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Shear transport far from equilibrium via holography

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In heavy-ion collisions, the quark-gluon plasma is produced far from equilibrium. This regime is currently inaccessible by direct quantum chromodynamics (QCD) computations. In a holographic context, we propose a general method to characterize transport properties based on well-defined two-point functions. We calculate shear transport and entropy far from equilibrium, defining a time-dependent ratio of shear viscosity to entropy density, η/s. Large deviations from its near-equilibrium value $1/4\pi$, up to a factor of 2.5, are found for realistic situations at the Large Hadron Collider. We predict the far-from-equilibrium time-dependence of η/s to substantially affect the evolution of the QCD plasma and to impact the extraction of QCD properties from flow coefficients in heavy-ion collision data.

1. Introduction

Non-equilibrium systems are abundant in nature and remain elusive despite various theoretical approaches [1–3]. Quantum systems far from equilibrium pose a harder problem yet. Such systems are, for instance, the rapidly expanding early universe, the quark-gluon plasma (QGP) generated in heavy-ion collisions [4], or condensed matter experiments in which external parameters are rapidly changed (quench) [5].

A hallmark of non-equilibrium systems is entropy production. Entropy production is a fundamental measure for time evolution. It occurs in conjunction with dissipative processes, for example the transfer of momentum in the direction transverse to a fluid flow, called shear transport. In hydrodynamics, the ability of a fluid for shear transport is quantified by the shear viscosity, η. In plasmas, the specific shear viscosity (the ratio of shear viscosity to entropy density, η/s) is a key property indicating how far a plasma deviates from an ideal fluid.

In this work, the central question is to what extent strong time-dependence and far-from-equilibrium physics affect shear transport in the QGP. That is, how large are the corrections to η/s?

Perturbative methods have successfully provided values for the QGP shear viscosity near equilibrium [6,7]. However, the corrections at leading and next-to-leading order in the quantum chromodynamics (QCD) coupling constant are large [8]. Hence, the challenge is much harder because the QGP is strongly coupled during much of its time evolution [9], a regime inaccessible to all perturbative methods.

Access to this question was granted with the advent of the gauge/gravity duality (aka holography, or AdS/CFT) [10]. For a generic plasma at strong coupling, an astonishingly low value of $\eta/s = 1/4\pi \times \hbar/k_B$ [11,12] was predicted.[1,2] Remarkably, this value was later experimentally found to be consistent with heavy-ion collision data [15–17] in conjunction with hydrodynamic modeling [18–20]. Since then, the ratio η/s has profoundly impacted the interpretation of heavy-ion collision data over the past decade [4]. Predictions by hydrodynamic simulations were improved by introducing a small but non-zero value of $\eta/s \leq 2.5 \times 1/4\pi$ [19,20]. For estimating the value of η/s from experimental data, the elliptic flow of charged particles received much attention [18–25].

During the early stage, the QCD matter created in a heavy-ion collision is expected to be out of equilibrium, even locally [26].

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[1] In the following we apply the so-called natural units by setting $c \equiv \hbar \equiv k_B \equiv 1$. Note that we explicitly keep the Newton gravitational constant G_N.

[2] The ratio η/s has a lower bound in all known systems when taking the hydrodynamic limit [12]. The value of that bound depends on the system [13,14]. In our model, the near-equilibrium value is $\eta/s = 1/4\pi$.

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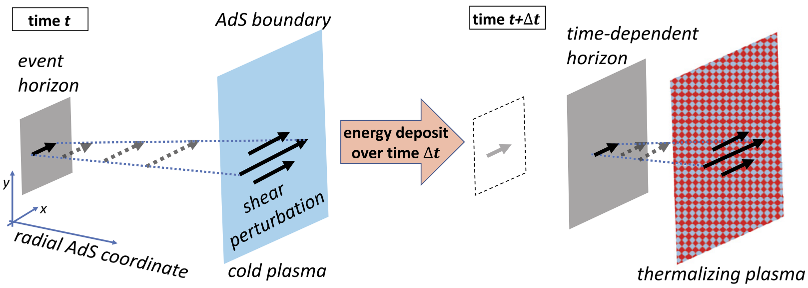

