mixes with a charged gaugino, creating a sever bound on the gaugino mass from the electroweak precision measurements. The neutral lepton component mixes with neutral gauginos and creates a heavy neutrino with a TeV scale mass. After EWSB the seesaw mechanism works to produce sub-eV scale neutrino masses. With the additional Higgs bidoublet, there are enough free parameters to reproduce realistic SM fermion mass matrices.

	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	$U(1)_{BL}$
$Q = \begin{pmatrix} u \\ d \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\mathbf{1}$	$1/3$
$Q^c = \begin{pmatrix} u^c \\ d^c \end{pmatrix}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{2}$	$-1/3$
$L = \begin{pmatrix} \nu \\ e \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	-1
$L^c = \begin{pmatrix} \nu^c \\ e^c \end{pmatrix}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	1
$\Phi_i = \begin{pmatrix} \phi^+ & \phi_1^0 \\ \phi_2^0 & \phi^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	0

Table 7.1.: Particle content of our SUSY LR model.

7.2 Particle Content

The particle content remains largely unchanged from the MSSM as can bee seen in Table 7.1. We extend the particle content in [59] by an extra Higgs bidoublet, which is necessary to obtain the realistic SM fermion mass matrices, otherwise there is no flavor mixing in the model. The superpotential can be written down (flavor sums implied) as

$$\begin{aligned} \mathcal{W} &= Y_q Q^T \tau_2 \Phi_1 \tau_2 Q^c + Y_q' Q^T \tau_2 \Phi_2 \tau_2 Q^c \\ &+ Y_e L^T \tau_2 \Phi_1 \tau_2 L^c + Y_e' L^T \tau_2 \Phi_2 \tau_2 L^c + \mu_{ii} \mathrm{Tr} \left(\Phi_i^T \tau_2 \Phi_i \tau_2 \right) , \end{aligned} \tag{7.1}$$

where we work the diagonal basis for the Higgs bidoublet without loss of generality. We can integrate a heavy Higgs bidoublet out at lower energies, and a lighter bidoublet to be approximately identified as the MSSM Higgs.

The scalar potential with soft SUSY breaking masses is given by

$$V_{soft} = m_{\tilde{L}}^2|\tilde{L}|^2+m_{\tilde{L}^c}^2|\tilde{L}^c|^2+m_{\tilde{Q}}^2|\tilde{Q}|^2+m_{\tilde{Q}^c}^2|\tilde{Q}^c|^2 + m_{ij}^2 \mathrm{Tr}\left(\Phi_i^\dagger \Phi_j\right) + B\mu_{ij}\mathrm{Tr}\left(\Phi_i^T \tau_2 \Phi_j \tau_2\right) . \quad (7.2)$$

Here we have omitted A-terms, for simplicity, since their effects are not important in the following discussions. While the SUSY mass term for the two bidoublet Higgs superfields μ_{ij} is diagonal in Eq. (7.1), here we have introduced the off-diagonal $B\mu_{ij}$ term, which will be tuned in order for the heavy Higgs bidoublet to develop a sizable VEV.

7.3 RGE Analysis and Radiative LR symmetry breaking

In our RGE analysis, we use a mixture of low energy data for the Standard Model gauge and Yukawa couplings mixed with high energy inputs inspired by the MSSM. For Yukawa couplings we only consider the 3rd generation. Using the RGEs of the SM [62] at the one-loop level we run them from $\mu = M_Z$ to $\mu = 1$ TeV. Taking the outputs of the previous SM RGE runnings at $\mu = 1$ TeV as inputs for the RGEs of the MSSM [63] at the one-loop level, we solve the MSSM RGEs until LR symmetry breaking scale v_R. In this section, we fix $v_R = 20$ TeV as a reference value. At the one-loop level the soft mass terms do not affect the runnings of the gauge and Yukawa couplings. At the LR symmetry breaking we have the relations between the hypercharge gauge coupling (g_Y) and the LR gauge couplings (g_R and g_{BL}) as

$$g_Y = g_R \sin\theta_R\,, \quad \tan\theta_R = 2\,\frac{g_{BL}}{g_R}\,. \quad (7.3)$$

In this analysis we choose, for simplicity, $\theta_R = 65^\circ$, $g_{BL} = 0.438$, and $g_R = 0.408$, which are evaluated at $v_R = 20$ TeV based on Eq. (7.3) from the known MSSM gauge couplings.

The values of the tau and top Yukawa couplings from the MSSM RGEs at $\mu = 20$ TeV are evaluated as $Y_\tau \simeq 0.01$ and $Y_t \simeq 0.8$. As a matter of simplicity we choose $Y_q = 0.7\, Y_t$ and $Y_{q'} = 0.3\, Y_t$ and $Y_{l'} = Y_l = Y_\tau/2$ as inputs at $\mu = 20$ TeV. We run the RGEs for the Yukawa couplings and gauge couplings (see Eqs. (A.41)-(A.47) in Appendix A) from 20 TeV up to a SUSY breaking mediation scale which we choose to be an intermediate scale $\mu = 10^{12}$ GeV, for simplicity. At the scale of 10^{12} GeV, we take all gaugino masses to be 2.5 TeV except for the $SU(2)_R$ gaugino which is 100 TeV to keep the guagino-lepton mixing within the current experimental bound. This bound will be discussed below. The RGE invariant relation in Eq. (A.42) is used for the gaugino masses. We calculate the RGE evolutions in Eqs. (A.48)-(A.53) for the soft masses at the one-loop level and run them down from $\mu = 10^{12}$ GeV to $\mu = 20$ TeV. We use the evaluated Yukawa and gauge couplings at $\mu = 10^{12}$ GeV as inputs into the soft mass RGEs. To realize the LR symmetry breaking the non-universal soft mass inputs are crucial. See Table 7.2 for our inputs at $\mu = 10^{12}$ GeV and outputs at $\mu = 20$ TeV.

Our choices for the masses are a result of straightforward numerical calculation of RGEs. At $\mu = 10^{12}$ GeV, g_{BL} is the largest coupling so Yukawas can be ignored except for the RGEs for the bidoublet Higgs mass squares. Because of this size, the sign in front of the D-term trace given in Eq. (A.54), which is involved in the RGEs of Eqs. (A.48)-(A.53), will dominate and could drive the soft mass square of $\tilde{L}^c$ negative at low energies.

