KNOWLEDGE SPILLOVERS AND ECONOMIC GROWTH

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ABSTRACT

Since the seminal work of Lucas (1988), uncompensated knowledge spillovers have been shown to play a critical role in the process of economic development. However, the standard Lucas model studies human capital accumulation in narrow settings with one industry and a closed-economy. This dissertation attempts to study the development process in richer settings. In particular, various types of spillovers are examined. Our work shows that intra-industry spillovers promote economic growth but inter-industry spillovers are more complex. Specifically, spillovers from human capital across sectors may lead to lower overall growth of consumption. In an open economy setting, the growth rates of human capital critically depend on variation across countries in educational productivity. In particular, if the growth rate of human capital is stronger abroad than domestically, human capital accumulation will decline at home. However, the magnitude of the problem depends on differences in regional external economies. In fact, such differences might actually cause the stock of human capital to decline over time. Our work also demonstrates that external economies from human capital have important implications for international trade, which provides additional linkages for economic activity across countries. Namely, increased spillovers at home will lead to a deterioration in the domestic terms of trade. Consequently, policies designed to affect the diffusion of knowledge will impact regional economic activity.

DEDICATION

I would like to dedicate this dissertation to Dr. Billy Helms. His encouragement and example have spurred me on to complete this task.

LIST OF ABBREVIATIONS AND SYMBOLS

α	Multiplicative inverse of the inter-temporal elasticity of substitution
β	Productivity of labor in country/sector 1
γ_i	Human capital spillovers within country/sector i
γ_i^*	Human capital spillovers flowing into country/sector i
δ	Rate of physical capital depreciation
η	Productivity of labor in country/sector 2
θ_x^B	Growth rate of variable x along the BGP
λ_i	Co-state variable for physical capital in sector i
μ_i	Co-state variable for human capital in sector i
ρ	Discount rate
ϕ	Efficiency of investment in type 1 human capital
χ	Efficiency of investment in type 2 human capital
A	Technology in country/sector 1
B	Technology in country/sector 2
BGP	Balanced Growth Path
c_i	Consumption of good i in country/sector i
c_1^A	Consumption by the home country of good A
c_1^B	Consumption by the home country of good B
c_2^A	Consumption by the foreign country of good A
c_2^B	Consumption by the foreign country of good B
CE	Competitive equilibrium
f_{k_i}	First derivative of the production function with respect to k_i
H	Current-valued Hamiltonian
H_i	Average stock of type i knowledge in the economy as a whole
h_i	Type i human capital of each individual

i	Subscript to distinguish between country/sector 1 and 2
k_i	Physical capital in country/sector i
MAR	Marshall Arrow Romer
P	Allocation under coordination
P_1	Allocation with coordination only in the home country
R&D	Research and development
t	Time period
U_i	Utility function in country/sector i
u_i	Time spent working in country/sector i
v_1	Time spent studying in sector 1
$\dot{x}$	Rate of change of variable x
y_i	Output in country/sector i

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CONTENTS

CHAPTER 1
INTRODUCTION

A great deal of research in macroeconomics in recent years stresses the role of human capital accumulation for promoting a country's level of economic development. In his seminal work, Lucas (1988) demonstrates that persistent accumulation of knowledge contributes to savings over time so that incomes and living standards continually improve. In particular, Lucas highlights the role of uncompensated knowledge spillovers in promoting economic growth.

Due to Lucas's work, a wave of research attempting to understand the source and impact of knowledge spillovers has been ignited. Several distinct types of knowledge spillovers have been identified in the literature. However, the standard Lucas model includes only one type of spillover in a closed-economy setting. The objective of this dissertation is to expand the Lucas framework to include several different types of knowledge spillovers and to provide insight into the effect that each specific type of spillover has on economic development.

Chapter two extends the original Lucas framework to include two sectors of production. This model features both intra-industry and inter-industry spillovers. Beginning with arguments put forth by Marshall (1890), many economists have argued that specialization and learning within individual sectors of the economy are the most important for economic growth. On the other hand, more recent work by Jacobs (1969) and Glaeser et al. (1992) stresses that inter-industry spillovers are more productive. Regardless of the direction of such claims, there has been little work attempting to study the roles of intra-industry and inter-industry spillovers in the endogenous growth process.

Standard models of human capital accumulation and growth are expressed in closed-economy settings. However, much empirical evidence demonstrates that economies are interconnected. In particular, spillovers of knowledge also extend beyond national borders. Chapter three explores the growth process in the presence of both domestic and international spillovers. This is accomplished through the development of a two-country version of the Lucas framework. The model allows the two countries to differ in several important ways. To begin, the model accounts for regional variation in the rate of economic growth due to differences in the productivity of human capital investment across countries. The degree of domestic and international spillovers is also allowed to vary across countries. Finally, several cases of economic coordination are examined, in which the degree of coordination is asymmetric across countries.

The framework contained in chapter three adds significant intricacy and flexibility to the original Lucas model. However, the implications of the model are still limited by the absence of trade. Chapter four extends the work of the previous chapter by placing the model in an environment of free trade. In particular, this allows for an examination of the effects of domestic and international knowledge spillovers on the terms of trade.

Together, the three models contained in this dissertation present new perspectives regarding the role of specific types of uncompensated knowledge spillovers in the development process. The final chapter contains a summary of the important results of these models.

CHAPTER 2

INTRA-INDUSTRY SPILLOVERS AND INTER-INDUSTRY SPILLOVERS IN A MODEL OF ECONOMIC GROWTH

2.1 Introduction

In response to growing dissatisfaction with exogenous growth models which attribute the source of growth to unexplained external factors, Romer (1986), Lucas (1988), and Rebelo (1991) developed important models that formalize the key determinants of economic growth. That is, the frameworks that they construct allow for cross country differences in rates of economic growth to be explained by decision making within the model. While diminishing returns to physical capital accumulation limited the ability of exogenous growth models to explain how perpetual economic growth takes place, Lucas (1988) incorporates human capital investment. The presence of this second factor of production alleviates the problem of diminishing returns to physical capital in neoclassical (exogenous) growth models. Consequently, Lucas refers to human capital as the "engine of growth" and argues that *uncompensated spillovers of knowledge* play an important role in the growth process.

Several distinct types of uncompensated knowledge spillovers have been identified in the literature. In particular, Marshall (1890) first introduced the idea of intra-industry spillovers by hypothesizing that industry concentration in a city allows for spillovers within the industry, leading to growth. This theory was further formalized by Arrow (1962) and Romer (1986). As such, this type of spillover became known as a Marshall-Arrow-Romer (MAR) spillover.[1] Porter (1990) also stressed the importance of intra-industry spillovers in the growth process.

[1] The descriptive label "MAR spillover" was introduced by Glaeser et al (1992) and is now widely accepted.

By comparison, Jacobs (1969) emphasized the key role that spillovers across industries play in the growth process. Her theories suggest that a city with a diverse range of industries will grow quickly due to the opportunities for inter-industry spillovers. Jacobs offered the anecdotal evidence of the industrially diverse city of Birmingham, England. Birmingham experienced consistent growth, as opposed to Manchester, a city heavily concentrated in the textile industry, which declined economically. Jacobs stressed that a local variety of industries will lead to higher growth due to the diverse set of ideas spilling over across sectors.

Regardless of the direction of previous claims – whether spillovers within industries or across industries are more important – there has been little work attempting to study the different roles of MAR spillovers and Jacobs externalities in the endogenous growth process. For example, the standard Lucas framework contains a single sector that produces a homogenous final product. Therefore, only one type of spillover can exist in this simple setting, and the model can only be used to understand the benefits of knowledge spillovers in general. A richer setting is necessary in order to analyze and compare different types of human capital spillovers. In order to address the roles of both types of spillovers in economic development, this paper extends the original Lucas framework to include two sectors of production.

In our two-sector framework, each sector produces a distinct good and agents have a preference for consumption of both goods. In contrast to the one-sector model of Lucas, agents must choose how to allocate their time between the two sectors. Production in each sector is enhanced not only by knowledge spillovers within the industry, but also by knowledge spilling over from the other industry.

Related Empirical Literature

Scherer (1982) presented evidence of the existence of inter-industry spillovers. His work traced industrial R&D expenditures and found that around seventy percent of inventions in a given industry are used outside that industry.

In comparison, Glaeser, Kallal, Scheinkman, and Schleifer (1992) also documented that inter-industry externalities take place. They focused on city-level data, due to the fact that cities bring about a high number of interactions between people. These interactions result in ample opportunity for knowledge spillovers. They compared industry employment data (in 1956 and 1987) to measure knowledge spillovers. It is assumed that increased employment is caused by increased productivity and that increased productivity comes from knowledge spillovers. The paper found that the data most closely matched Jacobs' theory that a city with industry diversity, as opposed to industry concentration, fosters growth.

Henderson, Kuncoro, and Turner (1995) used methodology similar to Glaeser et al. (1992) but introduced a distinction between mature and new, high tech industries. They created consistency by tracking the same eight industries in a panel of cities. In contrast, Glaeser et al. (1992) examined the six largest industries in each city, resulting in the use of a broad range of industries. Henderson et al. (1995) found intra-industry externalities for mature industries.[2] The authors found both intra- and inter-industry externalities for new, high tech industries. They conclude that inter-industry externalities (and hence, diversity) are required to attract new industries to a city. However, it is intra-industry externalities (concentration, which fosters "local trade secrets") which are important for maintaining industries.

The empirical evidence regarding intra- and inter-industry knowledge spillovers is inconclusive, in part due to the fact that knowledge spillovers are an intangible good. These spillovers are therefore very difficult, if not impossible, to trace. In light of the difficulty in measuring knowledge spillovers empirically, a convincing theoretical model is crucial.

The remainder of the paper is organized in the following manner. Section 2.2 describes fully the benchmark model. In the third section, the framework is altered in order to understand the possible effects of economic coordination.

[2]The authors do not distinguish between MAR and Porter externalities and refer to all intra-industry externalities as MAR externalities.

2.2 Benchmark Model

Our model extends the Lucas (1988) closed-economy single-sector framework by incorporating a second sector of production and allowing knowledge to spill over both within and across the two sectors. There is one country and each of the two sectors produces a distinct good. These sectors are simply referred to as "sector 1" and "sector 2." Knowledge (human capital) is specific to each sector. Agents are endowed with some positive amount of each type of human capital. Each agent must allocate his or her time between work and study in each of the two sectors. The time spent working is represented by $u_i(t)$ (where $i = 1, 2$, which represent each of the two sectors). The time spent studying in sector 1 is represented by $v_1(t)$, and by $1 - u_1(t) - u_2(t) - v_1(t)$ in sector 2. If the agent chooses to study in sector 1, then he or she will accumulate type 1 human capital, as represented by the evolution of human capital equation for sector 1:

$$\dot{h}_1(t) = \phi v_1(t) h_1(t) \tag{2.1}$$

where $h_1(t)$ is the type 1 human capital of each individual. $\phi > 0$ represents the growth rate of type 1 human capital if agents were to devote all of their time to human capital accumulation within sector 1.

If the agent chooses to study in sector 2, then he or she will accumulate type 2 human capital, as represented by the evolution of human capital equation for sector 2:

$$\dot{h}_2(t) = \chi \left(1 - u_1(t) - u_2(t) - v_1(t)\right) h_2(t) \tag{2.2}$$

where $\chi > 0$ represents the growth rate of type 2 human capital if agents were to devote all of their time to human capital accumulation within sector 2.

