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Effective theory of brane world with small tension

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The five dimensional theory compactified on S^1 with two ''branes'' (two domain walls) embedded in it is constructed, based on the field-theoretic mechanism to generate the ''brane.'' Some light states localized in the ''brane'' appear in the theory. One is the Nambu-Goldstone boson, which corresponds to the breaking of the translational invariance in the transverse direction of the ''brane.'' In addition, if the tension of the ''brane'' is smaller than the fundamental scale of the original theory, it is found that there may exist not only massless states but also some massive states lighter than the fundamental scale in the ''brane.'' We analyze the four dimensional effective theory by integrating out the freedom of the fifth dimension. We show that some effective couplings can be explicitly calculated. As one of our results, some effective couplings of the state localized in the ''brane'' to the higher Kaluza-Klein modes in the bulk are found to be suppressed by the width of the ''brane.'' The resultant suppression factor can be quantitatively different from the one analyzed by Bando *et al.* using the Nambu-Goto action, while they are qualitatively the same.

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I. INTRODUCTION

Recently, extra-dimensional theories have been intensively investigated. One of the crucial differences of recent theories with extra dimensions from old ones such as Kaluza-Klein theories [1], is the existence of the ''brane'' configuration which is embedded in the whole space-time dimensions. Then, we consider the case in which some fields (for example, fields in the standard model) reside on the ''brane'' and others (the graviton, for example) reside in the bulk. With this selection, which field resides where, a new possibility to solve the hierarchy problem without supersymmetry or technicolor was first pointed out in Ref. [2]. Other aspects of these theories have been discussed by many authors: phenomenology of the Kaluza-Klein (KK) modes [3], TeV scale unification [4], new interpretation of the fermion mass hierarchy [5], neutrino physics [6], cosmology and astrophysics [7].

Since the existence of the ''brane'' plays the most crucial role in the recent extra-dimensional theories, it is important to investigate how the ''brane'' is generated. Some mechanisms to generate the ''brane'' configuration are known in the field-theoretic point of view (domain wall) [8] or in the string theory (D-brane) [9]. In the following, let us discuss somewhat generically expected possibilities whatever the mechanism is.

Note that the ''brane'' configuration embedded in the extra dimensions breaks the translational invariance in the transverse direction of the ''brane.'' If the ''brane'' is spontaneously generated, there exits a Nambu-Goldstone (NG) mode in the ''brane.'' Thus, it is expected that the NG mode appears in the four dimensional effective theory.

In general, the ''brane,'' if it is the wall with zero width like the D-brane, is a fluctuating object from a quantum mechanical point of view. The width of the fluctuation is naturally given by M_F/Λ^2, where Λ and M_F are the ''brane'' tension and the fundamental scale of the original theory, respectively. If the ''brane'' tension is smaller than the fundamental scale, this effect should be considered in the effective theory. In fact, Bando *et al.* argued in Ref. [10] that the recoil effect of the fluctuating brane by the NG boson suppresses some effective couplings among matter on the brane and the higher KK modes in the bulk, and the suppression factor is found to be $\exp(-M_F^2\ m_{KK}^2\ /\Lambda^4)$, where m_{KK} is the mass of the KK mode.

Also, there is a possibility that not only massless states but also some massive states are localized in the brane. For example, consider the field-theoretic mechanism to generate the domain wall [8]. Some fields are localized by the potential wall in the direction of the extra dimensions. If the potential is deep enough and its width is larger than the inverse of the fundamental scale, there may exist a massive spectrum within a range smaller than the fundamental scale. In this case, the localized massive states should be considered in the four dimensional effective theory.