The running mass squared for $\tilde{L}_3^c$ is shown in Fig 7.1. We see that it becomes negative at low energies. Here we consider the case that the 3rd generation right-handed slepton doublet acquires the negative mass squared. The potential for $\tilde{L}_3^c$ is described as

$$V = m^2_{\tilde{L}_3^c} |\tilde{L}_3^c|^2 + \frac{1}{8}\left(g_R^2 + 4 g_{BL}^2\right) |\tilde{L}_3^c|^4 \, , \tag{7.4}$$

	$\mu = 20$ TeV	$\mu = 10^{12}$ TeV
$M^2_{\tilde{L}^c_1}$	$2.0 \times 10^9\,\text{GeV}^2$	$2.5 \times 10^9\,\text{GeV}^2$
$M^2_{\tilde{L}^c_2}$	$2.0 \times 10^9\,\text{GeV}^2$	$2.5 \times 10^9\,\text{GeV}^2$
$M^2_{\tilde{L}^c_3}$	$-4.7 \times 10^7\,\text{GeV}^2$	$2.1 \times 10^4\,\text{GeV}^2$
$M^2_{\tilde{Q}^c_3}$	$3.1 \times 10^9\,\text{GeV}^2$	$2.5 \times 10^9\,\text{GeV}^2$
$M^2_{\tilde{Q}_3}$	$1.3 \times 10^{10}\,\text{GeV}^2$	$1.4 \times 10^{10}\,\text{GeV}^2$
$M^2_{\tilde{L}_3}$	$4.2 \times 10^9\,\text{GeV}^2$	$2.5 \times 10^9\,\text{GeV}^2$
$M^2_{\Phi_1}$	$1.0 \times 10^6\,\text{GeV}^2$	$2.1 \times 10^8\,\text{GeV}^2$
$M^2_{\Phi_2}$	$3.4 \times 10^9\,\text{GeV}^2$	$2.5 \times 10^9\,\text{GeV}^2$
$M_{\tilde{g}}$	5000 GeV	2500 GeV
M_L	2300 GeV	2500 GeV
M_R	10^5 GeV	10^5 GeV
M_{BL}	800 GeV	2500GeV

Table 7.2.: List of soft masses at $\mu = 10^{12}$ GeV (inputs) and at $\mu = 20$ TeV (outputs). $M_{\tilde{g}}$, M_L, M_R and M_{BL} are gaugino masses corresponding to $SU(3)_c$, $SU(2)_L$, $SU(2)_R$ and $U(1)_{BL}$, respectively.

and the right-handed scalar neutrino $\tilde{N}^c_3$ develops its VEV at the potential minimum as $\langle \tilde{N}^c_3 \rangle = v_R/\sqrt{2}$, where

$$v_R = \sqrt{\frac{-8m^2_{\tilde{L}^c_3}}{g_R^2 + 4g_{BL}^2}}\,. \tag{7.5}$$

The numerical value in this model for the VEV is 20 TeV and $m^2_{\tilde{L}^c_1}$ is evaluated at 20 TeV. Since the $SU(2)_R \times U(1)_{BL}$ symmetry is broken by the $SU(2)_R$ doublet VEV, the gauge boson mass relations are very similar to those in the SM. One gauge boson remains massless which is identified as the $U(1)_Y$ gauge boson while the three massive ones and a charge

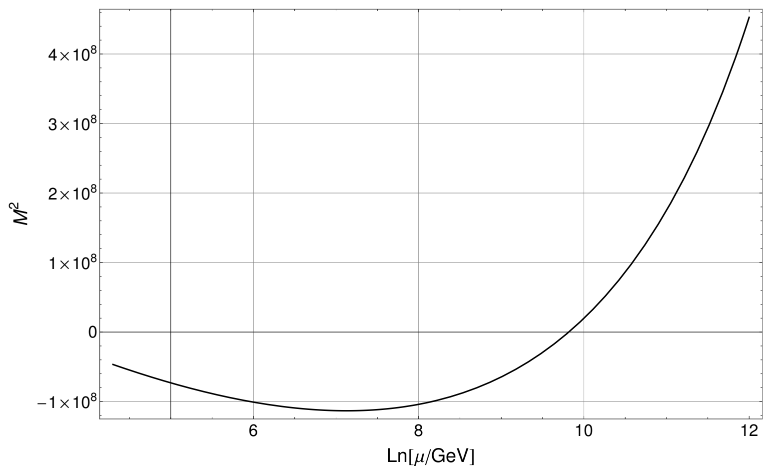

Fig. 7.1.: The RGE evolution of the soft mass squared for $\tilde{L}_3^c$, which becomes negative at low energies.

relation are

$$M_{W_R} = \frac{1}{2} g_R v_R \,, \tag{7.6}$$

$$M_{Z_R} = \frac{1}{2}\sqrt{g_R^2 + 4g_{BL}^2}\, v_R \,, \tag{7.7}$$

$$Q_Y = \frac{Q_{BL}}{2} - T_R^3 \,. \tag{7.8}$$

The gauge boson masses based on our runnings of the couplings and the above VEV come out to be 4.1 TeV and 9.6 TeV, respectively, which satisfies the LHC bound of $M_{W_R} \gtrsim 3$ TeV [50].

7.4 Mass bound on $SU(2)_R$ gaugino

In the above, we stated that there is a bound on the $SU(2)_R$ gaugino mass. This bound is unique to this model where the LR symmetry is broken by the VEV of the right-handed neutrino. After the breaking of the LR symmetry, the right-handed tau is mixed with the

$SU(2)_R$ gaugino. The relevant terms are

$$\mathcal{L} \supset M_R \tilde{\lambda}^+ \tilde{\lambda}^- + \frac{1}{\sqrt{2}} g_R v_R \tilde{\lambda}^- E^c = M_R \tilde{\lambda}^+ \tilde{\lambda}^- + \sqrt{2} M_{W_R} \tilde{\lambda}^- E^c \,. \tag{7.9}$$

We diagonalize the mass matrix as

$$\xi_1^+ = \cos\phi \tilde{\lambda}_R^+ + \sin\phi E^c \qquad \text{and} \qquad \xi_2^+ = \cos\phi E^c - \sin\phi \tilde{\lambda}_R^+ \tag{7.10}$$

with a mixing angle

$$\tan\phi = \frac{\sqrt{2} M_{W_R}}{M_R} \,. \tag{7.11}$$

The neutral current for the charged leptons in the SM is now modified as

$$J_Z^\mu = \frac{2 m_Z}{v} \left[\left(-\frac{1}{2} + \sin^2\theta_W \right) \overline{\tau_L} \gamma^\mu \tau_L + \sin^2\theta_W \cos^2\phi \, \overline{\tau_R} \gamma^\mu \tau_R \right] , \tag{7.12}$$

where $v = 246$ GeV, θ_W is the weak mixing angle, and $m_Z = 91.2$ GeV. Using the precision data at the LEP experiment for $Z \to \tau^+\tau^-$ decay width uncertainties, the modification of the weak neutral current must not change the width by more than $|\delta\Gamma| = 0.22$ MeV [41]. Using Eq. (7.12), we calculate the change of the decay width as

$$\delta\Gamma = \frac{m_Z^3 \sin^4\theta_W}{6\pi v^2} (\cos^4\phi - 1) \approx -\frac{m_Z^3 \sin^4\theta_W}{6\pi v^2} \left(\frac{4 M_{W_R}^2}{M_R^2} \right) , \tag{7.13}$$

where we have used Eq. (7.11) and $|\phi| \ll 1$. Now we interpret the LEP bound as $M_R \gtrsim 25 M_{W_R}$. At the scale of v_R=20 TeV we calculate M_{W_R} =4.1 TeV, so the mass $M_R = 100$ TeV shown in Table 7.2 is consistent with the LEP bound.