Fig. 1. Holographic setup: A rapid energy deposit over a time Δt turns a cold plasma into a thermalizing plasma far from equilibrium. On the left side, at time t, the gravity dual to the cold plasma is a nearly static black brane with a nearly constant event horizon, temperature, and entropy density. On the right side, at time $t+\Delta t$, the gravity dual to the thermalizing plasma far from equilibrium is a time-dependent black brane spacetime, from which we compute the time-dependent temperature and entropy density of the thermalizing plasma. We introduce a shear perturbation and compute the far-from-equilibrium analog of the specific shear viscosity, η/s.

This calls for more accurate values for the time evolution of η/s as input for hydrodynamic models. For QCD, a temperature-dependent η/s has been derived but, thus far, only in equilibrium approaches, most notably from the functional renormalization group (FRG) and lattice QCD [27,28], see also [29,30]. These results were then applied in hydrodynamic models, yielding predictions for various flow coefficients, v_n [31–34].

For model systems from kinetic theory and holography, studies in the far-from-equilibrium regime suggest that η/s receives contributions from higher-order viscous corrections. It shows explicit time dependence [26,35–41]. For details about the description of heavy-ion collisions by holographic means see e.g. [42–45].

In this work, we investigate the role of strong temporal gradients in the dynamics of η/s far from equilibrium. Our holographic model is based on a time-dependent charged black brane spacetime in Einstein-Maxwell theory coupled to time-dependent external sources [46–50]. This is illustrated in Fig. 1. The infall of matter causes the mass of the black brane to increase rapidly, corresponding to a change of temperature and entropy density in the dual field theory. To link this to hydrodynamically accessible quantities, we provide adequate out-of-equilibrium definitions.

A general method to investigate holographic systems far from equilibrium is introduced which can easily be adapted to other transport coefficients. This approach uses two-point functions in the time domain which are well-defined even in explicitly time-dependent regimes.

We show for the first time that strong temporal gradients alone induce large corrections to $\eta/s = 1/4\pi$ in holography. Varying our model parameters over a large range of values relevant to heavy-ion collisions, we demonstrate that these corrections to η/s are of order one in all cases.

2. Holographic model

We employ a superconformal Yang-Mills (in the following SYM) theory with $N_c \to \infty$ degrees of freedom in the limit of large 't Hooft coupling as a well established model for the QCD quark-gluon plasma in the strong coupling regime, see e.g. Ref. [11, 43–45]. Allowing for finite temperature and chemical potential, the AdS/CFT correspondence relates this field theory to Einstein-Maxwell (EM) theory in an asymptotically Anti-de Sitter (AdS) spacetime with the metric field g_{mn} and the $U(1)$ gauge field A_m [51]. The action reads

$$S = S_{\mathrm{EM}} + S_{\mathrm{matter}} \tag{1}$$

$$S_{\mathrm{EM}} = \int \mathrm{d}^4x \sqrt{-g}\left(\frac{1}{16\pi G_{\mathrm{N}}}(R+6) - \frac{1}{4}F^{mn}F_{mn}\right),$$

where R is the Ricci scalar associated with g_{mn}, which has the determinant g, and $F = \mathrm{d}A$ is the field strength tensor. Dimensionful quantities are scaled to the AdS radius. The gravitational system is 3+1 dimensional corresponding to a 2+1 dimensional field theory on the AdS boundary. The bulk gauge field A_m allows to charge the fluid and understand the effect of the chemical potential on the shear viscosity. It is, however, not essential for our setup to be driven out of equilibrium and can consistently be set to zero. Time dependence arises from the matter action S_{matter}.