We assume that human capital spillovers are present within each sector and also across sectors. γ_i represents the level of intra-industry spillovers and γ_i^* represents the level of inter-industry human capital spillovers that sector i experiences (that is, the spillover *into* sector i). Assume $\gamma_i, \gamma_i^* \geq 0$. These spillovers can enhance productivity, as can be seen in the production function for each sector:

$$\begin{aligned} y_1(t) &= f_1(k_1(t), u_1(t), h_1(t), H_1(t), H_2(t) \\ &= A\,(k_1(t))^{1-\beta}\,(u_1(t)h_1(t))^{\beta}\,(H_1(t))^{\gamma_1}\,(H_2(t))^{\gamma_1^*} \end{aligned} \tag{2.3}$$

$$\begin{aligned} y_2(t) &= f_2(k_2(t), u_2(t), h_2(t), H_1(t), H_2(t) \\ &= B\,(k_2(t))^{1-\eta}\,(u_2(t)h_2(t))^{\eta}\,(H_2(t))^{\gamma_2}\,(H_1(t))^{\gamma_2^*} \end{aligned} \tag{2.4}$$

where the average stock of type i knowledge in the economy as a whole is denoted by $H_i(t)$. In addition, $0 < \beta, \eta < 1$.

Consumption of good i will be represented by $c_i(t)$. Agents have a preference for both goods which is reflected in the lifetime utility maximization problem:

$$\underset{c_1(t);c_2(t)}{Max} \int_0^{\infty} e^{-\rho t}[\frac{c_1(t)^{1-\alpha}}{1-\alpha} + \frac{c_2(t)^{1-\alpha}}{1-\alpha}]dt \tag{2.5}$$

where $\frac{1}{\alpha} > 0$ is the inter-temporal elasticity of substitution between the two goods and $\rho > 0$ is the discount rate.

Good 1 is produced in sector 1 according to the production function given in equation (2.3). Part of the production is depreciated $(\delta k_1(t))$, part is reinvested $\left(\dot{k}_1(t)\right)$, and the remaining production is consumed. This relationship is expressed by the evolution of physical capital in sector 1:

$$\dot{k}_1(t) = A\,(k_1(t))^{1-\beta}\,(u_1(t)h_1(t))^{\beta}\,(H_1(t))^{\gamma_1}\,(H_2(t))^{\gamma_1^*} - \delta k_1(t) - c_1(t) \tag{2.6}$$

The evolution of physical capital in sector 2 is analogous:

$$\dot{k}_2(t) = B\,(k_2(t))^{1-\eta}\,(u_2(t)h_2(t))^{\eta}\,(H_2(t))^{\gamma_2}\,(H_1(t))^{\gamma_2^*} - \delta k_2(t) - c_2(t) \tag{2.7}$$

In order to solve the problem, we apply Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$
\begin{aligned}
H \;=\; & \frac{c_1(t)^{1-\alpha}}{1-\alpha}+\frac{c_2(t)^{1-\alpha}}{1-\alpha} \\
& +\lambda_1(t)\left(A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)h_1(t)\right)^{\beta}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}-\delta k_1(t)-c_1(t)\right) \\
& +\mu_1(t)\left(\phi v_1(t)h_1(t)\right) \qquad (2.8)\\
& +\lambda_2(t)\left(B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)h_2(t)\right)^{\eta}\left(H_2(t)\right)^{\gamma_2}\left(H_1(t)\right)^{\gamma_2^*}-\delta k_2(t)-c_2(t)\right) \\
& +\mu_2(t)\left(\chi\left(1-u_1(t)-u_2(t)-v_1(t)\right)h_2(t)\right)
\end{aligned}
$$

where $\lambda_i(t)$ represents the co-state variable for physical capital in sector i, and $\mu_i(t)$ represents the co-state variable for human capital in sector i.

First, the representative agent needs to decide the amount of each good to consume each period:

$$(c_1(t))^{-\alpha}=\lambda_1(t) \qquad (2.9)$$

$$(c_2(t))^{-\alpha}=\lambda_2(t) \qquad (2.10)$$

Since $\lambda_i(t)$ represents the value of an additional unit of physical capital in sector i, equations (2.9) and (2.10) represent the standard trade-off between consumption in the present and capital accumulation within each sector.

The representative agent must also divide time between production and learning in each of the two sectors. This decision leads to a series of no-arbitrage conditions. First, on the margin, the return to investment in physical and human capital must be the same within sector 1:

$$\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta-1}\left(h_1(t)\right)^{\beta}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}=\mu_1(t)\phi h_1(t) \qquad (2.11)$$

This equation represents an investment decision between the two types of capital within sector 1. On each side of the equation, the value of an additional unit of a specific type of capital is multiplied times the marginal number of units created through investment in that type of capital. According to equation (2.11), the marginal return to investment in each type of capital with sector 1 must be the same.

Equation (2.11) highlights an important feature of the model. The left side displays diminishing marginal return investment in physical capital. The right side reflects the linearity of the human capital accumulation formula, equation (2.1), because it indicates that the marginal return of time devoted to human capital accumulation is independent of the amount of time already invested in human capital accumulation. This provides interesting insight. If spillovers from either sector increase, then the marginal return to investment in physical capital increases. The return to investment in human capital is constant. Therefore, the time devoted to production should increase until the marginal return to investment in physical capital falls to the level of the marginal return to investment in human capital. Within sector 2, there is an analogous no-arbitrage relationship:

$$\lambda_2(t)\eta B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta-1}\left(h_2(t)\right)^{\eta}\left(H_2(t)\right)^{\gamma_2}\left(H_1(t)\right)^{\gamma_2^*} = \mu_2(t)\chi h_2(t) \tag{2.12}$$

Finally, the return to human capital investment must be the same across sectors:

$$\mu_1(t)\phi h_1(t) = \mu_2(t)\chi h_2(t) \tag{2.13}$$

Agents must allocate their time between investment in four different kinds of capital: physical capital in sector 1, human capital in sector 1, physical capital in sector 2, and human capital in sector 2. Together, the three no-arbitrage equations indicate that at the margin, the return to investment in each type of capital must be equal.

The Euler equations represent the capital gains or losses from physical and human capital over time:

$$\dot{\lambda}_1(t) = \lambda_1(t)(\rho - f_{k_1}(t) + \delta) \tag{2.14}$$

$$\dot{\lambda}_2(t) = \lambda_2(t)(\rho - f_{k_2}(t) + \delta) \tag{2.15}$$

$$\begin{aligned}\dot{\mu}_1(t) \quad = \quad & \rho\mu_1(t) - \lambda_1(t)A\beta\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta-1}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} \\ & -\mu_1(t)\phi v_1(t)\end{aligned} \tag{2.16}$$

$$\begin{aligned}\dot{\mu}_2(t) &= \rho\mu_2(t) - \lambda_2(t)B\eta\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta-1}\left(H_2(t)\right)^{\gamma_2}\left(H_1(t)\right)^{\gamma_2^*} \quad (2.17)\\ &-\mu_2(t)\chi\left(1-u_1(t)-u_2(t)-v_1(t)\right)\end{aligned}$$

Notably, physical capital is only used in the production sector, and so the Euler equations for physical capital are functions of the value of physical capital ($\lambda_1(t)$ and $\lambda_2(t)$). Human capital, on the other hand, is used for both production and for accumulating more human capital. Therefore, the Euler equations for human capital are functions of both the value of physical and human capital within each sector.

The no-arbitrage conditions for each sector, equations (2.11) and (2.12), specify that the marginal return to time spent in each sector must be equal. Therefore, these equations provide a relationship between the value of physical and human capital in each sector. These relationships can be utilized to transform the Euler equations for human capital into functions of only the value of human capital:

$$\dot{\mu}_1(t) = \mu_1(t)\left[\rho - \phi\left(u_1(t) + v_1(t)\right)\right] \quad (2.18)$$

$$\dot{\mu}_2(t) = \mu_2(t)\left[\rho - \chi\left(1 - u_1(t) - v_1(t)\right)\right] \quad (2.19)$$

Now, $\mu_i(t)$, the co-state variable for human capital in sector i, represents the value of an additional unit of human capital in sector i. Also remember that $u_1(t) + v_1(t)$ is the total time spent working and studying in sector 1 and $1 - u_1(t) - v_1(t)$ is the total time spent working and studying in sector 2. So then, according to equations (2.18) and (2.19), the growth rate of the marginal value of human capital in each sector is decreasing in the productivity of human capital accumulation. It is also decreasing in the total amount of time spent working and studying in that particular sector.

Increases in the efficiency of human capital investment or in the amount of time spent in the learning sector will lead to a greater accumulation of human capital. This results in a lower marginal value of human capital. Similarly, additional time invested in producing physical capital leads to a lower marginal value of physical capital. Since the purpose of human capital is to increase the production of physical capital (in the future), a fall in the marginal value of physical capital causes a decline in the marginal value of human capital.

In order to analyze the growth rate of the economy over time, we next convert all equations to growth equations. We then solve for the growth rate of each variable along the balanced growth path (BGP). Along the BGP, all growth rates must be constant. The growth rate of variable x along the BGP is represented by θ_x^B. At this point, we impose the equilibrium condition that the average stock of sector-specific knowledge will be equal to the representative agent's sector-specific human capital stock, that is $H_i = h_i$[3].

First, convert the two transformed Euler equations for human capital, equations (2.18) and (2.19), into growth equations and then combine these equations to find the following relationship:

$$\theta_{\mu_2}^B = \frac{\phi + \chi}{\phi}\rho - \chi - \frac{\chi}{\phi}\theta_{\mu_1}^B \tag{2.20}$$

This equation is interesting because we see a negative relationship between the growth of the co-state variables in each sector. The growth of the marginal value of human capital in sector 1 has a negative effect on the growth of the marginal value of human capital in sector 2. This negative effect is scaled by the ratio of relative productivity of human capital accumulation in each sector.

The origin of this effect comes from the fact that the agent splits time between the two sectors. From equations (2.18) and (2.19), time spent working or studying in a particular sector lowers the growth of the marginal value of human capital in that sector. Now, spending time in one sector means that time is not being spent in the other sector. Therefore, time spent in a particular sector has the effect of raising the growth of the marginal value of human capital in the *other* sector. This results in a negative relationship between the growth rates of the marginal value of human capital in the two sectors.

Equation (2.20) can then be substituted into the no-arbitrage condition for human capital across sectors, equation (2.13), to obtain growth rates of human capital co-state

[3]It is of course important that this condition was imposed after individual agents made their decisions regarding human capital accumulation.

variables in terms of the growth rates of human capital in each sector:

$$\theta_{\mu_1}^B = \rho - \frac{\phi\chi}{\phi+\chi} - \frac{\phi}{\phi+\chi}\left(\theta_{h_1}^B - \theta_{h_2}^B\right) \tag{2.21}$$

$$\theta_{\mu_2}^B = \rho - \frac{\phi\chi}{\phi+\chi} - \frac{\chi}{\phi+\chi}\left(\theta_{h_2}^B - \theta_{h_1}^B\right) \tag{2.22}$$

It is interesting that the growth of the marginal value of human capital in each sector is dependent on the growth of human capital in both sectors. The relationship between the growth of the marginal value of human capital and the human capital growth in the same sector is negative. This is because an increase in the stock of human capital in a sector causes the growth of the marginal value of human capital to fall. On the other hand, the co-state variable is positively affected by human capital growth in the other sector. This can be attributed to the fact that the representative agent has a preference for both goods.