7.5 SM fermion mass matrices

We first examine the neutral fermion sector to analyze the mixing between the gauginos and leptons from the SUSY gauge interaction after $\tilde{L}^c$ develops a nonzero VEV. The hypercharge $Q_Y = 0$ sector of the Lagrangian after LR symmetry breaking is

$$\mathcal{L} \supset g_{BL} v_R \nu^c \lambda_{BL} + \frac{1}{2} g_R v_R \nu^c \lambda_R^3 + \frac{1}{2} M_R \lambda_R^3 \lambda_R^3 + \frac{1}{2} M_{BL} \lambda_{BL} \lambda_{BL}, \tag{7.14}$$

where λ_R^3 is the gaugino corresponding to the $SU(2)_R$ generator T_R^3. The mass matrix after the LR symmetry breaking is found to be

$$M_{\tilde{\lambda}_R^3, \tilde{\lambda}_{BL}, \nu^c} = \begin{pmatrix} M_R & 0 & \frac{1}{2} g_R v_R \\ 0 & M_{BL} & g_{BL} v_R \\ \frac{1}{2} g_R v_R & g_{BL} v_R & 0 \end{pmatrix}. \tag{7.15}$$

Because of the LEP bound $M_R \gg M_{W_R}$, λ_R^3 is decoupled, while the right-handed neutrino (ν^c) acquires its Majorana mass of $\mathcal{O}(1 \text{ TeV})$ through the mixing with the B-L gaugino with M_{BL}, $g_R v_R$, $g_{BL} v_R = \mathcal{O}(1 \text{ TeV})$. With this right-handed neutrino mass of $\mathcal{O}(1 \text{ TeV})$, the seesaw mechanism works in our model.

After EWSB, the SM fermion mass matrices can be expressed as

$$M_t = \frac{1}{\sqrt{2}} Y_Q v_u + \frac{1}{\sqrt{2}} Y'_Q v'_u = M_Q + M'_Q \,, \tag{7.16}$$

$$M_b = \frac{1}{\sqrt{2}} Y_Q v_d + \frac{1}{\sqrt{2}} Y'_Q v'_d = c M_Q + c' M'_Q \,, \tag{7.17}$$

$$M_\nu^D = \frac{1}{\sqrt{2}} Y_L v_u + \frac{1}{\sqrt{2}} Y'_L v'_u = M_L + M'_L \,, \tag{7.18}$$

$$M_\tau = \frac{1}{\sqrt{2}} Y_L v_d + \frac{1}{\sqrt{2}} Y'_L v'_d = c M_L + c' M'_L \,, \tag{7.19}$$

where $c = v_d/v_u$ and $c' = v'_d/v'_u$, and we have considered the 3rd generation to simplify our discussion. Since there are two Higgs bidoublets creating four nonzero VEVs, they can all be paramterized on a 4-sphere, allowing for 3 free parameters under the constraint $v_u^2 + v_d^2 + v_u'^2 + v_d'^2 = (246)^2$ GeV^2. We tune Y'_L so that there is a cancellation in Eq. (7.18) to produce the neutrino Dirac mass, $M_\nu^D = \mathcal{O}(10^{-3}$ GeV), while allowing for the tau lepton Dirac mass $M_\tau = \mathcal{O}(1\,\text{GeV})$. In the quark sector we tune the quark Yukawa coupling, Y'_Q, so that there is a cancellation in Eq. (7.17) to produce $M_b = \mathcal{O}(1$ GeV) while the top quark mass equation produces $M_t = \mathcal{O}(100$ GeV). Our discussion here is easily extended to the three generation case, and we can reproduce realistic SM fermion mass matrices.

The Dirac mass term for the neutrinos will further mix with the Higgsinos and neutral gauginos from the EW sector as well to produce a neutralino mass matrix

$$\begin{pmatrix} 0 & \mu_{11} & 0 & 0 & Y_L\frac{v_R}{\sqrt{2}} & 0 & 0 & 0 \\ \mu_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_{22} & Y'_L\frac{v_R}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & \mu_{22} & 0 & 0 & 0 & 0 & 0 \\ Y_L\frac{v_R}{\sqrt{2}} & 0 & Y'_L\frac{v_R}{\sqrt{2}} & 0 & 0 & M_\nu^D & 0 & 0 \\ 0 & 0 & 0 & 0 & M_\nu^D & 0 & M_{W_R}\tan\theta_R & M_{W_R} \\ 0 & 0 & 0 & 0 & 0 & M_{W_R}\tan\theta_R & M_{BL} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{W_R} & 0 & M_R \end{pmatrix}. \tag{7.20}$$

For simplicity we took the one generation case. This can be easily extended to the 3 generation case by promoting the Yukawa couplings to 3×3 matrices. Since $M_R \gg M_{W_R}$, the $SU(2)_R$ gaugino is decoupled. To understand the seesaw mechanism in our model, we focus on the block-diagonal 3×3 matrix composed of the elements M_ν^D, $M_{W_R}\tan\theta_R$ and M_{BL}. Since $M_{W_R}\tan\theta_R$, $M_{BL} = \mathcal{O}(1$ TeV)$\gg M_\nu^D = \mathcal{O}(1$ MeV), we find a mass eigenvalue for the light neutrino as

$$m_\nu \simeq \frac{\left(M_\nu^D\right)^2}{M_{BL}} = \mathcal{O}(0.1\ \mathrm{eV}) \tag{7.21}$$

through the seesaw mechanism.[1]

[1] It is interesting to notice that if $M_{BL} \gg M_{W_R}$ the block-diagonal matrix has a "double seesaw" structure, leading to mass eigenvalues approximately given by $(M_\nu^D)^2/\tilde{M}$, $\tilde{M} \simeq (M_{W_R} \tan\theta_R)^2/M_{BL}$ and M_{BL}.

8 Conclusions

In this final chapter we state the conclusions for our two proposed models. Section 7.1 discusses Chapter 5 and Sections 7.2 discusses Chapter 6.