In this paper, the five dimensional theory compactified on S^1 with two domain walls embedded in it is constructed based on a concrete field-theoretic mechanism to generate the domain wall [8]. Because of the existence of the domain wall, the breakdown of the translational invariance in the direction of the fifth dimension leads to the presence of the NG boson in the wall, as far as the gravitational effect is negligible. In addition, if the width of the ''brane'' r_W is larger than $1/M_F$, we can find that some massive modes whose masses are smaller than the fundamental scale may be also localized in the wall. We analyze the four dimensional effective theory by integrating out the freedom of the fifth dimension. We show that some effective couplings can be explicitly calculated. As one of the results, some effective couplings of the state localized in the ''brane'' to the higher Kaluza-Klein modes in the bulk are found to be suppressed by the width of the ''brane,'' and the suppression factor is $\exp(-c\ r_W^2\ m_{KK}^2)$ with c the combination of the coupling constants. In the result of Ref. [10], r_W corresponds to the

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width of the brane fluctuation, $\sim M_F/\Lambda^2$. However, they are quantitatively different.

In Sec. II, we construct the five dimensional theory compactified on S^1 with two domain walls based on the field-theoretic mechanism to generate the domain wall. The massless localized state appears as the fluctuation mode from the background domain wall configuration. This is just the NG boson with respect to the breaking of the translational invariance in the direction of the fifth dimension. In Sec. III, we consider the fermion in the bulk which couples to the domain wall background. In addition to a localized chiral fermion known as the ''domain wall fermion,'' the existence of the localized massive states is discussed in the case that the wall width is larger than the inverse of the fundamental scale. We analyze the four dimensional effective theory in Sec. IV. The effective couplings among some fields in the wall or among fields in the walls and the bulk are explicitly calculated. Section V is devoted to summary and comments.

II. SETUP

First of all, we discuss the mechanism to generate the domain wall in five dimensional space-time [8]. Let us consider a real scalar field in five dimensions,

$$\mathcal{L}_{(5)}=\frac{1}{2}\partial_M\phi\partial^M\phi-V(\phi), \tag{1}$$

where

$$V(\phi)=\frac{m_\phi^4}{2\lambda}-m_\phi^2\phi^2+\frac{\lambda}{2}\phi^4. \tag{2}$$

Here $\phi=\phi(x,y)$, and $x=x^\mu$ $(\mu=0,1,2,3)$, and y is the coordinates of the four dimensional space-time and the fifth dimension, respectively. Note that ϕ and λ have mass dimensions 3/2 and -1, respectively. It is well known that there is a nontrivial background configuration $\phi_0(y)$ as a solution of the equation of motion, called the kink solution, in the direction of the fifth dimension:

$$\phi_0(y)=\frac{m_\phi}{\sqrt{\lambda}}\tanh[m_\phi y]. \tag{3}$$

Here, we fixed the kink center at $y=0$. The domain wall is generated by the vacuum expectation value of ϕ, and its energy density is given by

$$\rho(y)=\frac{1}{2}\left(\frac{d\phi_0}{dy}\right)^2+V(\phi_0)=\frac{m_\phi^4}{\lambda}\cosh^{-4}[m_\phi y]. \tag{4}$$

We define the ''brane'' tension of the wall as

$$\Lambda^4=\int_{-\infty}^{\infty}dy\ \ \rho(y)=\frac{4}{3}\frac{m_\phi^3}{\lambda}. \tag{5}$$

From Eq. (3) the width of the ''brane'' r_W is given by $1/m_\phi$. In the following discussion, we consider the case that the tension of the ''brane'' is smaller than the fundamental scale and/or the width of the wall are larger than the inverse of the fundamental scale, $m_\phi<M_F$.

In the WKB approximation, the spectrum of perturbation in the presence of the background kink can be found by solving the linearized equation of motion for the field $\phi(x,y)=\phi_0(y)+U_\eta(y)\eta(x)$. For the massless mode $\partial_{(4)}^2\eta(x)=0$, the solution is given by

$$U_\eta(y)=\frac{\sqrt{3m_\phi}}{2}\cosh^{-2}[m_\phi y]. \tag{6}$$

The kinetic term of $\eta(x)$ is canonically normalized by $U_\eta(y)$. This massless mode corresponds to the freedom of the shift of the kink solution in the direction of the fifth dimension, $\phi_0(y)\to\phi_0[y+\eta(x)]$. Once we fix the kink center at a point, the translational invariance in the direction of the fifth dimension is spontaneously broken. Therefore, the massless mode is the NG mode with respect to this break down, and the decay constant is given by $d\phi_0/dy\propto U_\eta(y)$. Also, there is one massive mode $\tilde{\eta}(x)$ localized around the kink center with mass squared $m^2=3m_\phi^2$ [11]. For this state, the solution of the linearized equation is given by

$$U_{\tilde{\eta}}(y)=\sqrt{\frac{3}{2}m_\phi}\sinh[m_\phi y]\cosh^{-2}[m_\phi y]. \tag{7}$$

The kinetic term of $\tilde{\eta}(x)$ is canonically normalized by $U_{\tilde{\eta}}(y)$.