A thermalizing field theory plasma is dual to a time-dependent metric background, i.e. a dynamic spacetime on the gravity side, see Fig. 1. Such a spacetime can be achieved by the infall of matter from the boundary toward the singularity. This matter gives rise to the gravitational stress-energy tensor as well as to the $U(1)$ current. These act as time-dependent sources in the Einstein-Maxwell equations. As the mass of the black brane increases, the apparent horizon radius and event horizon radius grow with time, see Fig. 1. This is realized in the charged time-dependent Vaidya black brane [46–50] with the solution

$$ds^2 = \frac{1}{z^2}\left(-f(v,z)\,\mathrm{d}v^2 - 2\,\mathrm{d}v\,\mathrm{d}z + \mathrm{d}x^2 + \mathrm{d}y^2\right), \tag{2}$$

$$A(v) = (\mu(v) - Q(v)\,z)\,\mathrm{d}v\,. \tag{3}$$

Here, the Eddington-Finkelstein coordinate v denotes the null time. At the AdS boundary, it equals the time in the field theory, $v = t$. We work with the inverse radial AdS coordinate, z, for which the AdS boundary is located at $z = 0$, and the singularity at $z \to \infty$. The time-dependent horizon position follows from the blackening factor, $f(v,z) = 1 - 2G_{\mathrm{N}}\,M(v)\,z^3 + 4\pi\,G_{\mathrm{N}}\,Q(v)^2\,z^4$. We fix the location of the apparent horizon to be at $z = 1$. This also fixes the event horizon location to $z = 1$.

The mass and charge densities of the brane, M and Q, are v-dependent. They can be chosen freely to match a given evolution of the temperature, T, and chemical potential, μ. For our calculations, we increase the mass by $\Delta M \propto 1 + \tanh(v/\Delta t)$ in a characteristic time scale Δt. The largest amount of mass (or charge) is falling into the black brane at a time $v = 0$. The background solution, Eq. (2), and thus the energy increase in the field theory, is homogeneous and isotropic along the boundary coordinates.

With the holography-based approach we investigate the dynamics of heavy-ion collisions at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC). In this letter, we particularly focus on the earliest stage, the heat-up phase, where the system is the farthest from equilibrium. One expects to reach a maximum temperature in the range of 300 MeV–600MeV within a time 0.15fm–0.6fm for central collisions at RHIC, while temperatures of 600MeV–900MeV within similar times at the LHC [52].

3. Entropy & shear far from equilibrium

In equilibrium, the entropy density of the black brane horizon corresponds to the entropy density of the dual field theory.

Fig. 1 illustrates a black brane and its dual field theory on the AdS boundary during an ongoing energy deposit according to the increasing mass function $M(v)$ in $f(v, z)$, Eq. (2). The black brane changes its surface area, i.e. the black brane entropy changes.

Out of equilibrium, however, this entropy is not equal to the field theory entropy. Instead, a well-suited measure for the entropy density of the boundary field theory is defined on the boundary itself and can be derived from the on-shell action. The latter follows from Eq. (1) by holographic renormalization [53]. We obtain the generating functional, the pressure $P = M/8\pi$, as a function of the temperature T and the chemical potential μ. Derived from holography, it is well-defined in a time-dependent scenario, see e.g. [54]. We define the time-dependent entropy density of the field theory as $s = (\partial P/\partial T)_\mu$. The generating functional yields $T(t)$ as well since holography assures direct access to $P(t)$ and $\mu(t)$.

Shear in a fluid near- or far-from-equilibrium is encoded in the spatial off-diagonal components of the stress-energy tensor, e.g. $\langle T^{xy} \rangle$. Near equilibrium, the retarded Green's function $G_R^{xy,xy}$ encodes shear transport and is related to the shear viscosity, η, via a Kubo formula,

$$\eta = \lim_{\omega \to 0} \lim_{\mathbf{k} \to \mathbf{0}} \frac{G_R^{xy,xy}(\omega, \mathbf{k})}{-\mathrm{i}\omega}\,, \tag{4}$$