8.1 R-parity Conserving Minimal Supersymmetric B-L Model

We have proposed a simple gauged $U(1)_{B-L}$ extension of the MSSM, where R-parity is conserved as usual in the MSSM. The global $B-L$ symmetry in the MSSM is gauged and three right-handed neutrino chiral multiplets are introduced, which make the model free from all gauge and gravitational anomalies. No $B-L$ Higgs field is introduced. We assign an even R-parity to one right-handed neutrino superfield Φ, while the other two right-handed neutrino superfields are odd as usual. The scalar component of Φ plays a role of the $B-L$ Higgs field to beak the $U(1)_{B-L}$ gauge symmetry through its negative mass squared which is radiatively generated by the RG evolution of soft SUSY breaking parameters. Therefore, the scale of the $U(1)_{B-L}$ symmetry breaking is controlled by the SUSY breaking parameters and naturally be at the (multi-)TeV scale. We have shown that this radiative symmetry breaking actually occurs with a suitable choice of model parameters. Because of our novel R-parity assignment, three light neutrinos are Dirac particles with one massless state. Since R-parity is conserved, the lightest neutralino is a prime candidate of the cosmological dark matter. Depending on its mass, the lighter Majorana mass eigenstate (χ_ℓ) of a mixture of the $B-L$ gaugino and the fermionic component of Φ (R-parity odd right-handed neutrino) appears as a new dark matter candidate. Assuming χ_ℓ is the lightest R-parity odd particle, we have calculated the dark matter relic abundance. When the mass of χ_ℓ is close to half of the Z' boson mass, the pair annihilation cross section of the dark matter particle is enhanced through the Z' boson resonance in the s-channel process and the observed dark matter relic abundance is reproduced. We have also discussed LHC phenomenology for the Dirac neutrinos. The Z' boson, once discovered at the LHC, will be a novel probe of the Dirac nature of the light neutrinos since its invisible decay processes include the final states with one massless (left-handed) neutrino and two Dirac neutrinos, in sharp contrast with the conventional $B-L$ extension of the SM or MSSM, where the right-handed neutrinos are heavy Majorana particles and decay to the weak gauge bosons/Higgs boson plus leptons. With a discovery of Z' boson, the High-Luminosity LHC may reveal the existence of the right-handed neutrino.

Since the neutrinos are Dirac particles in our model, their Dirac Yukawa coupling must be extremely small. It is an important issue how to naturally realize such a small Yukawa coupling, or a huge hierarchy between the neutrino Yukawa coupling and those of the other SM fermions, in a reasonable theoretical framework. In addition, the mass squared hierarchy between ϕ and the other right-handed neutrinos is crucial to achieve the radiative $B-L$ gauge symmetry breaking. Realizing this hierarchy in a natural way is an additional issue. In order to solve these hierarchy problems, we may extend the model to the brane-world framework with 5-dimensional warped space-time [64]. Arranging the bulk mass parameters for the bulk hypermultiplets corresponding to the matter and Higgs fields in the minimal SUSY $B-L$ model, we can obtain large hierarchy among parameters in 4-dimensional effective theory with mildly hierarchical parameters in the original 5-dimensional theory. This direction is worth investigating.

8.2 Radiative Breaking of the Minimal Supersymmetric Left-Right Model

We have considered a SUSY Left-Right symmetric model based on the gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{BL}$, where in addition to the quark and lepton superfields only two Higgs bidoublets are introduced. With suitable soft mass inputs at a SUSY breaking mediation scale, where scalar squared masses are all positive, we have found that a right-handed slepton doublet mass squared becomes negative in its RG evolution, and as a result, the LR symmetry is radiatively broken to the SM gauge group by a right-handed neutrino VEV. The right-handed neutrino VEV also generates a mass mixing between the $SU(2)_R$ gaugino and SM right-handed lepton. This is a unique feature of our model, and the mass mixing is severely constrained by the LEP electroweak precision data. We have found the mass ratio of $M_R \gtrsim 25 M_{W_R}$ from the LEP bound. Realistic SM fermion mass matrices can be reproduced by the introduction of the two Higgs bidoublets and suitable tunings of Yukawa matrices. The right-handed neutrinos acquire Majorana masses of $\mathcal{O}(1 \text{ TeV})$ through its mixing with the B-L gaugino, and the seesaw mechanism works to generate a light neutrino mass of sub-eV scale.

In our model, R-parity is also broken by the right-handed sneutrino VEV, so that the lightest superpartner (LSP) neutralino, which is the conventional dark matter candidate in SUSY models, becomes unstable and no longer remains a viable dark matter candidate. As discussed in [65, 61], even in the presence of R-parity violation, an unstable gravitino if it is the LSP has a lifetime longer than the age of the universe and can still be the dark matter candidate. Hence, as a simple way to incorporate a dark matter candidate in our model, we can consider the LSP gravitino scenario. However, with the given mass hierarchy $M_R = 100$ TeV $\gg M_{BL} = 800$ GeV, it is difficult to naturally provide the LSP gravitino in 4-dimensional supergravity mediated SUSY breaking. For a simple realization, we may consider a gravity mediated SUSY breaking in a warped 5-dimensional supergravity [66], where gravitino is always the LSP with a SUSY breaking mediation scale being "warped down" from the Planck mass. This gravity mediation at low energies fits the choice of the SUSY breaking mediation scale to be $\mu = 10^{12}$ GeV in our RGE analysis.

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A APPENDIX A: Renormalization Group Equations

A complete study can be found in [62] for the SM and [63] for the MSSM. In the scheme of GUT theories the $U(1)$ associated isn't the hypercharge exactly. Some literature uses the gauge group $U(1)$ normalization of $\frac{3}{5}g_y^2 = g_1^2$.

A.1 The Standard Model

A.1.1 Gauge couplings

The RGEs for the gauge couplings of the SM are

$$16\pi^2 \frac{dg_Y}{dt} = \tfrac{41}{6} g_Y^3, \tag{A.1}$$

$$16\pi^2 \frac{dg_L}{dt} = -\tfrac{19}{6} g_L^3, \tag{A.2}$$

$$16\pi^2 \frac{dg_c}{dt} = -7 g_c^3, \tag{A.3}$$

where $t = \ln \mu$ and μ is the energy scale.

A.1.2 Yukawa Couplings

Each bold Yukawa couplings is a 3x3 complex matrix. The indicies explicitly written in the main body of this document are understood. They are

$$\frac{d\mathbf{Y}_i}{dt} = \frac{\mathbf{Y}_i}{16\pi^2}\beta_i \tag{A.4}$$

where

$$\beta_u = \frac{3}{2}(|\mathbf{Y}_u|^2 - |\mathbf{Y}_d|^2) + \mathrm{Tr}[3|\mathbf{Y}_u|^2 + 3|\mathbf{Y}_d|^2 + |\mathbf{Y}_e|^2] - \left(\frac{17}{12}g_Y^2 + \frac{9}{4}g_L^2 + 8g_c^2\right), \tag{A.5}$$

$$\beta_d = \frac{3}{2}(|\mathbf{Y}_d|^2 - |\mathbf{Y}_u|^2) + \mathrm{Tr}[3|\mathbf{Y}_u|^2 + 3|\mathbf{Y}_d|^2 + |\mathbf{Y}_e|^2] - \left(\frac{5}{12}g_Y^2 + \frac{9}{4}g_L^2 + 8g_c^2\right), \tag{A.6}$$

$$\beta_e = \frac{3}{2}|\mathbf{Y}_e|^2 + \mathrm{Tr}[3|\mathbf{Y}_u|^2 + 3|\mathbf{Y}_d|^2 + |\mathbf{Y}_e|^2] - \left(\frac{45}{12}g_Y^2 + \frac{9}{4}g_L^2\right). \tag{A.7}$$

The RGE for the Higgs quartic coupling is

$$\frac{d\lambda}{dt} = \frac{\beta_\lambda}{16\pi^2} \tag{A.8}$$

where the one-loop beta function is

$$\begin{aligned}\beta_\lambda =& 12\lambda^2 - \left(3g_Y^2 + 9g_L^2\right)\lambda + \frac{9}{4}\left(\frac{1}{3}g_Y^4 + \frac{2}{3}g_Y^2 g_L^2 + g_L^4\right) \\ &+ 4\lambda\mathrm{Tr}[3|\mathbf{Y}_u|^2 + 3|\mathbf{Y}_d| + |\mathbf{Y}_e|^2] - 4\mathrm{Tr}[3|\mathbf{Y}_u|^4 + 3|\mathbf{Y}_d|^4 + |\mathbf{Y}_e|^4].\end{aligned} \tag{A.9}$$

A.2 The Minimal Supersymmetric Standard Model

In SUSY because of the degree of symmetry, there are no vertex corrections only wave function renormalization contributions. The additional factor of scalars running in loops produce slightly different equations.