Up to now, the compactification of the fifth dimension is not discussed. In the following discussion, we consider the five dimensional theory compactified on the S^1 with radius R, in which the domain wall is embedded. In this case, the kink solution discussed above is not a solution of the compactified theory, because it cannot satisfy the periodic boundary condition $\phi_0(y)=\phi_0(y+2\pi R)$. In order to get such a solution, we introduce the antikink solution centered at $y=\pi R$ and connect it to the kink solution with the dilute gas approximation. The approximate solution is given by

$$\phi_0(y)=\frac{m_\phi}{\sqrt{\lambda}}\tanh[m_\phi y]\tanh[m_\phi(\pi R-y)], \tag{8}$$

with the periodic boundary condition $\phi_0(y)=\phi_0(y+2\pi R)$. In order for this approximation to be justified, $r_W(\equiv 1/m_\phi)\ll\pi R$ is assumed. Since the distance between the kink center and the antikink center is very large compared with the width of each domain wall, the overlapping of two domain walls can be neglected. In the paper, we assume that the domain wall centered at $y=0$ is the ''brane'' in which we are living, and concentrate on only the domain wall around the kink center. In our case, the solution of Eq. (3) can be regarded as a good approximation. We do not have to pay attention to the existence of the other domain wall centered at $y=\pi R$ in the following. Now, the compactified five dimensional theory in which two domain walls are embedded has been constructed.[1]

III. DOMAIN WALL FERMIONS

In this section, consider the bulk fermion which has a Yukawa coupling to the background kink solution. It is well known that there is a localized chiral fermion in the domain wall [8]. If the width of the ''brane'' is larger than the inverse of the fundamental scale, we can find that not only the chiral fermion but also massive states lighter than the fundamental scale may be localized in the wall.

Let us consider the bulk fermion Lagrangian of the form

$$\mathcal{L}_{(5)}=\overline{\Psi}(x,y)[i\Gamma^M\partial_M+g_Y\phi_0(y)]\Psi(x,y), \tag{9}$$

where Ψ is a four component spinor in five dimensions, and the five-dimensional γ matrices are $\Gamma^\mu=\gamma^\mu$ for $\mu=0,\ldots,3$ and $\Gamma^4=i\gamma_5$, respectively. Here, we assume that g_Y is real and positive. Suppose that the fermion Ψ is described by Weyl fermions in four dimensional space-time such that

$$\Psi(x,y)=U_L(y)\psi_L(x)+U_R(y)\psi_R(x). \tag{10}$$

The above Lagrangian can be rewritten as

$$\begin{aligned}\mathcal{L}_{(5)}=&|U_L|^2\overline{\psi_L}i\gamma^\mu\partial_\mu\psi_L+|U_R|^2\overline{\psi_R i}\gamma^\mu\partial_\mu\psi_R+\overline{\psi_L}U_L^\dagger(-U_R'\\&+U_R\ \ g_Y\phi_0)\psi_R+\overline{\psi_R}U_R^\dagger(+U_L'+U_L\ \ g_Y\phi_0)\psi_L,\end{aligned} \tag{11}$$

where a prime denotes the derivative with respect to y. Considering the Dirac equation in four dimensions, $i\gamma^\mu\partial_\mu\psi_{L,R}=m\psi_{R,L}$, the equations of motion for $U_{L,R}$ are given by