Human capital growth in a sector will ultimately raise consumption growth of the good in that sector. This causes the good in the other sector to be in relatively short supply, which increases the value of additional human capital in the other sector.

An interesting relationship can be found between the working time in each sector by combining the evolution of human capital equations, the transformed Euler equations for human capital, equations (2.18) and (2.19), and the no-arbitrage for human capital across sectors:

$$u_1(t) = \frac{\chi}{\phi} u_2(t) \tag{2.23}$$

The agent determines the amount of time to spend working in each sector based on the relative productivity in human capital accumulation across sectors. It is rather interesting that the decision does not depend on β or η, the actual productivity of working time ($u_i h_i$) in each sector.

According to equation (2.23) if, for example, productivity in human capital accumulation is higher in sector 2 than sector 1, then the agent will spend more time in the sector 1 production sector than in the sector 2 production sector. Recall that agents have a preference for consuming both goods. More productivity in human capital accumulation in sector 2 benefits production in sector 2, which leads the representative agent to spend more time in sector 1 to try to increase production in that sector.

For simplicity, we will continue the analysis of sector 1, excluding some details for sector 2, as the derivations are analogous. Next, combine equation (2.9), the sector 1 consumption choice condition, with equation (2.14), the sector 1 Euler equation for physical capital. This yields the following growth rate of consumption:

$$\theta_{c_1}^B = \frac{f_{k_1} - (\rho + \delta)}{\alpha} \tag{2.24}$$

Therefore, in order for $\theta_{c_1}^B$ to be constant, f_{k_1} must be constant which implies:

$$\theta_{k_1}^B = \left(1 + \frac{\gamma_1}{\beta}\right)\theta_{h_1}^B + \frac{\gamma_1^*}{\beta}\theta_{h_2}^B \tag{2.25}$$

Then, using the evolution of physical capital, equation (2.6), we find that in order for the growth of physical capital to be constant, we must have:

$$\theta_{k_1}^B = \theta_{c_1}^B \tag{2.26}$$

Together, equations (2.25) and (2.26) form an important new expression for the growth of consumption of good 1, now in terms of the growth rates of human capital in each sector:

$$\theta_{c_1}^B = \left(1 + \frac{\gamma_1}{\beta}\right)\theta_{h_1}^B + \frac{\gamma_1^*}{\beta}\theta_{h_2}^B \tag{2.27}$$

This equation illustrates some of the important results of the model. Notably, the growth of human capital has a very strong effect on the growth of consumption, highlighting the role of human capital as the "engine of growth."

From the production function of sector 1, the coefficients $\frac{\gamma_1}{\beta}$ and $\frac{\gamma_1^*}{\beta}$ may be interpreted as the relative impact of each type of spillover. It is also worth noting that equation (2.27) seems to indicate that the two different types of spillovers affect consumption growth symmetrically. However, the impact of the spillover is dependent on the growth of human capital in each sector, which may not be symmetric. It is therefore important to examine the growth of sector-specific human capital more closely.

Next, substitute equations (2.9), (2.26), and (2.27), as well as the growth rate of the human capital co-state variable for sector 1, equation (2.21), into the no-arbitrage condition across productive assets within sector 1, equation (2.11).[4] This yields an expression for the growth rate of human capital in sector 1 in terms of the growth rate of human capital in sector 2:

$$\theta_{h_1}^B = \frac{\left(\frac{\phi\chi}{\phi+\chi} - \rho\right) - \left[\frac{\phi}{\phi+\chi} + (\alpha - 1)\frac{\gamma_1^*}{\beta}\right]\theta_{h_2}^B}{\frac{\chi}{\phi+\chi} + (\alpha - 1)\left(1 + \frac{\gamma_1}{\beta}\right)} \tag{2.28}$$

An analogous analysis can be performed for sector 2, with the resulting equations:

$$\theta_{c_2}^B = \theta_{h_2}^B + \frac{\gamma_2}{\eta}\theta_{h_2}^B + \frac{\gamma_2^*}{\eta}\theta_{h_1}^B \tag{2.29}$$

$$\theta_{h_2}^B = \frac{\left(\frac{\phi\chi}{\phi+\chi} - \rho\right) - \left[\frac{\chi}{\phi+\chi} + (\alpha - 1)\frac{\gamma_2^*}{\eta}\right]\theta_{h_1}^B}{\frac{\phi}{\phi+\chi} + (\alpha - 1)\left(1 + \frac{\gamma_2}{\eta}\right)} \tag{2.30}$$

We can now use equations (2.28) and (2.30) to solve for the growth of human capital in each sector. The growth equations for human capital can then be substituted into equations (2.27) and (2.29) to solve for the growth rates of consumption of each good. In turn, we have the following proposition:

[4] Along the BGP, the amount of time working in each sector must remain constant, that is, $\theta_{u_1}^B = \theta_{u_2}^B = \theta_{v_1}^B = 0$.

Proposition 2.1. The growth rates of human capital and consumption are as follows:

$$\theta^B_{h_1} = \left(\frac{\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}}{\begin{array}{c}\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right] \\ +(\alpha-1)\left[\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_2^*}{\eta}\frac{\gamma_1^*}{\beta}\right]\end{array}} \right) * \left(\frac{\phi\chi}{\phi+\chi} - \rho\right) \tag{2.31}$$

$$\theta^B_{h_2} = \left(\frac{\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}}{\begin{array}{c}\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right] \\ +(\alpha-1)\left[\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_2^*}{\eta}\frac{\gamma_1^*}{\beta}\right]\end{array}} \right) * \left(\frac{\phi\chi}{\phi+\chi} - \rho\right) \tag{2.32}$$

$$\theta^B_{c_1} = \theta^B_{c_2} = \left(\frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\begin{array}{c}\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right] \\ +(\alpha-1)\left[\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_2^*}{\eta}\frac{\gamma_1^*}{\beta}\right]\end{array}} \right) * \left(\frac{\phi\chi}{\phi+\chi} - \rho\right) \tag{2.33}$$

Before discussing the final results of the model and conditions for growth in the presence of both intra- and inter-industry knowledge spillovers, we will consider the results in the presence of only one type of spillover at a time. This will enhance our understanding of each type of spillover. We begin by considering a model where only intra-industry spillovers are present, yielding the following lemma:

Lemma 2.1. Assume that $\gamma_1^* = \gamma_2^* = 0$. Furthermore, let $\alpha > 1$ and $\frac{\phi\chi}{\phi+\chi} > \rho$. In this case, there exists a unique, balanced growth path. The growth rates of human capital and consumption are as follows:

$$\theta_{h_1}^B = \left(\frac{\left(1+\frac{\gamma_2}{\eta}\right)}{\frac{\phi}{\phi+\chi}\left(1+\frac{\gamma_1}{\beta}\right)+\frac{\chi}{\phi+\chi}\left(1+\frac{\gamma_2}{\eta}\right)+(\alpha-1)\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)} \right) \left(\frac{\phi\chi}{\phi+\chi} - \rho \right) \tag{2.34}$$

$$\theta_{h_2}^B = \left(\frac{\left(1+\frac{\gamma_1}{\beta}\right)}{\frac{\phi}{\phi+\chi}\left(1+\frac{\gamma_1}{\beta}\right)+\frac{\chi}{\phi+\chi}\left(1+\frac{\gamma_2}{\eta}\right)+(\alpha-1)\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)} \right) \left(\frac{\phi\chi}{\phi+\chi} - \rho \right) \tag{2.35}$$

$$\theta_{c_1}^B = \theta_{c_2}^B = \left(\frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)}{\frac{\phi}{\phi+\chi}\left(1+\frac{\gamma_1}{\beta}\right)+\frac{\chi}{\phi+\chi}\left(1+\frac{\gamma_2}{\eta}\right)+(\alpha-1)\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)} \right) \left(\frac{\phi\chi}{\phi+\chi} - \rho \right) \tag{2.36}$$

$\theta_{h_1}^B$ is positive and is decreasing in γ_1 and increasing in γ_2. $\theta_{h_2}^B$ is positive and is decreasing in γ_2 and increasing in γ_1. $\theta_{c_1}^B = \theta_{c_2}^B$ is positive and increasing in γ_1 and γ_2.

First, we will discuss the growth rates of human capital, which are given in equations (2.34) and (2.35). It is interesting to note that in a model without inter-industry spillovers, the interaction between the two sectors is limited to the fact that agents have a preference for consumption of both goods. An increase in intra-industry spillovers does not directly affect the other sector.

When the spillover within a sector increases, this causes a direct increase in productivity within the sector. This, in turn, increases the incentive to produce and lowers the incentive for individuals to accumulate human capital. The lower incentive leads to a decline in the growth of human capital in the sector. The free-riding problem will eventually lead to a fall in the growth of the average stock of knowledge in the sector. This results in less external benefit and a decrease in productivity growth, and ultimately increased incentive to accumulate human capital. The secondary effect dampens the original negative effect on the growth of human capital.

Next, we will consider equations (2.36), the growth of consumption. As in the original Lucas framework, in equilibrium, the increase in productivity caused by knowledge spillovers overwhelms the negative effect of free-riding. So, the overall effect on the growth of consumption is positive. In contrast to the single-sector Lucas model, agents in our model divide their time not only between work and study, but also between the two sectors. Due to the diminishing marginal utility of each good, increased productivity in one sector will cause agents to increase the time they spend working and studying in the other sector. This creates a positive effect on human capital and consumption growth within the other sector.

We move to consider the case where only inter-industry spillovers are present in the following lemma:

Lemma 2.2. Assume that $\gamma_1 = \gamma_2 = 0$*. Furthermore, let* $\alpha > 1$ *and* $\frac{\phi\chi}{\phi+\chi} > \rho$*. Also assume that* $\gamma_1^* < \beta$ *and* $\gamma_2^* < \eta$*. In this case, there exists a unique, balanced growth path. The growth rates of human capital and consumption are as follows:*

$$\theta_{h_1}^B = \frac{1-\frac{\gamma_1^*}{\beta}}{\frac{\phi}{\phi+\chi}\left[1-\frac{\gamma_2^*}{\eta}\right]+\frac{\chi}{\phi+\chi}\left[1-\frac{\gamma_1^*}{\beta}\right]+(\alpha-1)\left[1-\frac{\gamma_2^*}{\eta}\frac{\gamma_1^*}{\beta}\right]} * \left(\frac{\phi\chi}{\phi+\chi}-\rho\right) \tag{2.37}$$

$$\theta_{h_2}^B = \frac{1-\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[1-\frac{\gamma_2^*}{\eta}\right]+\frac{\chi}{\phi+\chi}\left[1-\frac{\gamma_1^*}{\beta}\right]+(\alpha-1)\left[1-\frac{\gamma_2^*}{\eta}\frac{\gamma_1^*}{\beta}\right]} * \left(\frac{\phi\chi}{\phi+\chi}-\rho\right) \tag{2.38}$$

$$\theta_{c_1}^B = \theta_{c_2}^B = \frac{1-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[1-\frac{\gamma_2^*}{\eta}\right]+\frac{\chi}{\phi+\chi}\left[1-\frac{\gamma_1^*}{\beta}\right]+(\alpha-1)\left[1-\frac{\gamma_2^*}{\eta}\frac{\gamma_1^*}{\beta}\right]} * \left(\frac{\phi\chi}{\phi+\chi}-\rho\right) \tag{2.39}$$

$\theta_{h_1}^B$ *is positive and is decreasing in* γ_1^* *and increasing in* γ_2^**.* $\theta_{h_2}^B$ *is positive and is decreasing in* γ_2^* *and increasing in* γ_1^**.* $\theta_{c_1}^B = \theta_{c_2}^B$ *is positive.*

(a) If $\chi > \phi\frac{\gamma_2^*}{\eta}$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is increasing in* γ_1^**. If* $\chi < \phi\frac{\gamma_2^*}{\eta}$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is decreasing in* γ_1^**.*

(b) If $\phi > \frac{\gamma_1^*}{\beta}\chi$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is increasing in* γ_2^**. If* $\phi < \frac{\gamma_1^*}{\beta}\chi$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is decreasing in* γ_2^**.*

In the case of intra-industry spillovers, the effect of one sector on the other sector is indirect. In a model with inter-industry spillovers, the actions of agents in one sector directly affect the other sector through the inter-industry spillover. The dynamics of models including inter-industry spillovers are therefore more complex. We will consider first the effects on the growth rates of human capital, equations (2.37) and (2.38). We will then consider the ultimate effects on the growth of consumption, equation (2.39).