A.2.1 Gauge Couplings

The gauge couplings RGEs for the MSSM are

$$16\pi^2 \frac{dg_Y}{dt} = 11 g_Y^3, \tag{A.10}$$

$$16\pi^2 \frac{dg_L}{dt} = g_L^3, \tag{A.11}$$

$$16\pi^2 \frac{dg_c}{dt} = -3 g_c^3. \tag{A.12}$$

A.2.2 Yukawa Couplings

The Yukawa couplings follow the same form as (A.4) with beta functions

$$\beta_u = 3|\mathbf{Y}_u|^2 + |\mathbf{Y}_d|^2 + 3\mathrm{Tr}[|\mathbf{Y}_u|^2] - \left(\frac{13}{9} g_Y^2 + 3 g_L^2 + \frac{16}{3} g_c^2\right), \tag{A.13}$$

$$\beta_d = 3|\mathbf{Y}_d|^2 + |\mathbf{Y}_u|^2 + \mathrm{Tr}[3|\mathbf{Y}_d|^2 + |\mathbf{Y}_e|^2] - \left(\frac{7}{9} g_Y^2 + 3 g_L^2 + \frac{16}{3} g_c^2\right), \tag{A.14}$$

$$\beta_e = 3|\mathbf{Y}_e|^2 + \mathrm{Tr}[3|\mathbf{Y}_d|^2 + |\mathbf{Y}_e|^2] - \left(3 g_Y^2 + 3 g_L^2\right). \tag{A.15}$$

A.2.3 Soft SUSY couplings

The trilinear and soft masses are calculated as

$$\begin{aligned} \frac{dA_e^{ij}}{dt} = & \frac{1}{16\pi^2}\Big[\, 4(\mathbf{Y}_e\mathbf{Y}_e^\dagger)^{ik} A_e^{kj} \frac{Y_e^{kj}}{Y_e^{ij}} + 5 A_e^{ik} \frac{Y_e^{ik}}{Y_e^{ij}} (\mathbf{Y}_e^\dagger \mathbf{Y}_e)^{kj} - 3 \frac{A_e^{ij}}{Y_e^{ij}} (\mathbf{Y}_e \mathbf{Y}_e^\dagger \mathbf{Y}_e)^{ij} \\ & + 2(A_e^{km}|Y_e^{km}|^2 + 3 A_d^{km}|Y_d^{km}|^2) - 6(g_Y^2 M_Y + g_L^2 M_L)\,\Big], \end{aligned} \tag{A.16}$$

$$\begin{aligned} \frac{dA_d^{ij}}{dt} = & \frac{1}{16\pi^2}\Big[\, 4(\mathbf{Y}_d\mathbf{Y}_d^\dagger)^{ik} A_d^{kj} \frac{Y_d^{kj}}{Y_d^{ij}} + 5 A_d^{ik} \frac{Y_d^{ik}}{Y_d^{ij}} (\mathbf{Y}_d^\dagger \mathbf{Y}_d)^{kj} - 3 \frac{A_d^{ij}}{Y_d^{ij}} (\mathbf{Y}_d \mathbf{Y}_d^\dagger \mathbf{Y}_d)^{ij} \\ & + (A_d^{ik} - A_d^{ij})(\mathbf{Y}_u^\dagger \mathbf{Y}_u)^{kj} \frac{Y_d^{ik}}{Y_d^{ij}} + 2(\mathbf{Y}_d \mathbf{Y}_u^\dagger)^{ik} A_u^{kj} \frac{Y_u^{kj}}{Y_d^{ij}} + 2(A_e^{km}|Y_e^{km}|^2 \\ & + 3 A_d^{km}|Y_d^{km}|^2) - \frac{14}{9} g_Y^2 M_Y - 6 g_L^2 M_L - \frac{32}{3} g_c^2 M_c\,\Big], \end{aligned} \tag{A.17}$$

$$
\begin{aligned}
\frac{dA_u^{ij}}{dt} &= \frac{1}{16\pi^2}[\,4(\mathbf{Y}_u\mathbf{Y}_u^\dagger)^{ik}A_u^{kj}\frac{Y_u^{kj}}{Y_u^{ij}} + 5A_u^{ik}\frac{Y_u^{ik}}{Y_u^{ij}}(\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{kj} - 3\frac{A_u^{ij}}{Y_u^{ij}}(\mathbf{Y}_u\mathbf{Y}_u^\dagger\mathbf{Y}_u)^{ij} \\
&\quad + (A_u^{ik} - A_u^{ij})(\mathbf{Y}_d^\dagger\mathbf{Y}_d)^{kj}\frac{Y_u^{ik}}{Y_u^{ij}} + 2(\mathbf{Y}_u\mathbf{Y}_d^\dagger)^{ik}A_d^{kj}\frac{Y_d^{kj}}{Y_u^{ij}} + 6A_u^{km}|Y_u^{km}|^2 \\
&\quad - \frac{26}{9}g_Y^2M_Y - 6g_L^2M_L - \frac{32}{3}g_c^2M_c\,]\,, \qquad \text{(A.18)}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dm_{h_u}^2}{dt} &= \frac{1}{8\pi^2}[\sum_{i,j}3|Y_u^{ji}|^2(m_{h_u}^2 + m_{\tilde{q}_i}^2 + m_{\tilde{u}_j^c}^2 + |A_u^{ji}|^2) + \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} - g_Y^2M_Y^2 \\
&\quad - 3g_L^2M_L^2\,]\,, \qquad \text{(A.19)} \\
\frac{dm_{h_d}^2}{dt} &= \frac{1}{8\pi^2}[\sum_{i,j}\Big(|Y_e^{ji}|^2(m_{h_d}^2 + m_{\tilde{l}_i}^2 + m_{e_j}^2 + |A_e^{ji}|^2) + 3|Y_d^{ji}|^2(m_{h_d}^2 + m_{\tilde{q}_i}^2 + m_{\tilde{d}_j^c}^2 \\
&\quad + |A_d^{ji}|^2)\Big) - \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} - g_Y^2M_Y^2 - 3g_L^2M_L^2\,]\,, \qquad \text{(A.20)} \\
\frac{dm_{\tilde{e}_i^c}^2}{dt} &= \frac{1}{8\pi^2}[\sum_j 2|Y_e^{ij}|^2(m_{h_d}^2 + m_{\tilde{e}_i^c}^2 + m_{\tilde{l}_j}^2 + |A_e^{ij}|^2) \\
&\quad + g_Y^2\mathrm{Tr}\{Ym^2\} - 4g_Y^2M_Y^2\,]\,, \qquad \text{(A.21)} \\
\frac{dm_{\tilde{l}_i}^2}{dt} &= \frac{1}{8\pi^2}[\sum_j |Y_e^{ji}|^2(m_{h_d}^2 + m_{\tilde{l}_i}^2 + m_{\tilde{e}_j^c}^2 + |A_e^{ji}|^2) - \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} \\
&\quad - g_Y^2M_Y^2 - 3g_L^2M_L^2\,]\,, \qquad \text{(A.22)} \\
\frac{dm_{\tilde{d}_i^c}^2}{dt} &= \frac{1}{8\pi^2}[\sum_j 2|Y_d^{ij}|^2(m_{h_d}^2 + m_{\tilde{d}_i^c}^2 + m_{\tilde{q}_j}^2 + |A_d^{ij}|^2) + \frac{1}{3}g_Y^2\mathrm{Tr}\{Ym^2\} \\
&\quad - \frac{4}{9}g_Y^2M_Y^2 - \frac{16}{3}g_c^2M_c^2\,]\,, \qquad \text{(A.23)} \\
\frac{dm_{\tilde{u}_i^c}^2}{dt} &= \frac{1}{8\pi^2}[\sum_j 2|Y_u^{ij}|^2(m_{h_u}^2 + m_{\tilde{u}_i^c}^2 + m_{\tilde{q}_j^c}^2 + |A_u^{ij}|^2) - \frac{2}{3}g_Y^2\mathrm{Tr}\{Ym^2\} \\
&\quad - \frac{16}{9}g_Y^2M_Y^2 - \frac{16}{3}g_c^2M_c^2\,]\,, \qquad \text{(A.24)}
\end{aligned}
$$