$$U_L''+(m^2+g_Y\phi_0'-g_Y^2\phi_0^2)U_L=0, \tag{12}$$

$$U_R''+(m^2-g_Y\phi_0'-g_Y^2\phi_0^2)U_R=0. \tag{13}$$

It is useful to rewrite these equations into the form

$$U_L''+\left(E+\frac{V_L}{\cosh^2[m_\phi y]}\right)U_L=0, \tag{14}$$

$$U_R''+\left(E+\frac{V_R}{\cosh^2[m_\phi y]}\right)U_R=0, \tag{15}$$

where

$$E=m^2-\frac{g_Y^2}{\lambda}m_\phi^2, \tag{16}$$

$$V_L=\frac{g_Y}{\sqrt{\lambda}}m_\phi^2\left(\frac{g_Y}{\sqrt{\lambda}}+1\right), \tag{17}$$

$$V_R=\frac{g_Y}{\sqrt{\lambda}}m_\phi^2\left(\frac{g_Y}{\sqrt{\lambda}}-1\right). \tag{18}$$

Note that these equations can be regarded as (1+1)-dimensional Schrödinger equations with the potential $-V_{L,R}\cosh^{-2}[m_\phi y]$.

We are interested in the bound state which is the solution satisfying two boundary conditions; $U_{L,R}(0)$ is finite and $U_{L,R}(y)\to 0$ for $|y|\to\infty$. Such a solution is given by using the hypergeometric function $F[a,b;c;z]$. For $U_L(y)$, we find the following mass eigenvalues:

$$m^2=m_\phi^2 n_L\left(2\frac{g_Y}{\sqrt{\lambda}}-n_L\right),\quad n_L=0,1,2,\ldots<g_Y/\sqrt{\lambda}. \tag{19}$$

The eigenstates for even numbers of $n_L=2n_L'$ ($n_L'=0,1,2,\ldots$) and odd numbers of $n_L=2n_L''+1$ ($n_L''=0,1,2,\ldots$) are given by (up to normalization factor)

$$\begin{aligned}U_L^{n_L'}(y)=&(\cosh[m_\phi y])^{-\frac{g_Y}{\sqrt{\lambda}}}\\&\times F\left[-n_L',-\frac{g_Y}{\sqrt{\lambda}}+n_L';1/2;1-\cosh^2[m_\phi y]\right]\end{aligned} \tag{20}$$

and

$$\begin{aligned}U_L^{n_L''}(y)=&\sinh[m_\phi y](\cosh[m_\phi y])^{-g_Y/\sqrt{\lambda}}F\Bigg[-n_L'',-\frac{g_Y}{\sqrt{\lambda}}+n_L''\\&+1;3/2;1-\cosh^2[m_\phi y]\Bigg],\end{aligned} \tag{21}$$

respectively. Note that, if $g_Y/\sqrt{\lambda}>1$, there exists at least one massive bound state localized around the kink center. The mass eigenvalues are smaller than the fundamental scale with our assumption $m_\phi\ll M_F$, and, therefore, such states should be considered in the effective theory.

The solution for $U_R(y)$ can be found by the same manner. The mass eigenvalue for $U_R(y)$ is given by replacing n_L to n_R+1 in the results for U_L:

$$\begin{aligned}m^2=&m_\phi^2(n_R+1)\left(2\frac{g_Y}{\sqrt{\lambda}}-(n_R+1)\right),\\&n_R=0,1,2,\ldots<g_Y/\sqrt{\lambda}-1.\end{aligned} \tag{22}$$

[1]This configuration where a pair of kink and antikink exists on the compact space is not stable for the small fluctuation. However, it is a difficult problem to construct a stable configuration with some kink solutions on a compact space, and it is out of our scope. Here we assume the existence of the mechanism of the stabilization.

The eigenstates for even numbers of $n_R=2n'_R$ ($n'_R=0,1,2,\ldots$) and odd numbers of $n_R=2n''_R+1$ ($n''_R=0,1,2,\ldots$) are given by (up to normalization factor)

$$U_R^{n'_R}(y)=(\cosh[m_\phi y])^{-g_Y/\sqrt{\lambda}+1}F\left[-n'_R,-\frac{g_Y}{\sqrt{\lambda}}+n'_R+1;1/2;1-\cosh^2[m_\phi y]\right] \quad (23)$$