An increase in the inter-industry spillovers into sector 1 causes an increase in productivity within sector 1 and thus causes a decline in the growth of human capital in sector 1. The resulting loss in human capital accumulation in sector 1 directly hurts the productivity in sector 2 through the inter-industry spillover. This ultimately gives agents more incentive to accumulate sector 2 type knowledge. The external benefit on sector 1 of the resulting growth in human capital in sector 2 causes even more free-riding within sector 1.

As described in the previous paragraph, in the presence of greater inter-industry spillovers, the secondary sequence of reactions magnifies the free-rider problem. Thus, the decline in human capital growth is accelerated (a multiplier effect). This leads to the possibility of a negative effect on consumption growth in the presence of asymmetry.

Specifically, an increase in the spillover into sector 1 will have a negative effect on consumption growth if two conditions are met. The first condition is that the efficiency of human capital investment in sector 1 is high relative to the efficiency of human capital investment in sector 2. The second condition is that the spillover into sector 2 has a very strong relative impact on production (i.e. $\frac{\gamma_2^*}{\eta}$ is high). If these two conditions hold, then the losses caused by free-riding will be exacerbated.

When higher inter-industry spillovers into sector 1 allow agents to free-ride off of the average level of sector knowledge in sector 2, then agents spend less time accumulating sector 1 type human capital. If the efficiency of human capital investment is high in sector 1, then the free-riding will cause a large loss in human capital accumulation. If, at the same time, the inter-industry spillover into sector 2 plays a substantial role in production, then the decline in human capital accumulation in sector 1 will have a large negative impact on sector 2. This leads to an overall decline in the consumption growth of both goods.

It is not difficult to believe that asymmetries regularly occur in economies. Consider as an example a dominant, high growth sector and a second smaller, dependent sector. In the high growth sector, the return to investment in human capital accumulation is very high. In contrast, human capital accumulates slowly in the smaller sector. The small sector is instead very dependent on knowledge that spills over from the other sector as a source of growth. So then, the human capital that is being accumulated in the high growth sector is actually the "engine of growth" for both sectors.

If, for any reason, agents in the high growth sector decrease the time they devote to human capital accumulation, this will have a strong negative impact on both sectors. Accordingly, if spillovers into the high growth sector increase, causing a free-rider problem, consumption growth will decline.

Finally, we return to equations (2.31), (2.32), and (2.33) and consider in more detail the case where both types of spillovers are present. This yields the following lemma:

Lemma 2.3. Let $\alpha > 1$ *and* $\frac{\phi\chi}{\phi+\chi} > \rho$*. Also assume that* $1 + \frac{\gamma_2}{\eta} > \frac{\gamma_1^*}{\beta}$ and $1 + \frac{\gamma_1}{\beta} > \frac{\gamma_2^*}{\eta}$. *In this case, there exists a unique, balanced growth path.* $\theta_{h_1}^B$ *is positive and is decreasing in* γ_1 *and* γ_1^* *and increasing in* γ_2 *and* γ_2^*. $\theta_{h_2}^B$ *is positive and is decreasing in* γ_2 *and* γ_2^* *and increasing in* γ_1 *and* γ_1^*. $\theta_{c_1}^B = \theta_{c_2}^B$ *is positive.*

(a) If $\chi > \phi\frac{\gamma_2^*}{\eta+\gamma_2}$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is increasing in* γ_1 and γ_1^*. *If* $\chi < \phi\frac{\gamma_2^*}{\eta+\gamma_2}$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is decreasing in* γ_1 and γ_1^*.

(b) If $\phi > \chi\frac{\gamma_1^*}{\beta+\gamma_1}$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is increasing in* γ_2 and γ_2^*. *If* $\phi < \chi\frac{\gamma_1^*}{\beta+\gamma_1}$*, then* $\theta_{c_1}^B = \theta_{c_2}^B$ *is decreasing in* γ_2 and γ_2^*.

We begin by analyzing equations (2.31) and (2.32), the growth rates of human capital.

To understand the intuition that drives Lemma 2.3, consider the production functions for each sector. Spillovers within sector 1 and from sector 2 to sector 1 increase productivity in sector 1 and therefore hurt the growth of human capital in that sector. The spillovers allow agents in sector 1 to free ride off of others' knowledge, thereby lowering their own incentive to acquire human capital. The effect on human capital accumulation in sector 1 affects workers in sector 2 because they also benefit from sector 1 human capital through the inter-industry spillover. Since agents in sector 2 have less human capital on which to free-ride, they have a greater incentive to acquire human capital. So the growth rate of human capital in sector 2 increases as a result of the increase in sector 1 spillovers. The intuition for sector 2 spillovers is analogous.

Next, we consider the growth of consumption, equation (2.32). The overall effect on the growth rates of consumption depends on the relative productivity of human capital accumulation within each sector. In most cases, the effect on the growth of consumption is positive. However, the effect on consumption growth of an increase of sector 1 spillovers will sometimes be negative.

Negative effects on consumption growth will occur when two conditions hold simultaneously. The first condition is that sector 1 efficiency in human capital investment is high relative to sector 2 efficiency in human capital investment. The second condition is that the relative importance for production of the spillover into sector 2 is high in relation to both the spillover within sector 2 and the impact of the individual agent's human capital on production (i.e. $\frac{\gamma_2^*}{\eta+\gamma_2}$ is high).

If the first condition holds, then a decrease in time spent in human capital accumulation due to free-riding will have a strong negative effect on the growth of human capital in sector 1. This will ultimately affect consumption. If the second condition also holds, then the inter-industry spillover into sector 2 is relatively large. Therefore, the sharp decline in the growth of human capital in sector 1 has a strong negative effect on productivity in sector 2. Hence, when the two conditions hold, spillovers into sector 1 will have a negative effect on the growth of consumption. The analysis for sector 2 spillovers is analogous.

To summarize the conclusions from the model to this point, the framework aims to study the various roles of intra-industry or MAR externalities versus inter-industry or Jacobs externalities in the development process. As articulated by Lucas, there are potential growth-promoting effects from human capital spillovers, but their presence also leads to inefficiences from free-rider problems. Notably, both types of spillovers simultaneously affect economic activity in each sector of our framework. Interestingly, the results imply that intra-industry spillovers promote economic growth, as argued by Marshall and others. However, the role of inter-industry spillovers may not be as beneficial as Jacobs suggested. In particular, spillovers from human capital across sectors may lead to lower overall growth of consumption.

2.3 Economic Coordination

In the benchmark model, the representative agent does not consider the impact of his or her actions on the average stock of human capital in each sector. Instead he or she takes these average stocks as given. In contrast, a model of economic coordination will be developed in this section. In this framework, a national planner coordinates decision making. The planner considers fully the impact that individual human capital investment decisions have on the average stock of human capital in each sector – and ultimately on production through the spillovers. This is modeled by setting the average stock of type i knowledge in each sector, $H_i(t)$, equal to the amount of individual human capital stock in the sector, $h_i(t)$.

Significant deviations from the competitive model will be noted in our analysis. In order to solve the planner's problem, we apply Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$\begin{aligned} H = {} & \frac{c_1(t)^{1-\alpha}}{1-\alpha} + \frac{c_2(t)^{1-\alpha}}{1-\alpha} \\ & +\lambda_1(t) \left(A \left(k_1(t)\right)^{1-\beta} \left(u_1(t)\right)^{\beta} \left(h_1(t)\right)^{\beta+\gamma_1} \left(h_2(t)\right)^{\gamma_1^*} - \delta k_1(t) - c_1(t) \right) \\ & +\mu_1(t)\phi v_1(t) h_1(t) \\ & +\lambda_2(t) \left(B \left(k_2(t)\right)^{1-\eta} \left(u_2(t)\right)^{\eta} \left(h_2(t)\right)^{\eta+\gamma_2} \left(h_1(t)\right)^{\gamma_2^*} - \delta k_2(t) - c_2(t) \right) \\ & +\mu_2(t) \left(\chi \left(1 - u_1(t) - u_2(t) - v_1(t)\right) h_2(t) \right) \end{aligned} \tag{2.40}$$

In determining how to allocate time spent working and studying in each sector, the planner considers the return to investment in each of the four types of capital: sector 1 type physical and human capital and sector 2 type physical and human capital. This decision leads to a series of no-arbitrage conditions:

$$\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta-1}\left(h_1(t)\right)^{\beta+\gamma_1}\left(h_2(t)\right)^{\gamma_1^*}=\mu_1(t)\phi h_1(t) \tag{2.41}$$

$$\lambda_2(t)\eta B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta-1}\left(h_2(t)\right)^{\eta+\gamma_2}\left(h_1(t)\right)^{\gamma_2^*}=\mu_2(t)\chi h_2(t) \tag{2.42}$$

Note that the no-arbitrage condition for human capital investment across sectors is the same as in the competitive model, equation (2.13).