$$\frac{dm^2_{\tilde{q}_i}}{dt} = \frac{1}{8\pi^2}[\sum_{i,j}\Big(|Y_u^{ji}|^2(m^2_{h_u}+m^2_{\tilde{q}_i}+m^2_{\tilde{u}^c_j}+|A_u^{ji}|^2)+|Y_d^{ji}|^2(m^2_{h_d}+m^2_{\tilde{q}_i}+m^2_{\tilde{d}^c_j}$$
$$+|A_d^{ji}|^2)\Big)+\frac{1}{6}g_Y^2\mathrm{Tr}\{Ym^2\}-\frac{1}{9}g_Y^2M_Y^2-3g_L^2M_L^2-\frac{16}{3}g_c^2M_c^2\ ]\ , \tag{A.25}$$

$$\mathrm{Tr}\{Ym^2\} = \sum_{i=1}^{n_g}(m^2_{\tilde{q}_i}-2m^2_{\tilde{u}^c_i}+m^2_{\tilde{d}^c_i}-m^2_{\tilde{l}_i}+m^2_{\tilde{e}^c_i})+m^2_{h_u}-m^2_{h_d}\ . \tag{A.26}$$

A.3 The $B-L$ Model

A.3.1 Gauge Couplings

The RGEs for the gauge couplings are

$$16\pi^2\frac{d\,g_i}{d(\ln\mu)} = b_i g_i^3\,, \tag{A.27}$$

where $b_i = (16, 11, 1, -3)$ for $U(1)_{BL}\times U(1)_Y\times SU(2)_L\times SU(3)_c$ respectively. The gaugino masses can be simply defined using the RGE invariant quantity

$$\frac{d}{d(\ln\mu)}\left(\frac{M_i}{g_i^2}\right) = 0\,. \tag{A.28}$$

A.3.2 Yukawaw Couplings

RGEs for the Yukawa couplings at the one-loop level are described as

$$16\pi^2\frac{d\mathbf{Y}_i}{d(\ln\mu)} = \mathbf{Y}_i\beta_i\,, \tag{A.29}$$

where the beta functions for each Yukawa are defined as

$$\beta_{Y_u} = 3\mathbf{Y}_u^\dagger \mathbf{Y}_u + Y_d^\dagger \mathbf{Y}_d + Tr\left(3\mathbf{Y}_u^\dagger \mathbf{Y}_u + \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu\right) - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{1}{9}\left(4g_{\mathrm{BL}}^2 + 13g_Y^2\right) \tag{A.30}$$

$$\beta_{Y_d} = \mathbf{Y}_u^\dagger \mathbf{Y}_u + 3\mathbf{Y}_d^\dagger \mathbf{Y}_d + Tr\left(3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e\right) - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{1}{9}\left(4{g_{\mathrm{BL}}}^2 + 7{g_Y}^2\right)] \tag{A.31}$$

$$\beta_{Y_e} = \mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + 3\mathbf{Y}_e^\dagger \mathbf{Y}_e + Tr\left(3\mathbf{Y}_d^\dagger \mathbf{Y}_d + \mathbf{Y}_e^\dagger \mathbf{Y}_e\right) - 3g_2^2 - \left(4{g_{\mathrm{BL}}}^2 + 3{g_Y}^2\right) \tag{A.32}$$

$$\beta_{Y_\nu} = 3\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu + \mathbf{Y}_e^\dagger \mathbf{Y}_e + Tr\left(3\mathbf{Y}_u^\dagger \mathbf{Y}_u + Y_\nu^\dagger \mathbf{Y}_\nu\right) - \frac{16}{3} g_3^2 - 3g_2^2 - \left(4{g_{\mathrm{BL}}}^2 + {g_Y}^2\right)] \tag{A.33}$$