and

$$U_R^{n''_R}(y)=\sinh[m_\phi y](\cosh[m_\phi y])^{-g_Y/\sqrt{\lambda}+1}F\left[-n''_R,-\frac{g_Y}{\sqrt{\lambda}}+n''_R+2;3/2;1-\cosh^2[m_\phi y]\right], \quad (24)$$

respectively. Dirac fermions with masses $m_n^2=m_\phi^2 n\,(2g_Y/\sqrt{\lambda}-n)$ ($n=1,2,\ldots$) are composed by the nth left-handed and the $(n-1)$th right-handed fermions, $\mathcal{L}_{mass}=-m_n\overline{\psi_L^n}\psi_R^{n-1}$.

Note that the case $n_L=0$ corresponds to a massless state, and the left-handed chiral fermion is localized around the kink center,

$$\Psi^{n=0}(x,y)=U_L^{n_L=0}(y)\,\psi_L^0(x), \quad (25)$$

where

$$U_L^{n_L=0}(y)=\left(\frac{m_\phi\Gamma(g_Y/\sqrt{\lambda}+\frac{1}{2})}{\sqrt{\pi}\Gamma(g_Y/\sqrt{\lambda})}\right)^{1/2}(\cosh[m_\phi y])^{-g_Y/\sqrt{\lambda}}. \quad (26)$$

On the other hand, there is no solution for the right-handed chiral fermion which satisfies the boundary conditions. In fact, the right-handed chiral fermion is expected to be localized around the antikink center, $y=\pi R$. The massive bound states are also localized there. These eigenstates can be given from the above result by exchanging the chirality $L\leftrightarrow R$.

Since the fifth dimension is compactified on S^1, there are KK modes of the fermion which are not localized. Unfortunately, we cannot exactly solve Eqs. (14) and (15) without the second boundary condition. However, if $E\gg V_{L,R}$, the potential terms can be neglected, and these equations is reduced to the usual KK mode equations. In this case, the bulk fermion can be expanded by the series of the KK modes:

$$\Psi_{L,R}(x,y)\sim\frac{1}{\sqrt{2\pi R}}\sum_n \psi_{L,R}^n(x)\; e^{i(n/R)y}. \quad (27)$$

The masses of the KK modes are given by $\tilde{m}_n^2\simeq(n/R)^2+g_Y^2m_\phi^2/\lambda$ with integer n.

IV. EFFECTIVE THEORY IN FOUR DIMENSIONS

In the previous sections, all states which we are interested in were given. Now, let us analyze the four dimensional effective theory by integrating out the freedom of the fifth dimension. We first consider the effective coupling among the domain wall fermions and the KK modes in the bulk. Suppose that the domain wall fermion has the gauge charge and the gauge boson resides in the bulk. The gauge boson is described by the series of expansion of the KK modes as

$$A_\mu(x,y)=\frac{1}{\sqrt{2\pi R}}\sum_n A_\mu^n(x)\; e^{i(n/R)y}, \quad (28)$$

with the mass squared $m_{KK}^2=(n/R)^2$.

The interaction among two domain wall chiral fermions and the bulk gauge boson is of the form

$$\mathcal{L}_{int}^{(5)}=g\;\overline{\Psi_L^0(x,y)}\,\gamma^M A_M(x,y)\Psi_L^0(x,y), \quad (29)$$

where g is the gauge coupling constant with mass dimension $-1/2$. The effective gauge coupling in the four dimensional theory, defined as

$$\mathcal{L}_{int}^{(4)}=\sum_n g_n^{eff}\;\overline{\psi_L^0}(x)\,\gamma^\mu A_\mu(x,y)\,\psi_L^0(x), \quad (30)$$

is given by

$$\begin{aligned} g_n^{eff}&=\frac{g}{\sqrt{2\pi R}}\int_{-\pi R}^{\pi R}dy\;|U_L^0(y)|^2\;e^{i(n/R)y}\\ &\sim\frac{g}{\sqrt{2\pi R}}\left(\frac{m_\phi\Gamma(g_Y/\sqrt{\lambda}+\frac{1}{2})}{\sqrt{\pi}\Gamma(g_Y/\sqrt{\lambda})}\right)\\ &\quad\times\int_{-\infty}^{\infty}dy\;(\cosh[m_\phi y])^{-2g_Y/\sqrt{\lambda}}\;e^{i(n/R)y}. \end{aligned} \quad (31)$$