The Euler equations for physical capital will be analogous to those found for the competitive equilibrium. However, the Euler equations for human capital are significantly different than in the competitive model, since the planner considers the full effect of human capital decisions. The new Euler equations for human capital are presented here:

$$\begin{aligned}\dot{\mu}_1(t) &= \rho\mu_1(t)-\lambda_1(t)A\left(\beta+\gamma_1\right)\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta+\gamma_1-1}\left(h_2(t)\right)^{\gamma_1^*} \\ &\quad -\mu_1(t)\phi v_1(t)-\lambda_2(t)B\gamma_2^*\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta+\gamma_2}\left(h_1(t)\right)^{\gamma_2^*-1}\end{aligned} \tag{2.43}$$

$$\begin{aligned}\dot{\mu}_2(t) &= \rho\mu_2(t)-\lambda_2(t)B\left(\eta+\gamma_2\right)\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta+\gamma_2-1}\left(h_1(t)\right)^{\gamma_2^*} \\ &\quad -\mu_2(t)\chi\left(1-u_1(t)-u_2(t)-v_1(t)\right) \\ &\quad -\lambda_1(t)A\gamma_1^*\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta+\gamma_1}\left(h_2(t)\right)^{\gamma_1^*-1}\end{aligned} \tag{2.44}$$

As in the competitive model, the no-arbitrage conditions for each sector, equations (2.41) and (2.42), provide a relationship between the value of physical and human capital. This can be utilized to transform the Euler equations for human capital into functions of only the value of human capital. The equations are presented here as growth rates:

$$\theta_{\mu_1}^B=\rho-\phi\left(u_1(t)+v_1(t)+\frac{\gamma_1}{\beta}u_1(t)+\frac{\gamma_2^*}{\eta}u_2(t)\right) \tag{2.45}$$

$$\theta_{\mu_2}^{B} = \rho - \chi\left((1 - u_1(t) - v_1(t)) + \frac{\gamma_2}{\eta}u_2(t) + \frac{\gamma_1^*}{\beta}u_1(t)\right) \tag{2.46}$$

Similar to the competitive case, the transformed Euler equations for human capital depict the negative relationship between the efficiency of human capital investment and the growth of the marginal value of human capital. However, in the presence of a national planner, the negative effect is magnified through the spillovers.

Consider equation (2.45). Recall that $\frac{\gamma_1}{\beta}$ represents the relative importance to sector 1 production of the intra-industry spillover in sector 1. In equation (2.45), this term is multiplied by $u_1(t)$, the time spent working in sector 1, and so this product represents the full benefit that the average stock of human capital in sector 1 has on sector 1 production through the spillover. Similarly, the term $\frac{\gamma_2^*}{\eta}u_2(t)$ represents the full benefit that the average stock of human capital in sector 1 has on sector 2 production through the spillover.

Through the inclusion of these terms, the planner considers not only the direct benefit of sector 1 human capital on sector 1 production, but also the full benefit that human capital in sector 1 has on production in both sectors through the spillovers. The planner recognizes that the spillovers increase the value of human capital, and therefore decrease the marginal value of human capital.

The transformed Euler equations can be combined with the evolution of human capital equations and the no-arbitrage equation for working time across sectors. This forms a relationship comparing the time spent in production across the two sectors:

$$u_1(t) = \left(\frac{\chi\left(1 + \frac{\gamma_2}{\eta}\right) - \phi\frac{\gamma_2^*}{\eta}}{\phi\left(1 + \frac{\gamma_1}{\beta}\right) - \chi\frac{\gamma_1^*}{\beta}}\right)u_2(t) \tag{2.47}$$

In order to ensure that there is a positive amount of time spent working in each sector, $\chi\left(1 + \frac{\gamma_2}{\eta}\right) > \phi\frac{\gamma_2^*}{\eta}$ and $\phi\left(1 + \frac{\gamma_1}{\beta}\right) > \chi\frac{\gamma_1^*}{\beta}$.

In the competitive case, the relationship between the amount of time spent working in each sector was solely determined by the ratio of the efficiency of human capital investment in each sector. Specifically, agents choose to spend more time working in the sector with the lower efficiency of human capital investment, in order to smooth production (and therefore consumption) across the two sectors.

In contrast, equation (2.47) demonstrates that in deciding how to optimally allocate their time, the planner takes into account the spillovers. The planner considers the direct impact of the individual human capital accumulated through investment of time in the learning sector, but also the full impact of the average stock of human capital on production through the spillovers. In equation (2.47), consider the case when the coefficient on $u_2(t)$ is greater than 1. In this instance, the time dedicated to human capital accumulation in sector 2 has a greater positive impact on production across sectors than the time dedicated to human capital accumulation in sector 1. In this situation, the planner will allocate more time to production in sector 1 than in sector 2.

Next, substitute the evolution of human capital equations into equation (2.47) to obtain an expression for $u_2(t)$ in terms of $\theta^B_{h_1}$ and $\theta^B_{h_2}$:

$$u_2(t) = \left(\frac{\frac{1}{\phi\chi}\left[\phi\left(1+\frac{\gamma_1}{\beta}\right) - \chi\frac{\gamma_1^*}{\beta}\right]}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} \right) * \left[\frac{\phi\chi}{\phi+\chi} - \frac{\chi}{\phi+\chi}\theta^B_{h_1} - \frac{\phi}{\phi+\chi}\theta^B_{h_2}\right] \tag{2.48}$$

The time spent in the production sector in 2 has a negative relationship with human capital growth in both sectors. The same relationship exists for time spent working in sector 1. If more time is invested in accumulating physical capital, then obviously less time will be devoted to human capital accumulation, and the growth of human capital will fall.

Finally, use equations (2.47) and (2.48) and the evolution of human capital equation for sector 1, equation (2.1), to rewrite equation (2.45) and obtain an equation for $\theta^B_{\mu_2}$ only in terms of $\theta^B_{h_1}$ and $\theta^B_{h_2}$:

$$\theta^B_{\mu_2} = \rho - \theta^B_{h_2} - \left(\frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} \right) \left[\frac{\phi\chi}{\phi+\chi} - \frac{\chi}{\phi+\chi}\theta^B_{h_1} - \frac{\phi}{\phi+\chi}\theta^B_{h_2}\right] \tag{2.49}$$

The analogous equation for sector 1 can be found by combining equation (2.49) with the no-arbitrage relationship across sectors, equation (2.13):

$$\theta_{\mu_1}^B = \rho - \theta_{h_1}^B - \left(\frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} \right) \left[\frac{\phi\chi}{\phi+\chi} - \frac{\chi}{\phi+\chi}\theta_{h_1}^B - \frac{\phi}{\phi+\chi}\theta_{h_2}^B \right] \tag{2.50}$$

Next, substitute equations (2.9), (2.26), (2.27), and (2.50) into the no-arbitrage condition across productive assets within sector 1, equation (2.41). This yields an expression for the growth rate of human capital in sector 1 in terms of the growth rate of human capital in sector 2:

$$\theta_{h_1}^B = \frac{\left(\frac{\phi\chi}{\phi+\chi} \frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} - \rho \right) - \left[\frac{\phi}{\phi+\chi} \frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} + (\alpha-1)\frac{\gamma_1^*}{\beta} \right] \theta_{h_2}^B}{\frac{\chi}{\phi+\chi} \frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} + (\alpha-1)\left(1+\frac{\gamma_1}{\beta}\right)} \tag{2.51}$$

An analogous analysis can be performed for sector 2, with the resulting equations:

$$\theta_{c_2}^B = \theta_{h_2}^B + \frac{\gamma_2}{\eta}\theta_{h_2}^B + \frac{\gamma_2^*}{\eta}\theta_{h_1}^B \tag{2.52}$$

$$\theta_{h_2}^B = \frac{\left(\frac{\phi\chi}{\phi+\chi} \frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} - \rho \right) - \left[\frac{\chi}{\phi+\chi} \frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} + (\alpha-1)\frac{\gamma_2^*}{\eta} \right] \theta_{h_1}^B}{\frac{\phi}{\phi+\chi} \frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} + (\alpha-1)\left(1+\frac{\gamma_2}{\eta}\right)} \tag{2.53}$$

We can now use equations (2.51) and (2.53) to solve for the growth of human capital in each sector, yielding the following proposition:

Proposition 2.2. Let $\alpha > 0$ *and* $\frac{\phi\chi}{\phi+\chi} > \rho$. *Also assume that* $1 + \frac{\gamma_2}{\eta} > \frac{\gamma_1^*}{\beta}$, $1 + \frac{\gamma_1}{\beta} > \frac{\gamma_2^*}{\eta}$, $\chi\left(1 + \frac{\gamma_2}{\eta}\right) > \phi\frac{\gamma_2^*}{\eta}$, and $\phi\left(1 + \frac{\gamma_1}{\beta}\right) > \chi\frac{\gamma_1^*}{\beta}$. *The growth rates of human capital are as follows:*

$$\theta_{h_1}^B(P) = \frac{\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}}{\alpha\left[\left(1 + \frac{\gamma_1}{\beta}\right)\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]} * \left(\frac{\phi\chi}{\phi+\chi}\frac{\left(1 + \frac{\gamma_1}{\beta}\right)\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1 + \frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} - \rho\right) > \theta_{h_1}^B(CE) \tag{2.54}$$

$$\theta_{h_2}^B(P) = \frac{\left(1 + \frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}}{\alpha\left[\left(1 + \frac{\gamma_1}{\beta}\right)\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]} * \left(\frac{\phi\chi}{\phi+\chi}\frac{\left(1 + \frac{\gamma_1}{\beta}\right)\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1 + \frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right] + \frac{\chi}{\phi+\chi}\left[\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]} - \rho\right) > \theta_{h_2}^B(CE) \tag{2.55}$$

$$\theta_{h_1}^B(P) - \theta_{h_1}^B(CE) = \frac{\left(1 + \frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}}{\left(1 + \frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}}\left[\theta_{h_2}^B(P) - \theta_{h_2}^B(CE)\right] \tag{2.56}$$

As shown in equations (2.54) and (2.55), the presence of a planner causes the growth rates of human capital in both sectors to increase. This gives evidence of a free-rider problem in the competitive case.

Additionally, according to equations (2.56), if spillovers into sector 2 play a greater role in sector 2 production than the role that spillovers play in sector 1 production (i.e. $\frac{\gamma_2}{\eta} + \frac{\gamma_2^*}{\eta} > \frac{\gamma_1}{\beta} + \frac{\gamma_1^*}{\beta}$), then the increase in human capital in sector 1 due to the planner is greater than the increase in human capital in sector 2. This is due to the consumption smoothing preference of agents. The planner recognizes that the spillovers will increase sector 2 production. Therefore, the planner increases time devoted to human capital accumulation in sector 1. This will allow sector 1 to experience similar productivity growth.

The following lemma describes that effect on the growth of human capital in each sector caused by changes in the magnitude of each spillover:

Lemma 2.4. Assume $\frac{\phi}{\phi+\chi}\frac{\phi\chi}{\phi+\chi} > \left(1+\frac{\gamma_2}{\eta}\right)\rho$. $\theta^B_{h_1}(P)$ *is decreasing in* γ_1 *and* γ_1^* *and increasing in* γ_2 *and* γ_2^*. *Assume* $\frac{\chi}{\phi+\chi}\frac{\phi\chi}{\phi+\chi} > \left(1+\frac{\gamma_1}{\beta}\right)\rho$. $\theta^B_{h_2}(P)$ *is decreasing in* γ_2 *and* γ_2^* *and increasing in* γ_1 *and* γ_1^*.

As in the competitive case, spillovers into a particular sector have a negative impact on human capital growth in that sector. Spillovers into the other sector have a positive impact on human capital growth. Since a planner is present, the negative impact on growth cannot be caused by free-riding. As discussed previously, it is due to the strong preference for consumption smoothing. From the outset, the planner considers the full effects of spillovers on production. The planner recognizes the positive effect of spillovers on a particular sector and allocates the time accordingly. This allows for the representative agent to be able to consume the same amount of each good.