A.3.3 Soft SUSY Masses

We describe the soft masses as ignoring trilinear terms from the neutrinos

$$\begin{aligned}
\frac{dm^2_{h_u}}{dt} &= \frac{1}{8\pi^2}[\sum_{i,j} 3|Y_u^{ji}|^2(m^2_{h_u} + m^2_{\tilde{q}_i} + m^2_{\tilde{u}^c_j} + |A_u^{ji}|^2) + |Y_\nu^{ji}|^2(m^2_{h_u} + m^2_{\tilde{l}_i} + m^2_{\tilde{N}^c_j}) \\
&\quad + \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} - g_Y^2M_Y^2 + \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} - g_Y^2M_Y^2 - 3g_L^2M_L^2\ ] \ , \qquad \text{(A.34)} \\
\frac{dm^2_{h_d}}{dt} &= \frac{1}{8\pi^2}[\sum_{i,j}\Big(|Y_e^{ji}|^2(m^2_{h_d} + m^2_{\tilde{l}_i} + m^2_{e_j} + |A_e^{ji}|^2) + 3|Y_d^{ji}|^2(m^2_{h_d} + m^2_{\tilde{q}_i} + m^2_{\tilde{d}^c_j} + |A_d^{ji}|^2)\Big) \\
&\quad - \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} - g_Y^2M_Y^2 - 3g_L^2M_L^2\ ] \ , \qquad \text{(A.35)} \\
\frac{dm^2_{\tilde{e}^c_i}}{dt} &= \frac{1}{8\pi^2}[\sum_{j} 2|Y_e^{ij}|^2(m^2_{h_d} + m^2_{\tilde{e}^c_i} + m^2_{\tilde{l}_j} + |A_e^{ij}|^2) \\
&\quad + g_Y^2\mathrm{Tr}\{Ym^2\} + g_{BL}^2\mathrm{Tr}\{Q_{BL}m^2\} - 4g_Y^2M_Y^2 - 4g_{BL}^2M_{BL}^2\ ] \ , \qquad \text{(A.36)} \\
\frac{dm^2_{\tilde{l}_i}}{dt} &= \frac{1}{8\pi^2}[\sum_{j}|Y_e^{ji}|^2(m^2_{h_d} + m^2_{\tilde{l}_i} + m^2_{\tilde{e}^c_j} + |A_e^{ji}|^2) + |Y_\nu^{ji}|^2(m^2_{h_u} + m^2_{\tilde{l}_i} + m^2_{\tilde{N}^c_j}) - \frac{1}{2}g_Y^2\mathrm{Tr}\{Ym^2\} \\
&\quad - g_{BL}^2\mathrm{Tr}\{Q_{BL}m^2\} - g_Y^2M_Y^2 - 3g_L^2M_L^2 - 2g_{BL}^2M_{BL}^2\ ] \ , \qquad \text{(A.37)} \\
\frac{dm^2_{\tilde{d}^c_i}}{dt} &= \frac{1}{8\pi^2}[\sum_{j} 2|Y_d^{ij}|^2(m^2_{h_d} + m^2_{\tilde{d}^c_i} + m^2_{\tilde{q}_j} + |A_d^{ij}|^2) + \frac{1}{3}g_Y^2\mathrm{Tr}\{Ym^2\} \\
&\quad - \frac{1}{3}g_{BL}^2\mathrm{Tr}\{Q_{BL}m^2\} - \frac{4}{9}g_Y^2M_Y^2 - \frac{16}{3}g_c^2M_c^2 - \frac{4}{9}g_{BL}^2M_{BL}^2\ ] \ , \qquad \text{(A.38)} \\
\frac{dm^2_{\tilde{u}^c_i}}{dt} &= \frac{1}{8\pi^2}[\sum_{j} 2|Y_u^{ij}|^2(m^2_{h_u} + m^2_{\tilde{u}^c_i} + m^2_{\tilde{q}^c_j} + |A_u^{ij}|^2) - \frac{2}{3}g_Y^2\mathrm{Tr}\{Ym^2\} - \frac{1}{3}g_{BL}^2\mathrm{Tr}\{Q_{BL}m^2\} \\
&\quad - \frac{16}{9}g_Y^2M_Y^2 - \frac{16}{3}g_c^2M_c^2 - \frac{4}{9}g_{BL}^2M_{BL}^2\ ] \ . \qquad \text{(A.39)}
\end{aligned}$$

We define the hypercharge trace the same as in (A.26) and the $B - L$ trace as

$$\mathrm{Tr}\{Q_{BL}m^2\} = \sum_{i=1}^{n_g}(2m^2_{\tilde{q}_i} - m^2_{\tilde{u}^c_i} - m^2_{\tilde{d}^c_i} - 2m^2_{\tilde{l}_i} + m^2_{\tilde{e}^c_i} + m^2_{\tilde{N}^c_i}). \qquad \text{(A.40)}$$

A.4 The Left-Right Model

A.4.1 Gauge Couplings

The RGEs for the gauge couplings are

$$16\pi^2 \frac{d\,g_i}{d(\ln\mu)} \quad = b_i g_i^3\,, \tag{A.41}$$

where $b_i = (16, 1, 1, -3)$ for $U(1)_{BL} \times SU(2)_L \times SU(2)_R \times SU(3)_c$ respectively. The gaugino masses can be simply defined using the RGE invariant quantity

$$\frac{d}{d(\ln\mu)}\left(\frac{M_i}{g_i^2}\right) = 0\,. \tag{A.42}$$

A.4.2 Yukawaw Couplings

RGEs for the Yukawa couplings at the one-loop level are described as

$$16\pi^2 \frac{d\mathbf{Y}_i}{d(\ln\mu)} = \mathbf{Y}_i\beta_i\,, \tag{A.43}$$

where the beta functions for each Yukawa are defined as

$$\begin{aligned} \beta_q &= 4\mathbf{Y}_q^\dagger\mathbf{Y}_q + \mathrm{Tr}[3\mathbf{Y}_{\mathrm{q}}^\dagger\mathbf{Y}_{\mathrm{q}} + \mathbf{Y}_{\mathrm{l}}^\dagger\mathbf{Y}_{\mathrm{l}} + (3\mathbf{Y}_{\mathrm{q}}^\dagger\mathbf{Y}'_{\mathrm{q}} + \mathbf{Y}_{\mathrm{l}}^\dagger\mathbf{Y}'_{\mathrm{l}} + \mathrm{h.c.})] \\ &- \left(\frac{4}{9}g_{BL}^2 + 3g_R^2 + 3g_L^2 + \frac{16}{3}g_3^2\right), \end{aligned} \tag{A.44}$$

$$\begin{aligned} \beta_{q'} &= 4\mathbf{Y'}_q^\dagger\mathbf{Y'}_q + \mathrm{Tr}[3\mathbf{Y'}_{\mathrm{q}}^\dagger\mathbf{Y'}_{\mathrm{q}} + \mathbf{Y'}_{\mathrm{l}}^\dagger\mathbf{Y'}_{\mathrm{l}} + (3\mathbf{Y}_{\mathrm{q}}^\dagger\mathbf{Y}'_{\mathrm{q}} + \mathbf{Y}_{\mathrm{l}}^\dagger\mathbf{Y}'_{\mathrm{l}} + \mathrm{h.c.})] \\ &- \left(\frac{4}{9}g_{BL}^2 + 3g_R^2 + 3g_L^2 + \frac{16}{3}g_3^2\right), \end{aligned} \tag{A.45}$$

$$
\begin{aligned}
\beta_l &= 4\mathbf{Y}_l^\dagger\mathbf{Y}_l + \mathrm{Tr}[3\mathbf{Y}_{\rm q}^\dagger\mathbf{Y}_{\rm q} + \mathbf{Y}_{\rm l}^\dagger\mathbf{Y}_{\rm l} + (3\mathbf{Y}_{\rm q}^\dagger\mathbf{Y}'_{\rm q} + \mathbf{Y}_{\rm l}^\dagger\mathbf{Y}'_{\rm l} + \text{h.c.})] \\
&- \left(4g_{BL}^2 + 3g_R^2 + 3g_L^2\right), \qquad \text{(A.46)}\\
\beta_{l'} &= 4\mathbf{Y'}_l^\dagger\mathbf{Y'}_l + \mathrm{Tr}[3\mathbf{Y'}_{\rm q}^\dagger\mathbf{Y'}_{\rm q} + \mathbf{Y'}_{\rm l}^\dagger\mathbf{Y'}_{\rm l} + (3\mathbf{Y}_{\rm q}^\dagger\mathbf{Y}'_{\rm q} + \mathbf{Y}_{\rm l}^\dagger\mathbf{Y}'_{\rm l} + \text{h.c.})] \\
&- \left(4g_{BL}^2 + 3g_R^2 + 3g_L^2\right). \qquad \text{(A.47)}
\end{aligned}
$$