Here we assumed $(g_Y/\sqrt{\lambda})m_\phi\gg1/(\pi R)$, because of the asymptotic behavior $(\cosh[m_\phi y])^{-2g_Y/\sqrt{\lambda}}\sim\exp[-2(g_Y/\sqrt{\lambda})m_\phi|y|]$ for large $|y|\gg1$. It is useful to define the dimensionless coupling constant $\bar{g}\equiv g/\sqrt{2\pi R}$, since we get $g_n^{eff}=g/\sqrt{2\pi R}$ for $n=0$.

After integration with respect to the fifth dimension y, we get the effective coupling in the four dimensional theory, which is very well fitted by the function

$$g_n^{eff}\sim\bar{g}\exp\left[-\frac{1}{4}\frac{\sqrt{\lambda}}{g_Y}\left(\frac{n}{m_\phi R}\right)^2\right]. \quad (32)$$

Note that the effective coupling is suppressed for the higher KK modes with large n. This means the restoration of the translational invariance in the direction of the fifth dimension for such KK modes.

The KK modes with mass n/R much smaller than the inverse of the domain width $1/r_W(\sim m_\phi)$ feel the wall thin and hard, and the momentum conservation in the direction of the fifth dimension is hardly broken. As a result, the couplings become almost universal. On the other hand, modes with mass much larger than $1/r_W$ feel the wall thick and soft, and are not aware of the localization of the matter in the domain wall. Therefore, the breaking of the momentum conservation becomes restored as n becomes large.

Bando *et al.* [10] discussed the effective gauge coupling between the KK mode of the gauge boson in bulk and the matter localized in the ''brane'' by using the induced metric based on the Nambu-Goto action [12]. They showed the effective gauge coupling takes of the form as

$$g_n^{eff}=\bar{g}\exp\left[-\frac{1}{2}\left(\frac{n}{R}\right)^2\frac{M_F^2}{\Lambda^4}\right]. \tag{33}$$

This suppression factor comes from the quadratically divergent radiative correction through the NG boson loops to this vertex. Our result is qualitatively consistent with this result, since the quantum fluctuation width of the the brane $\sim M_F/\Lambda^2$ in the Nambu-Goto action corresponds to the width of the wall. However, our result is quantitatively different from theirs. For example, when $g_Y^2\sim\lambda\sim 1/M_F$, the effective coupling becomes

$$g_n^{eff}=\bar{g}\exp\left[-\frac{4}{9}\left(\frac{n}{R}\right)^2\frac{M_F^{2/3}}{\Lambda^{8/3}}\right]. \tag{34}$$

Next, let us consider the interactions among the localized states in the wall. Since these interactions are not suppressed by a volume factor r_W/R and are related to the mechanism to generate the ''brane,'' they are interesting from the phenomenological point of view. As one example, let us discuss the Yukawa interaction of the chiral fermion to the massive states and the NG boson in the Lagrangian of Eq. (9):

$$\begin{aligned}\mathcal{L}_{int}^{(5)}&=g_Y(U_\eta(y)\eta(x))\overline{\Psi_R^{n_R}(x,y)}\Psi_L^0(x,y)\\&=g_Y(U_\eta(y)U_L^{0\dagger}(y)U_R^{n_R}(y))\eta(x)\overline{\psi_R^{n_R}(x)}\psi_L^0(x).\end{aligned} \tag{35}$$

Then the effective Yukawa coupling in four dimensions is defined as

$$g_Y^{eff}(n_R)=g_Y\int_{-\infty}^{\infty}dy\ \ U_\eta(y)U_L^{0\dagger}(y)U_R^{n_R}(y). \tag{36}$$