The growth equations for human capital can be substituted into equations (2.27) and (2.29) to solve for the growth rates of consumption of each good. This yields the following proposition:

Proposition 2.3. Let $\alpha > 0$ *and* $\frac{\phi\chi}{\phi+\chi} > \rho$. Also assume that $1+\frac{\gamma_2}{\eta} > \frac{\gamma_1^*}{\beta}$,$1+\frac{\gamma_1}{\beta} > \frac{\gamma_2^*}{\eta}$, $\chi\left(1+\frac{\gamma_2}{\eta}\right) > \phi\frac{\gamma_2^*}{\eta}$, *and* $\phi\left(1+\frac{\gamma_1}{\beta}\right) > \chi\frac{\gamma_1^*}{\beta}$. *In this case, there exists a unique, balanced growth path and growth rates of consumption are positive. The growth rates of consumption are as follows:*

$$\begin{aligned}\theta^B_{c_1}(P) &= \theta^B_{c_2}(P) \\ &= \frac{1}{\alpha}\left(\frac{\phi\chi}{\phi+\chi}\frac{\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}{\frac{\phi}{\phi+\chi}\left[\left(1+\frac{\gamma_1}{\beta}\right)-\frac{\gamma_2^*}{\eta}\right]+\frac{\chi}{\phi+\chi}\left[\left(1+\frac{\gamma_2}{\eta}\right)-\frac{\gamma_1^*}{\beta}\right]}-\rho\right) \\ &> \theta^B_{c_1}(CE) = \theta^B_{c_2}(CE)\end{aligned} \tag{2.57}$$

In the planner's allocation, growth of consumption is higher than the competitive equilibrium, as shown in equation (2.57). This highlights the benefits of economic coordination through the elimination of free-riding. A social planner is not realistic, but improving upon the competitive equilibrium is possible.

The final step of our analysis is to consider the effect on the growth of consumption due to changes in the magnitude of each spillover:

Lemma 2.5. $\theta^B_{c1}(P) = \theta^B_{c2}(P)$ *is increasing in* γ_1, γ_2, γ_1^*, and γ_2^*.

All spillovers have a positive effect on the growth of consumption. As seen in Lemma 2.4, spillover effects on human capital growth are not always positive. However, the net effect is positive, as seen by the growth rates of consumption. This implies that when the growth of human capital in one sector falls in response to a change in spillovers, then the human capital in the other sector increases by a greater amount. As stated previously, consumers have a strong preference for consumption smoothing. When one of the spillovers increases, this directly benefits production in one of the sectors. In response, the planner determines the most optimal allocation of time across sectors in order to insure consumption growth of both goods.

2.4 Conclusion

The groundbreaking work of Lucas (1988) highlights the importance of uncompensated knowledge spillovers in economic development. Due to Lucas's work, a wave of research attempting to understand the source and impact of knowledge spillovers has been ignited. For example, beginning with arguments put forth by Marshall (1890), many economists have argued that specialization and learning within individual sectors of the economy are the most important for economic growth. On the other hand, more recent work by Jacobs (1969) and Glaeser et al. (1992) stresses that inter-industry spillovers are more beneficial. Regardless of the direction of such claims, there has been little work attempting to study the roles of intra-industry and inter-industry spillovers in the endogenous growth process. Consequently, this paper extends the original Lucas framework to include two sectors of production. Each sector produces a distinct good and agents have a preference for consumption of both goods.

Interestingly, intra-industry spillovers are always growth enhancing. In this manner, the model supports the initial claims by Marshall (1890). However, the effects of inter-industry spillovers may not be as beneficial as Jacobs suggested. That is, inter-industry spillovers introduce more complex interactions between the two sectors. In fact, inter-industry spillovers can lead to negative growth in the presence of significant asymmetries across sectors in the productivity of human capital accumulation. Moreover, the framework indicates that public policies should target sectors in which human capital investment is most productive.

CHAPTER 3

HETEROGENEOUS REGIONAL EXTERNAL ECONOMIES UNDER ASYMMETRIC COORDINATION

3.1 Introduction

This chapter seeks to study the development process among countries in an interconnected world. That is, it recognizes that economic activity extends beyond borders in ways that can profoundly influence economic growth across the world economy. Moreover, the world is composed of countries at very different stages of economic development.

It is hard to ignore the potential for such asymmetries to affect outcomes in very different ways. For example, Coe, Helpman, and Hoffmaister (1997) find that developing countries can achieve higher productivity growth by trading with countries in which there is more research and development activity. In addition, Miller and Upadhyay (2000) point out that countries which have more export orientation also tend to have higher rates of productivity.

In order to investigate how connections across countries affect the development process, our framework is built upon the Lucas (1988) model in which human capital is the "engine of growth." In particular, Lucas posits that *uncompensated spillovers of knowledge* have an important influence on the rate of economic growth within countries. However, the Lucas model looks at economic activity in complete isolation – there is a single representative agent making decisions in a closed economy.

While the closed-economy Lucas framework provides a useful benchmark, it fails to address how economic activity across interconnected economies affects growth rates across the world. Notably, Jaffe, Trajtenberg, and Henderson (1997) find evidence indicating that knowledge spillovers extend beyond borders. In addition, the extent of ideas transmission depends on the degree to which countries participate in international markets.

There have been previous attempts to extend the Lucas framework to regional settings involving multiple countries. For example, Holod and Reed (2004) look at a model which incorporates both local and regional external economies. However, each country is a mirror image of the other. That is, spillovers are the same in each country. By comparison, Holod and Reed (2009) study an asymmetric world in which there is variation in spillovers both within and across borders.

While Holod and Reed (2009) claim to study the development process in the presence of asymmetry, the asymmetries are limited to the *degree to which spillovers occur internally and regionally.* Such variation may be relevant to the development process across the world, but it fails to recognize that the productivity of the human capital sector varies in significant ways across countries. Consequently, the various states of the education sector are likely to affect the ability of countries to achieve higher rates of economic growth.

Moreover, it is also recognized that policymaking across countries varies in significant ways. For example, Easterly (1993) argues that there are significant distortions in resource allocation in developing countries which lead to relatively low rates of growth compared to the developed world. In particular, he states: "Although tax distortions are also significant in industrial countries [King and Fullerton (1984)], these examples suggest another order of magnitude for developing country distortions." This implies that asymmetries in the efficiency of investment policies also play a critical role in the variation of growth rates across the world.

In light of these issues, this paper constructs a human capital-based model of endogenous growth with two countries. In order to provide motivation for a setting in which countries are at different stages of the development process, we consider that the productivity of the human capital sector varies across countries. As in Holod and Reed (2009), the extent of regional external economies is asymmetric in each country. Notably, we find that differences in the productivity of the human capital sector play a significant role in the rates of economic growth – and, that regional external economies may distort incentives in each economy. In particular, if the growth rate of human capital accumulation is stronger abroad than domestically, the difference acts as a drag on human capital investment in the domestic economy. In fact, such differences might actually cause the stock of human capital to decline over time.

The paper proceeds by studying how variation in the efficiency of policymaking across countries plays an important role in the development process in the world economy. To do so, we consider that there is a planner in place in the domestic economy but not in the foreign country. Interestingly, the presence of a domestic planner improves human capital investment in the home country but can exacerbate inefficiencies in the foreign economy. As human capital accumulation improves with a domestic planner, there is a greater stock of knowledge upon which individuals in the foreign country can "free-ride." Thus, as spillovers of knowledge can take place regionally and flow across countries, so also can the effects of economic coordination. Only symmetric coordination – reflecting improved policymaking in both countries – will lead to greater human capital investment in the presence of heterogeneous regional external economies.

The remainder of the essay is as follows: Section 3.2 presents the benchmark model in which there are regional external economies but each country follows the competitive balanced growth path. Section 3.3 studies a setting in which there is asymmetric coordination in human capital investment across countries. That is, there is a planner in the domestic economy but not in the foreign country. Section 3.4 looks at economic outcomes in the presence of symmetric coordination. Section 3.5 provides some concluding remarks.

3.2 Benchmark Model

There are two countries: home and foreign. Throughout the paper, home country variables will be specified with the subscript "1" while foreign country variables will be denoted with the subscript "2." In each location, individuals are endowed with some positive amount of country-specific knowledge. Each agent in each country has the opportunity to either work or study. The time spent working is represented by $u_i(t)$ (where $i = 1, 2$).

If the agent chooses to study, then he will accumulate human capital, as described by the following evolution equations:

$$\dot{h}_1(t) = \phi(1 - u_1(t))h_1(t) \tag{3.1}$$

$$\dot{h}_2(t) = \chi(1 - u_2(t))h_2(t) \tag{3.2}$$

where h_i is the level of human capital of each individual. The growth rate of human capital in the home country depends on the parameter $\phi > 0$. In turn, $\chi > 0$ represents the growth rate of human capital in the foreign country if agents were to devote all of their time to human capital accumulation.

There are spillovers of human capital that occur both within and across countries. For example, γ_i represents the level of domestic human capital spillovers in country i. By comparison, γ_i^* reflects the extent that spillovers cross borders into country i. Both parameters are non-negative. The production functions for each country show that knowledge spillovers can add to productivity in each economy:

$$\begin{aligned} y_1(t) &= f_1(k_1(t), u_1(t), h_1(t), H_1(t), H_2(t)) \\ &= A\,(k_1(t))^{1-\beta}\,(u_1(t)h_1(t))^{\beta}\,(H_1(t))^{\gamma_1}\,(H_2(t))^{\gamma_1^*} \end{aligned} \tag{3.3}$$

$$\begin{aligned} y_2(t) &= f_2\,(k_2(t), u_2(t), h_2(t), H_1(t), H_2(t)) \\ &= B\,(k_2(t))^{1-\eta}\,(u_2(t)h_2(t))^{\eta}\,(H_2(t))^{\gamma_2}\,(H_1(t))^{\gamma_2^*} \end{aligned} \tag{3.4}$$

where the stock of human capital in each economy is denoted by $H_i(t)$. Let $0 < \beta, \eta < 1$.

Furthermore, agents have the following lifetime utility function in each country:

$$U_i(t) = \int_0^{\infty} e^{-\rho t} \left[\frac{(c_i(t))^{1-\alpha}}{1-\alpha} \right] dt \tag{3.5}$$

where $\frac{1}{\alpha} > 0$ is the intertemporal elasticity of substitution and $\rho > 0$ is the discount rate.

We will concentrate on presenting the decision making and results for human capital investment and economic growth in the home country. The problem of agents in the foreign country is analogous to the problem facing agents in the home country. In particular, agents in the home country solve the following:

$$\underset{c_1(t)}{Max} \int_0^{\infty} e^{-\rho t} \left[\frac{(c_1(t))^{1-\alpha}}{1-\alpha} \right] dt \tag{3.6}$$

subject to the evolution of physical capital in equation (3.7) below and the evolution of human capital as previously specified in equation (3.1):

$$\dot{k}_1(t) = A\left(k_1(t)\right)^{1-\beta} \left(u_1(t)\right)^{\beta} \left(h_1(t)\right)^{\beta} \left(H_1(t)\right)^{\gamma_1} \left(H_2(t)\right)^{\gamma_1^*} - \delta k_1(t) - c_1(t) \tag{3.7}$$

In order to solve the problem, we apply Pontryagin's maximum principle, which revolves around the following current-valued Hamiltonian:

$$\begin{aligned} H &= \frac{\left(c_1\left(t\right)\right)^{1-\alpha}}{1-\alpha} \\ &+\lambda_1(t)[A\left(k_1(t)\right)^{1-\beta} \left(h_1(t)\right)^{\beta} \left(H_1(t)\right)^{\gamma_1} \left(H_2(t)\right)^{\gamma_1^*} - \delta k_1(t) - c_1(t)] \\ &+\mu_1(t)[\phi(1-u_1(t))h_1(t)] \end{aligned} \tag{3.8}$$

where $\lambda_1\left(t\right)$ and $\mu_1\left(t\right)$ represent the co-state variables for physical and human capital.