with Fourier space frequency ω and momentum $\mathbf{k}$. Far from equilibrium, the shear correlator is well-defined in position space. However, time translation invariance is violated and hence plane waves, $e^{-ik\cdot x}$, are no solution basis. Thus, we work in position space, computing the 2-point function using linear response theory. We introduce a perturbation of the metric, $h_{xy}^{(0)}$, which sources the operator T^{xy}. For a localized source, $h_{xy}^{(0)}(\tau) \propto \delta(\tau - t_1)$, the system response yields the correlator directly,

$$\begin{aligned} \langle T^{xy}(t_2) \rangle_{\delta_{t_1}} &\propto -\int \mathrm{d}\tau\, G_R^{xy,xy}(\tau, t_2)\, \delta(\tau - t_1) \\ &= -G_R^{xy,xy}(t_1, t_2)\,, \end{aligned} \tag{5}$$

where we suppressed the spatial dependence. Invoking the holographic correspondence, we perturb the metric, Eq. (2), with a fluctuation h_{xy} [54–57]. This yields a differential boundary value problem for h_{xy} in v and z. It can be cast into an ordinary differential equation in z at an initial time. Two boundary conditions are imposed: a δ-source at $z = 0$, and ingoing solutions at $z = 1$. The resulting solution is propagated forward in time. Finally, the solution for h_{xy} at time t_2 yields the desired correlator after holographic renormalization [58,59].

This retarded Green's function in the time domain is a well-defined object even far from equilibrium. The counterpart in the frequency domain is constructed via a generalization of the Fourier transformation, namely a Wigner transformation, to allow for time dependence [60]: The frequency ω still corresponds to the relative time between source and response, $t_2 - t_1$. But, in contrast to Fourier, the Wigner-transformed correlator depends on the average time $t_{avg} = (t_1 + t_2)/2$. We consider all contributions above order 10^{-10} in the numerical implementation.

By applying Eq. (4) to any average time, we define our measure for shear transport far from equilibrium,

$$\eta(t_{avg}) = \lim_{\omega \to 0} \lim_{\mathbf{k} \to \mathbf{0}} \frac{G_R^{xy,xy}(\omega, \mathbf{k}, t_{avg})}{-\mathrm{i}\omega}\,. \tag{6}$$

When evaluated in an equilibrium state, Eq. (6) reduces to the known expression for η given in Eq. (4). This procedure can be adopted easily to other transport phenomena.

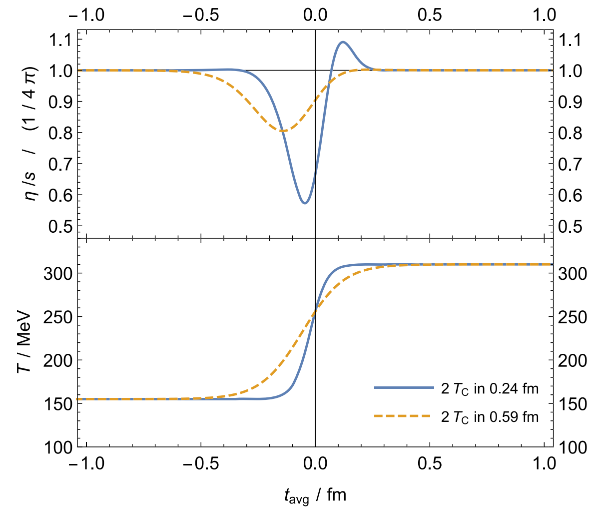

Fig. 2. Time evolution of η/s. Lower panel: Two sample temperature profiles $T(t_{avg})$ with heat-up to $2T_C$ within 0.24fm and 0.59fm. Upper panel: η/s reacts with a marked minimum. The thin black line represents the near-equilibrium value, $1/4\pi$.