A.4.3 Soft SUSY couplings

The soft mass RGEs are

$$
\begin{aligned}
8\pi^2\frac{dm^2_{\tilde{Q}_i}}{d(\ln\mu)} &= \sum_{j,k}|Y_Q^{ijk}|^2\Big(m^2_{\tilde{Q}_i} + m^2_{\tilde{Q}^c_j} + m^2_{\Phi_k}\Big) \\
&+ \frac{1}{3}g_{BL}^2\mathrm{Tr}[Q_{BL}m^2] - \frac{4}{9}g_{BL}^2M_{BL}^2 - 3g_L^2M_L^2 - \frac{16}{3}g_3^2M_3^2, \qquad \text{(A.48)}\\
8\pi^2\frac{dm^2_{\tilde{Q}^c_i}}{d(\ln\mu)} &= \sum_{j,k}|Y_Q^{ijk}|^2\Big(m^2_{\tilde{Q}_i} + m^2_{\tilde{Q}^c_j} + m^2_{\Phi_k}\Big) \\
&- \frac{1}{3}g_{BL}^2\mathrm{Tr}[Q_{BL}m^2] - \frac{4}{9}g_{BL}^2M_{BL}^2 - 3g_R^2M_R^2 - \frac{16}{3}g_3^2M_3^2, \qquad \text{(A.49)}\\
8\pi^2\frac{dm^2_{\tilde{L}_i}}{d(\ln\mu)} &= \sum_{j,k}|Y_L^{ijk}|^2\Big(m^2_{\tilde{L}_i} + m^2_{\tilde{L}^c_j} + m^2_{\Phi_k}\Big) \\
&- g_{BL}^2\mathrm{Tr}[Q_{BL}m^2] - 4g_{BL}^2M_{BL}^2 - 3g_L^2M_L^2, \qquad \text{(A.50)}\\
8\pi^2\frac{dm^2_{\tilde{L}_i{}^c}}{d(\ln\mu)} &= \sum_{j,k}|Y_L^{ijk}|^2\Big(m^2_{\tilde{L}_i} + m^2_{\tilde{L}^c_j} + m^2_{\Phi_k}\Big) \\
&+ g_{BL}^2\mathrm{Tr}[Q_{BL}m^2] - 4g_{BL}^2M_{BL}^2 - 3g_R^2M_R^2, \qquad \text{(A.51)}\\
8\pi^2\frac{dm^2_{\Phi_1}}{d(\ln\mu)} &= 3\sum_{i,j}|Y_Q^{ij}|^2\Big(m^2_{\tilde{Q}_i} + m^2_{\tilde{Q}^c_j} + m^2_{\Phi_1}\Big) + \sum_{i,j}|Y_L^{ij}|^2\Big(m^2_{\tilde{L}_i} + m^2_{\tilde{L}^c_j} + m^2_{\Phi_1}\Big) \\
&- 3g_L^2M_L^2 - 3g_R^2M_R^2, \qquad \text{(A.52)}\\
8\pi^2\frac{dm^2_{\Phi_2}}{d(\ln\mu)} &= 3\sum_{i,j}|Y'^{ij}_Q|^2\Big(m^2_{\tilde{Q}_i} + m^2_{\tilde{Q}^c_j} + m^2_{\Phi_2}\Big) + \sum_{i,j}|Y'^{ij}_L|^2\Big(m^2_{\tilde{L}_i} + m^2_{\tilde{L}^c_j} + m^2_{\Phi_2}\Big) \\
&- 3g_L^2M_L^2 - 3g_R^2M_R^2. \qquad \text{(A.53)}
\end{aligned}
$$

For equations (A.48)-(A.53), the trace terms are defined as

$$\mathrm{Tr}\left[Q_{BL} m^2\right] = 2 \sum_i \left(m^2_{\tilde{Q}_i} - m^2_{\tilde{Q}^c_i} - m^2_{\tilde{L}_i} + m^2_{\tilde{L}^c_i} \right) . \tag{A.54}$$

B APPENDIX B: SUSY Algebra

B.1 Grassman Algebra

The superspace coordinates θ and $\bar{\theta}$ are chiral coordinates. We can explicitly write the spinor index, θ^A and $\bar{\theta}_{\dot{A}}$ where $\theta^A\theta_A = \bar{\theta}_{\dot{A}}\bar{\theta}^{\dot{A}} = 0$. In the main body of this document we use covariant bilinear notation where $\theta\theta \neq 0$. Three or more factors of θ or $\bar{\theta}$ will be zero. We use the Pauli Matrices, σ^μ as well the following identities:

$$\epsilon^{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \tag{B.1}$$

$$\epsilon_{AB} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{B.2}$$

$$\theta^A\theta^B = -\frac{1}{2}\epsilon^{AB}\theta\theta, \tag{B.3}$$

$$\theta_A\theta_B = \frac{1}{2}\epsilon_{AB}\theta\theta, \tag{B.4}$$

$$\bar{\theta}_{\dot{A}}\bar{\theta}_{\dot{B}} = -\frac{1}{2}\epsilon_{\dot{A}\dot{B}}\bar{\theta}\bar{\theta}, \tag{B.5}$$

$$\bar{\theta}^{\dot{A}}\bar{\theta}^{\dot{B}} = \frac{1}{2}\epsilon^{\dot{A}\dot{B}}\bar{\theta}\bar{\theta}, \tag{B.6}$$

$$\theta\sigma^\mu\bar{\theta} = \theta^A\sigma^\mu_{A\dot{B}}\bar{\theta}^{\dot{B}}, \tag{B.7}$$

$$d^2\theta = -\frac{1}{4}d\theta^A d\theta_A, \tag{B.8}$$

$$d^2\bar{\theta} = -\frac{1}{4}d\bar{\theta}_{\dot{A}}d\bar{\theta}^{\dot{A}}, \tag{B.9}$$

$$d^4\theta = d^2\theta d^2\bar{\theta}, \tag{B.10}$$

$$\int d^2\theta = \int d^2\bar{\theta} = \int d^2\theta\theta^A = \int d^2\bar{\theta}\bar{\theta}_{\dot{A}} = 0, \tag{B.11}$$

$$\int d^2\theta\theta\theta = \int d^2\bar{\theta}\bar{\theta}\bar{\theta} = 1, \tag{B.12}$$

$$\int d^4\theta\theta\theta\bar{\theta}\bar{\theta} = 1. \tag{B.13}$$

The mass dimension of θ and $\bar{\theta}$ are -1/2 while $d\theta$ and $d\bar{\theta}$ are +1/2.