First, agents choose how much to consume in each period:

$$\left(c_1\left(t\right)\right)^{-\alpha} = \lambda_1\left(t\right) \tag{3.9}$$

The first-order condition for consumption reflects a standard intertemporal consumption choice problem in the presence of physical capital investment. On the one hand, an increase in current consumption contemporaneously raises lifetime utility, depending on the marginal utility of consumption. On the other hand, an increase in current consumption pulls resources away from investment. The valuation from the loss of physical capital depends on the co-state variable, $\lambda_1\left(t\right)$.

Individuals must also decide how to allocate their time between the production and learning sectors which yields the following no-arbitrage condition:

$$\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta-1}\left(h_1(t)\right)^{\beta}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}=\mu_1(t)\phi h_1(t) \tag{3.10}$$

Equation (3.10) reflects that at the margin, the returns to physical capital and human capital must be the same. An increase in working time produces additional physical capital when the marginal valuation of physical capital is equal to $\lambda_1(t)$. The opportunity cost from time spent working involves the loss of human capital which has marginal valuation $\mu_1(t)$. Thus, the choice of working time reflects that the returns from accumulating more of either type of capital must be equal.

Moreover, it is noteworthy that the left-hand side of equation (3.10) is limited by diminishing marginal returns to production, but the right-hand side indicates that the marginal return to time devoted to human capital accumulation is independent of the amount of time already spent accumulating human capital. That is, the linearity of the human capital accumulation function, equation (3.1), indicates there are constant returns to human capital investment.

Equation (3.10) drives some of the main results of this paper. For example, if spillovers increase at home, productivity at home will increase. In turn, agents will shift time to the production sector until the marginal return from production is equal to the marginal return from learning. Stated differently, we see that agents have more incentive to produce as they can "free-ride" on the human capital stock at home. Similar comments apply to regional spillovers.

The Euler equations represent the capital gains or losses from physical and human capital over time:

$$\dot{\lambda}_1(t)=\lambda_1(t)(\rho-f_{k_1}(t)+\delta) \tag{3.11}$$

$$\begin{aligned}\dot{\mu}_1(t) &= \rho\mu_1(t)-\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta-1}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}\\ &\quad -\mu_1(t)\phi(1-u_1(t))\end{aligned} \tag{3.12}$$

Interestingly, the Euler equation for physical capital primarily depends on the marginal valuation of physical capital ($\lambda_1(t)$). This takes place because physical capital is only used in the production sector. However, the Euler equation for human capital is more complex. Though human capital is the sole input in the human capital sector, it is also used in the production sector. Consequently, the Euler equation depends on both co-state variables.

Yet, the no-arbitrage condition, equation (3.10), provides important information about the relative values of human capital and physical capital. Interestingly, it can be used to rewrite the Euler equation for human capital in terms of only the value of human capital:

$$\dot{\mu}_1(t) = \mu_1(t)\,(\rho - \phi) \tag{3.13}$$

With this background, we are now in a position to turn to the primary goal of the paper – to study the impact of regional external economies in the presence of asymmetry in the productivity of the education sector across countries. To do so, we will begin by converting all equations which depend on the levels of variables at each point in time to growth equations. We will then proceed to solve for the growth rate of each variable along the balanced growth path, where the growth rates of all variables must be constant. The growth rate of variable x along the BGP is represented by θ_x^B.

First, we write a new equation for the growth rate of consumption by combining the consumption choice condition, equation (3.9), with the Euler equation for physical capital, equation (3.11):

$$\theta_{c_1}^B = \frac{f_{k_1} - (\rho + \delta)}{\alpha} \tag{3.14}$$

Therefore, in order for $\theta_{c_1}^B$ to be constant, f_{k_1}must be constant. We next impose the condition that, in equilibrium, the average stock of knowledge within each country will be equal to the representative agent's human capital stock, explicitly: $H_i = h_i$. This yields:

$$\theta_{k_1}^B = \left(1 + \frac{\gamma_1}{\beta}\right)\theta_{h_1}^B + \frac{\gamma_1^*}{\beta}\theta_{h_2}^B \tag{3.15}$$

According to the evolution of physical capital, equation (3.7), physical capital can only be constant if the following condition holds:

$$\theta_{k_1}^B = \theta_{c_1}^B \tag{3.16}$$

Together, equations (3.15) and (3.16) form a new expression for the growth of consumption, now in terms of each country's growth rate of human capital:

$$\theta_{c_1}^B = \left(1 + \frac{\gamma_1}{\beta}\right)\theta_{h_1}^B + \frac{\gamma_1^*}{\beta}\theta_{h_2}^B \tag{3.17}$$

In the absence of human capital spillovers, the growth rate of consumption would simply be equal to the growth rate of human capital in the country where that good is produced. Consequently, human capital is the "engine of growth."

However, equation (3.17) also shows the direct impact of regional spillovers. The coefficients $\frac{\gamma_1}{\beta}$ and $\frac{\gamma_1^*}{\beta}$ can be understood by considering the production function of the home country, equation (3.3). Notably, they represent the productivity impact from each type of spillover relative to the role of an individual's human capital stock.

By substituting equation (3.9), the consumption choice equation; equation (3.13), the transformed Euler equation for human capital; and equations (3.16) and (3.17) into equation (3.10), the no-arbitrage condition, we obtain an equation for the growth rate of human capital in the home country in terms of the growth rate of human capital in the foreign country:[1]

$$\theta_{h_1}^B = \frac{(\phi - \rho) - (\alpha - 1)\frac{\gamma_1^*}{\beta}\theta_{h_2}^B}{(\alpha - 1)\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha - 1}\right)} \tag{3.18}$$

Interestingly, equation (3.18) demonstrates how the presence of regional external economies can distort incentives to invest in human capital across countries. As the regional external economies in the home country are stronger (γ_1^* is larger), the growth rate of human capital in the home country will be smaller, assuming $\alpha > 1$.

An analogous analysis can be performed for the foreign country with the resulting

[1]Note that along the BGP, the amount of time spent in each sector must remain constant, that is, $\theta_{u_1}^B = 0$.

equations:

$$\lambda_2(t)\eta B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta-1}\left(h_2(t)\right)^{\eta}\left(H_2(t)\right)^{\gamma_2}\left(H_1(t)\right)^{\gamma_2^*}=\mu_2(t)\chi h_2(t) \tag{3.19}$$

$$\begin{aligned}\dot{\mu}_2(t) &= \rho\mu_2(t)-\eta\lambda_2(t)B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta-1}\left(H_2(t)\right)^{\gamma_2}\left(H_1(t)\right)^{\gamma_2^*}\\ &\quad -\mu_2(t)\chi(1-u_2(t))\end{aligned} \tag{3.20}$$

$$\dot{\mu}_2(t)=\mu_2(t)\left(\rho-\chi\right) \tag{3.21}$$

$$\theta_{c_2}^B=\left(1+\frac{\gamma_2}{\eta}\right)\theta_{h_2}^B+\frac{\gamma_2^*}{\eta}\theta_{h_1}^B \tag{3.22}$$

$$\theta_{h_2}^B=\frac{(\chi-\rho)-(\alpha-1)\frac{\gamma_2^*}{\eta}\theta_{h_1}^B}{(\alpha-1)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)} \tag{3.23}$$

For the remainder of the essay, we impose the following: $\alpha>1$, $\phi>\rho$, and $\chi>\rho$. Also, $\frac{\gamma_1^*}{\beta}<\left(1+\frac{\gamma_2}{\eta}\right)$ and $\frac{\gamma_2^*}{\eta}<\left(1+\frac{\gamma_1}{\beta}\right)$. These assumptions ensure a unique BGP.

Finally, equations (3.18) and (3.23) can be used to solve for the growth rate of human capital in each country. This leads to the following proposition:

Proposition 3.1. The growth rates of human capital in each country are as follows:

$$\theta_{h_1}^B=\frac{\left[\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\right](\phi-\rho)-\frac{\gamma_1^*}{\beta}(\chi-\phi)}{(\alpha-1)\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]} \tag{3.24}$$

$$\theta_{h_2}^B=\frac{\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)-\frac{\gamma_2^*}{\eta}\right](\chi-\rho)-\frac{\gamma_2^*}{\eta}(\phi-\chi)}{(\alpha-1)\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]} \tag{3.25}$$

If the productivity of the education sector in each country is the same, equations (3.24) and (3.25) would be the same as Holod and Reed (2009). However, it is not reasonable to assume that education sectors across countries are identical. Consequently, our framework provides numerous insights regarding how asymmetry in human capital production plays an important role in the presence of regional variation in external economies. We elaborate on our insights immediately below.

To begin, equation (3.24) demonstrates that the efficiency of human capital investment in the home country is still the main determinant of human capital growth at home. However, asymmetries in the education sectors can also be quite important as observed by the term $-\frac{\gamma_1^*}{\beta}(\chi - \phi)$.

Interestingly, the term shows that the differential can act as a drag on human capital investment. If human capital productivity is larger in the foreign country than at home, then the incentives to invest in domestic human capital would be weaker. Consequently, regional variation in the productivity of the human capital sector can greatly affect human capital accumulation across countries.

We turn to the role of domestic and regional spillovers in the following lemma:

Lemma 3.1. Suppose that $\left(\frac{\gamma_2}{\eta} + \frac{\alpha}{\alpha-1}\right)(\phi - \rho) > \frac{\gamma_1^*}{\beta}(\chi - \rho)$ *and* $\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha-1}\right)(\chi - \rho) > \frac{\gamma_2^*}{\eta}(\phi - \rho)$. *Under these conditions,* $\theta_{h_1}^B$ *is positive and is decreasing in both* γ_1 *and* γ_1^*. *However, it is increasing in* γ_2 *and* γ_2^*. $\theta_{h_2}^B$ *is positive and is decreasing in* γ_2 *and* γ_2^*, *but increasing in* γ_1 *and* γ_1^*.

We begin by discussing the various assumptions imposed in Lemma 3.1. First, consider that the productivity of the educational sector is identical across countries, as in Holod and Reed (2009). In that case, the role of regional spillovers would be more straightforward – a sufficient condition for the first condition in Lemma 3.1 would only require that $\gamma_1^* < \beta$.

However, in the presence of regional variation in the productivity of the human capital sector, the condition is more complicated. In particular, if productivity of the education sector is stronger regionally than domestically, there is less support for the condition to hold. In fact, in such a setting, it would not apply if there are minimal spillovers within the foreign country.

To understand the intuition that drives Lemma 3.1, consider the production functions in each country. Spillovers into the home country augment home production. This allows agents in the home country to free-ride off of others' knowledge, thereby lowering their own incentive to acquire human capital. Therefore, the growth of human capital in the home country is negatively affected by home spillovers.

Similarly, foreign spillovers cause free-riding in the foreign country and negatively impact human capital growth in the foreign country. The loss in human capital growth in the foreign country is felt by the home country through the international spillover. The resulting decline in production growth raises the incentive for agents in the home country to accumulate human capital. As a result, greater foreign spillovers have a positive effect on the growth of human capital in the home country.