4. Results

To apply our model to the early pre-equilibrium phase of heavy-ion collisions at RHIC and LHC, we start at the critical temperature of the QGP phase transition, $T_C = 155$MeV, and raise the temperature by a factor of 2, 4, 6.5, and 10. The duration required for the temperature to increase between 5% and 95% of the peak temperature is referred to as heat-up time. This corresponds to $3.5\,\Delta t$ in our notation, which we choose to be 0.24fm and 0.59fm, respectively. For now, we consider collisions at vanishing baryochemical potential, $\mu = 0$, adequate for collisions at RHIC and LHC.

The computation of the time-dependent shear viscosity and entropy density allows us to study the ratio $\eta(t_{avg})/s(t_{avg})$ in the far-from-equilibrium regime. Using the definition of the entropy density provided above, s shows a monotonic increase with time. However, the shear viscosity as a function of time, Eq. (6), shows an initial dip, then increases monotonically before it approaches the equilibrium value at late times. For some parameter combinations, the shear viscosity overshoots the equilibrium value and then asymptotes from above.

Fig. 2 presents explicit examples of the evolution of η/s, together with the associated temperature profiles with peak temperature $2T_C$ and two different heat-up times. For early and late times, the ratio reaches $1/4\pi$ in agreement with the near-equilibrium value of holographic plasmas dual to Einstein gravity. During the heat-up phase, however, we find a significant decrease with a prominent minimum, lying 20%–60% below the equilibrium value.

Both, time and temperature, determine how far the system is driven out of equilibrium: Decreasing the heat-up duration or increasing the peak temperature lead to a lower and more pronounced minimum.

In Fig. 3, we display $\eta/s(T)$, which combines the data for $T(t_{avg})$ with $\eta/s(t_{avg})$. The curves at heat-up times 0.24fm and 0.59fm enclose the region which is relevant to heavy-ion collisions. We find a universal behavior: Starting at the critical temperature, the curves bend down, reach their minima around $T = 1.3\,T_C$, and rise toward the equilibrium value.

With increasing peak temperatures, the curves for a given heat-up time become similar, especially near their minima. This implies that the curve at $T = 10\,T_C$ (lower dotted, red) provides a lower bound for η/s at each value of T. Furthermore, it shows that the influence of the peak temperature on η/s saturates. Thus, at large

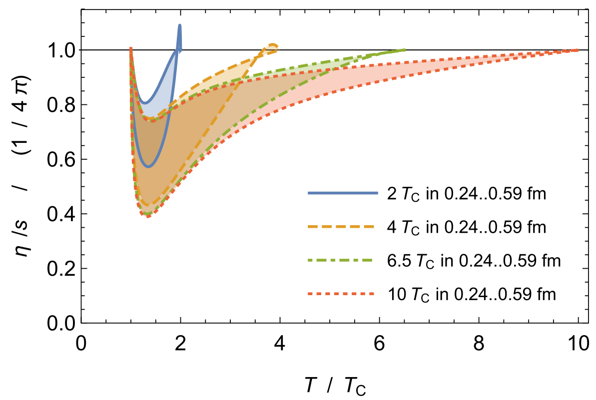

Fig. 3. Dependence of η/s on the instantaneous temperature, $T(t_{\mathrm{avg}})$. The shaded areas indicate the values arising from a sweep over a range of heat-up times for a fixed peak temperature.

peak temperatures $T \gtrsim 4\,T_{\mathrm{C}}$, the time span is the dominant out-of-equilibrium parameter which dictates the smallest possible value of η/s in a dynamical evolution.

We stress that all the effects discussed here also hold true at non-zero baryochemical potential μ. A negligible shift of the system response to later t_{avg} occurs in line with previous holography results [61,62]. These curves are not displayed.

5. Discussion

We have shown that η/s receives significant corrections up to a factor of 2.5 from strong temporal gradients in a holographic model far from equilibrium, cf. Fig. 2. For all peak temperatures and heat-up times in the reach of RHIC and LHC, these corrections are of order one, cf. Fig. 3. Holographic models successfully capture universal properties of high-temperature states in gauge theories [9,11,12,63]. Hence, we expect universal corrections to η/s of order one resulting from far-from-equilibrium physics in strongly coupled plasmas, in particular in the QCD quark-gluon plasma.