Note that $U_\eta(y)$ and $U_L^0(y)$ are even functions of y; the effective coupling remains nonzero only in the case that n_R takes even numbers. As already discussed, for $n_R=2n_R'$ $(n_R'=0,1,2,\dots)$, $U_R^{n_R'}(y)$ is given by

$$U_R^{n_R'}=N_R^{n_R'}(\cosh[m_\phi y])^{-g_Y/\sqrt{\lambda}+1}F\left[-n_R',-\frac{g_Y}{\sqrt{\lambda}}+n_R'+1;1/2;1-\cosh^2[m_\phi y]\right], \tag{37}$$

where $N_R^{n_R'}$ is the normalization factor by which the kinetic term of $\psi_R^{n_R'}$ is canonically normalized. The normalization factor is given by (up to phase)

$$N_R^{n_R'}=m_\phi^{1/2}\left(\int_{-\infty}^{\infty}dy\ \ (\cosh y)^{-2(g_Y/\sqrt{\lambda}-1)}F\left[-n_R',-\frac{g_Y}{\sqrt{\lambda}}+n_R'+1;1/2;1-\cosh^2 y\right]\right)^{-1/2}. \tag{38}$$

The explicit description of the effective Yukawa coupling is of the form

$$g_Y^{eff}(n_R')=\sqrt{\frac{3}{4}}\ g_Y\ m_\phi^{1/2}\ W\left(n_R',r\equiv\frac{g_Y}{\sqrt{\lambda}}\right), \tag{39}$$

where

$$\begin{aligned}W(n_R',r)=&\left(\frac{\Gamma(r+1/2)}{\sqrt{\pi}\Gamma(r)}\right)^{1/2}\int_{-\infty}^{\infty}dy\ \ (\cosh y)^{-2r-1}F[-n_R',\\&-r+n_R'+1;1/2;1-\cosh^2 y]\\&\times\left(\int_{-\infty}^{\infty}dy\ \ (\cosh y)^{-2(r-1)}F[-n_R',-r+n_R'\right.\\&\left.+1;1/2;1-\cosh^2 y]\right)^{-1/2}.\end{aligned} \tag{40}$$

By numerical calculations, we can find that the effective coupling for the lowest state $n_R'=0$ is the largest. For example, $W(n_R'=0,r)\sim 1$, $W(n_R'=1,r)\sim 0.1$, and $W(n_R'\geqslant 2,r)\ll 0.1$ for $r\gg 1$. In this case, $g_Y^{eff}(n_R'=0)\sim 0.3$ for $m_\phi/M_F=0.1$ and $g_Y=1/\sqrt{M_F}$.

We do not address the explicit results of other effective couplings. One can easily calculate them by using the formulas given in the paper. To do this, rewrite the original Lagrangian by using the fields in four dimensions and integrate out with respect to the fifth dimension. The result depends on the shape of the wave function in the direction of the fifth dimension. For example, the couplings of the higher massive bound states to the higher KK modes are more suppressed than that of the chiral fermion, since the width of the wall becomes larger for the higher bound states.

V. SUMMARY AND COMMENTS

In summary, we have constructed the five dimensional theory, in which the domain wall is embedded, based on the concrete field-theoretic mechanism to generate the domain wall. Because of the existence of the domain wall and the breakdown of the translational invariance in the direction of the fifth dimension, the NG boson appears in the wall. If the tension of the wall is smaller than the fundamental scale, there exist not only the massless states but also the massive bound states in the wall. They have masses smaller than the fundamental scale, and appear to be considered in the effective theory.

We have analyzed the four dimensional effective theory by integrating out the freedom of the fifth dimension. Some effective couplings have been explicitly calculated. We find that the effective couplings among two localized states and the higher KK modes, whose masses n/R are larger than the inverse of the wall width $1/r_W$, are exponentially suppressed as $\exp[-c(nr_W/R)^2]$, so that the breaking of translational invariance becomes restored for such modes. The effective Yukawa couplings among one massless state, one massive bound state, and the NG boson are also calculated, and it is found that the effective Yukawa coupling including the lightest massive state is the largest one. One can easily calculate other effective couplings by using the formulas given in this paper.