Considerations of an effective shear viscosity applied to holography suggest a drop of comparable size, e.g. [26,35]. Ref. [26] extracts hydrodynamic attractors from evolution equations and fits a first-order hydrodynamic ansatz to them. Ref. [35] resums higher-order viscosities which it identifies from the ringdown spectrum of an excited black hole in the dual theory. Our approach infers the specific shear viscosity directly from the concrete holographic system by using retarded Green's functions. The applicability of Green's functions far from equilibrium and the validity of the flucutation-dissipation theorem were checked holographically [60,64–66] and applied to the drag coefficient of heavy quarks [67]. We employ a Wigner transformation, which combines information from a time interval around t_{avg} in a consistent way. Sampling all relevant times t_{rel} ensures that the shear transport measure, Eq. (6), takes into account the time needed for information to propagate from the horizon to the boundary.

Kinetic theory models corroborate the expectation of universal corrections of order one from the weak coupling perspective. Far from equilibrium, higher order viscous corrections manifest themselves in a reduction of the specific shear viscosity [26,36–40]. If non-linearities are considered in such a renormalization, the shear viscosity even depends on the fluid's past evolution [41].

In general, the holographic value $1/4\pi$ is modified only in special situations. When implementing a strong anisotropy, for instance, one component of the shear viscosity tensor is reduced by a similar amount as in our study [68]. When correcting for a finite number of degrees of freedom and a finite 't Hooft coupling, η/s may decrease as well [13,14,69]. The impact on the specific shear viscosity is smaller than the far-from-equilibrium impact presented here.

When the system relaxes to equilibrium, our model yields the correct near-equilibrium value, i.e. $1/4\pi$. This asymptotics is in agreement with the late-time value of η/s in an analytic study on a thin-shell Vaidya setup [70]. It agrees as well with the late-time value of η/s in a numeric study on the onset of boost-invariant flow in an AdS setup [71]. In contrast to those works, we compute η/s also far from equilibrium, proposing an adequate definition.

Note that the time-independent holographic value of the specific shear viscosity is $\eta/s = 1/4\pi$ in 2+1 dimensions, just like in 3+1 dimensions. The shear tensor structure appearing in the hydrodynamic constitutive relation for the 2+1-dimensional energy-momentum tensor is identical to that in 3+1 dimensions [72,73]. Furthermore, in holographic models, the known logarithmic long-time tails caused by thermal fluctuations in 2+1 dimensional hydrodynamics are suppressed by the number of colors, $1/N_{\mathrm{c}}$ with $N_{\mathrm{c}} \to \infty$ [74–78]. Therefore, the third spatial direction does not influence our holographic result for η/s in the present system [51].

Up to now, we have not explained why it is reasonable to work far from equilibrium in hydrodynamics, which is normally used near equilibrium [79]. Hydrodynamics has become the standard tool to interpret experiments at RHIC and LHC [23,80,81]. Furthermore, it has been shown to work extremely well for theoretical models far from equilibrium [26,57,82–85]. The reason for this remains a fundamental question, although theoretical explanations based on resummation and resurgence have been proposed [26,36,41,85–87]. A separate observation is that Fig. 2 displays an η/s which reacts in a non-linear way to the temperature rise. This non-linear growth may be related to the chaotic evolution of black holes as presented in Refs. [88,89].

The proposed method to characterize transport properties far from equilibrium is applicable to a large range of coefficients. The specific shear viscosity is changed drastically. This fact directly impacts the generation of flow in the early collision phase. Therefore, it is compulsory to include the time dependence of η/s in order to extract the properties of QCD matter produced at RHIC and LHC with high accuracy. Such a sharpened analysis allows for a deeper understanding of the QGP and might open the possibility for new theoretical insight into time dependence far from equilibrium at the microscopic level.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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