In the phenomenological point of view, the suppression of the effective coupling including the KK modes is unfortunate, since this suppression makes the effect of the Kaluza-Klein modes in experiments loose as discussed in Ref. [10]. However, there is the Yukawa coupling including the NG boson in our effective theory. There is a process, for example, $\overline{\psi_L^0}\ \psi_L^0 \to \eta\eta$, intermediated by the bound state fermions. Such a process may become important in a search for the existence of the extra dimension and the domain wall or in cosmology and astrophysics.

Our discussion in the five dimensional theory can be extended to more higher dimensional theories. For example, in the six dimensional theory, we can use the vortex solution [13] to generate the domain wall. The same features of the effective theory discussed above can be expected, if the tension of the wall is smaller than the fundamental scale.

Finally, we would like to give some comments. Our theory discussed above has not been gauged, in other words, gravity has not been discussed. If we include the gravitational effect in our discussion, the NG boson is expected to be absorbed by gauge fields, some components of graviton in five dimensions, and to disappear in the effective theory. This fact was briefly discussed in Ref. [14] considering the fluctuation from the flat background metric, and they claimed that $g_{\mu 5}$ components absorb the NG boson and obtain masses such that $m_g^2 \sim \Lambda^4/M_{pl}^2$, where M_{pl} is the Planck mass. However, note that it is a highly nontrivial problem whether we can find a solution of the five dimensional universe including some domain walls. If the existence of the domain wall crucially affects the behavior of the background metric, the metric no longer becomes flat (as such an example, see Ref. [15]). In this case, it seems to be not so easy to understand which field is absorbed without a concrete solution.

Let us see the effect of the gravity for the five dimensional theory in which the domain walls are embedded. In this setup, suppose that there exists a solution of the Einstein equation with the metric of the form

$$ds^2 = \sigma(y)\,\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{41}$$

by which the Poincaré invariance in the four dimensions is ensured. Here the metric $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is used. The (μ,ν) component of the Einstein equation around the domain wall (we fix the center of the domain wall at $y=0$) is found to be

$$\frac{d^2\sigma(y)}{dy^2}\bigg/\sigma(y) = \frac{16\pi}{3M_F^3}\rho(y), \tag{42}$$

where $\rho(y) = \frac{1}{2}[d\phi_0(y)/dy]^2 + V(\phi_0)$ is the energy density of the bulk scalar field which generates the domain wall. Here, we assumed that the domain wall dominantly contributes to the energy momentum tenser, and thus the local behavior of the metric is controlled by the existence of the domain wall.

Regarding the domain wall as a thin object like the D-brane, we discuss the effective theory on the brane based on the Nambu-Goto action $S = -\int d^4x \Lambda^4 \sqrt{\hat{g}}$ by using the induced metric $\hat{g}(x,Y(x))$ [12]. The expansion of the determinant of the metric gives the action for the field $Y(x)$, which parametrizes the brane fluctuation,

$$S = \int d^4x \Lambda^4 \sqrt{\bar{g}}\left(\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu Y(x)\partial_\nu Y(x) - \frac{1}{4}\bar{g}^{\mu\nu}\bar{g}_{\mu\nu,55}Y(x)^2 + \cdots\right), \tag{43}$$

where the index 5 denotes the coordinate of the fifth dimension, and $\bar{g}_{\mu\nu} = g_{\mu\nu}(x,y=0)$ is the background metric at $y=0$. If the background metric is flat, $Y(x)$ is the massless state, which is just the NG boson. However, the nontrivial background metric seems to make the field massive. Using the metric assumed above and the Einstein equation (42), the mass term of $Y(x)$ is found to be $m_Y^2 \sim \rho(0)/M_F^3$. From the results given in Sec. II, we can expect $m_Y^2 \sim \Lambda^4 m_\phi/M_F^3$, which is larger than m_g^2. Since $Y(x)$ gets the mass term, it seems to not be the NG boson itself. The would-be NG boson may be another field or a combination of $Y(x)$ and other fields.

If there is another contribution to the energy momentum tensor, which is dominant compared with that of the domain wall, the mass term may be little related to the existence of the domain wall. In this case, $Y(x)$ may have much smaller mass or be massless and the would-be NG boson. In any case, to make a correct discussion, it seems to be essential to find a concrete solution of the extra dimensional theory with the domain wall configuration. Further work is needed.

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