Lemma 3.1 gives further insight regarding equation (3.24). Specifically, the international spillover into the home country negatively affects human capital growth at home due to free-riding. If the foreign country is very efficient in human capital investment, then the incentive for home agents to free-ride off of foreign knowledge will be high. Thus, the negative effect on human capital growth of human capital spilling over from the foreign to the home country will be magnified.

Again, it must be recognized that the assumptions in Lemma 3.1 require relatively similar productivity in human capital accumulation across countries. Otherwise, the impact of regional variation will not be nearly as clear as suggested by Holod and Reed (2009). In this manner, our framework demonstrates that regional variation may greatly alter the way that human capital spillovers affect human capital investment in endogenous growth models.

To draw further insights, we turn to the model's implications for the growth rates of consumption. The growth equations for human capital can be substituted into equation (3.17) and the analogous equation for the foreign country to solve for the growth rates of the consumption in each country. These growth equations are stated in the following proposition:

Proposition 3.2. A unique, balanced growth path exists, and the growth rates of consumption in both countries are positive. The growth rates of consumption in each country are as follows:

$$\theta_{c_1}^B = \frac{\left[\left(1+\frac{\gamma_1}{\beta}\right)\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\left(\frac{\gamma_2^*}{\eta}-\frac{1}{\alpha-1}\right)\right](\phi-\rho)+\frac{1}{\alpha-1}\frac{\gamma_1^*}{\beta}(\chi-\phi)}{(\alpha-1)\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]} \tag{3.26}$$

$$\theta_{c_2}^B = \frac{\left[\left(1+\frac{\gamma_2}{\eta}\right)\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)-\left(\frac{\gamma_1^*}{\beta}-\frac{1}{\alpha-1}\right)\frac{\gamma_2^*}{\eta}\right](\chi-\rho)+\frac{1}{\alpha-1}\frac{\gamma_2^*}{\eta}(\phi-\chi)}{(\alpha-1)\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]} \tag{3.27}$$

As was the case for equations (3.24) and (3.25), we see that equations (3.26) and (3.27) contain an additional term based on the differential in the efficiency of human capital investment across countries. This term disappears in the case of symmetric efficiency in human capital investment.

According to equation (3.26), there is a strong positive correlation between the growth of consumption in the home country and the efficiency of human capital investment in the home country. In addition, the productivity of human capital investment in the foreign country positively affects home consumption through the home international spillover.

Proposition 3.2 confirms the important role of the efficiency of human capital investment within a country for the growth process. However, the proposition also documents the growth-enhancing effects of the efficiency of human capital investment in the other country. In order to promote economic growth, a nation should encourage domestic human capital accumulation. In addition, growth of consumption will be higher when nations interact with other countries that have highly productive educational sectors.

The following lemma describes the effects of each type of spillovers on the growth rates of consumption.

Lemma 3.2. Assume $\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)(\phi-\rho) > \frac{\gamma_1^*}{\beta}(\chi-\rho)$ *and* $\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)(\chi-\rho) > \frac{\gamma_2^*}{\eta}(\phi-\rho)$. $\theta_{c_1}^B$ *is increasing in* γ_1 *and* γ_1^* *and decreasing in* γ_2 *and* γ_2^*. $\theta_{c_2}^B$ *is increasing in* γ_2 *and* γ_2^* *and decreasing in* γ_1 *and* γ_1^*.

Lemma 3.2 requires the same assumptions as Lemma 3.1. As with the growth of human capital, under these conditions, the magnitude of the effect on consumption growth is different than in the case of symmetric productivity of human capital accumulation across countries. Moreover, the effects of regional spillovers will only be the same as the symmetric case in limited scenarios.

Growth in home country spillovers will cause a direct increase in domestic production. As noted in Lemma 3.1, home country spillovers will decrease the growth of home human capital and increase the growth of foreign human capital. However, the effect on home human capital is stronger. The negative effects that home spillovers have on human capital accumulation in the home country are overwhelmed by the positive effects on productivity. Therefore, the overall effect on consumption in the home country is positive. This result echoes the results of the original Lucas model.

Foreign country spillovers, on the other hand, have a negative effect on home consumption. The free-riding caused by these spillovers in the foreign country negatively impacts the home country through the international spillover. However, home country production receives no benefit from these spillovers. Hence, increased spillovers into the foreign country negatively impact consumption at home.

In contrast to the original closed-economy Lucas model in which knowledge spillovers were always found to be growth-enhancing, the open-economy framework in the presence of asymmetries leads to the possibility of negative growth resulting from some types of knowledge spillovers. As stated above, increased foreign country spillovers were found to have a negative effect on growth in the home country.

The effects of spillovers on consumption found in this article give interesting implications for government policies concerning information sharing and trade. Home spillovers are growth enhancing and should therefore be encouraged. For example, home domestic spillovers could be augmented through adjustments in patent laws or through enhanced labor mobility. Home international spillovers increase when a country becomes more export oriented or when immigration flows increase. In contrast, foreign spillovers decrease growth at home. So, then, if trade partnerships are made with countries that are very export-oriented or have extensive domestic information sharing, home growth could decrease. The findings of this article draw attention to the potential negative impacts of knowledge spillovers that did not exist in the original closed-economy Lucas model.

We will next consider an important special case. The following lemma describes the situation when one condition of Lemma 3.1 and Lemma 3.2 is not met:

Lemma 3.3. Assume $\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)(\phi-\rho) < \frac{\gamma_1^*}{\beta}(\chi-\rho)$[2]. $\theta_{h_1}^B$ *is negative and is decreasing in* γ_2^* *and* γ_1^* *and increasing in* γ_1 *and* γ_2. $\theta_{h_2}^B$ *is positive and is decreasing in* γ_1 *and* γ_2 *and increasing in* γ_2^* *and* γ_1^*. $\theta_{c_1}^B$ *is increasing in* γ_2^* *and* γ_1^* *and decreasing in* γ_1 *and* γ_2. $\theta_{c_2}^B$ *is increasing in* γ_1 *and* γ_2 *and decreasing in* γ_2^* *and* γ_1^*.

The assumption in Lemma 3.3 requires strong asymmetry in the efficiency of human capital investment across countries. The principal consequence of this assumption is negative human capital growth in the home country. This is a novel result. Previous models have documented the negative effects of some types of spillovers on the growth of human capital. However, no other model has suggested the possibility of a deterioration in the human capital stock over time. Note that even in this case, consumption growth is still positive in both countries.

In order for the assumption in Lemma 3.3 to hold, a number of conditions must be met simultaneously. These conditions describe a situation in which the home country has become extremely dependent on human capital spilling over from the foreign country, to the extent that the average human capital stock in the home country is actually shrinking over time.

The first condition is that the foreign country must be significantly more productive in human capital investment than the home country. Second, the spillover from the foreign country into the home country must be high. Together, these two conditions encourage home agents to depend on foreign human capital, rather than studying themselves.

Third, the spillover within the foreign country must be low, which will further encourage human capital accumulation in the foreign country. Fourth, the discount rate must be relatively high. This will encourage home agents to spend more time in the

[2]This assumption implies that $\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)-\frac{\gamma_2^*}{\eta}\right](\chi-\rho) > \frac{\gamma_2^*}{\eta}(\phi-\chi)$ must be true.

production sector, taking advantage of the spillovers from the foreign country, and less time accumulating their own human capital, which could enhance future consumption. Finally, $\alpha > 1$ is significantly large.

The decline in human capital growth in the home country will affect both countries. The average stock of human capital in the home country affects home production through the home domestic spillover (γ_1), and it affects foreign production through the foreign international spillover (γ_2^*). Negative human capital growth in the home country will cause the effects of these two spillovers on human capital and consumption to be opposite of the effects in the standard case of positive home human capital growth described in Lemma 3.2.

As an example, suppose that the home domestic spillover increases. When home human capital growth is negative, the increased spillover actually causes a decline in production growth at home. Lower production growth leads to lower consumption growth in the home country. Home agents then have a greater incentive to accumulate more human capital. Thus, the final effect of the increase in the home domestic spillover is an increase in the growth of home human capital.

Foreign human capital affects the foreign country through the foreign domestic spillover (γ_2) and the home country through the home international spillover (γ_1^*). Since foreign human capital growth is positive under the assumptions of Lemma 3.3, changes in these two spillovers have the same effects on human capital and consumption as the standard case in Lemma 3.2.

The following lemma describes a second special case:

Lemma 3.4. Assume $\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha-1}\right)(\chi - \rho) < \frac{\gamma_2^*}{\eta}(\phi - \rho)$.[3] $\theta_{h_1}^B$ *is positive and is decreasing in* γ_1 *and* γ_2 *and increasing in* γ_1^* *and* γ_2^*. $\theta_{h_2}^B$ *is negative and is decreasing in* γ_1^* *and* γ_2^* *and increasing in* γ_1 *and* γ_2. $\theta_{c_1}^B$ *is increasing in* γ_1 *and* γ_2 *and decreasing in* γ_1^* *and* γ_2^*. $\theta_{c_2}^B$ *is increasing in* γ_1^* *and* γ_2^* *and decreasing in* γ_1 *and* γ_2.

The assumptions of Lemma 3.4 lead to the special case in which foreign human capital

[3]This assumption implies that $\left[\left(\frac{\gamma_2}{\eta} + \frac{\alpha}{\alpha-1}\right) - \frac{\gamma_1^*}{\beta}\right](\phi - \rho) > \frac{\gamma_1^*}{\beta}(\chi - \phi)$ must be true.

growth is negative. The repercussions of a decline in foreign human capital growth are analogous to the results described in Lemma 3.3, caused by negative human capital growth at home.

3.3 Asymmetric Coordination

We will now consider a model of asymmetric coordination, that is, a social planner is introduced in the home country, but the foreign country will remain competitive. The analysis will focus on the changes within the home country caused by a social planner.

In order to solve the problem, we apply Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$\begin{aligned} H &= \frac{(c_1(t))^{1-\alpha}}{1-\alpha} \\ &\quad +\lambda_1(t)\left[A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta+\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}-\delta k_1(t)-c_1(t)\right] \\ &\quad +\mu_1\left(t\right)\left[\phi(1-u_1(t))h_1(t)\right] \end{aligned} \tag{3.28}$$

The social planner will decide how to allocate time between the production and learning sectors, yielding the following no-arbitrage condition:

$$\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta-1}\left(h_1(t)\right)^{\beta+\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}=\mu_1(t)\phi h_1(t) \tag{3.29}$$

The human capital Euler equation represents the capital gains or losses from human capital over time:

$$\begin{aligned} \dot{\mu}_1(t) &= \rho\mu_1(t)-\lambda_1(t)A\left(\beta+\gamma_1\right)\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta+\gamma_1-1}\left(H_2(t)\right)^{\gamma_1^*} \\ &\quad -\mu_1(t)\phi(1-u_1(t)) \end{aligned} \tag{3.30}$$

As in the competitive model, the no-arbitrage condition, equation (3.29), provides a relationship between the values of physical and human capital, which can be utilized to transform the Euler equation for human capital into functions of only the value of human capital. The resulting equation can then be further transformed using the evolution of human capital, equation (3.1). This leads to the following equation for the growth of the human capital co-state variable:

$$\theta_{\mu_1}^B = \rho - \left(1 + \frac{\gamma_1}{\beta}\right)\phi + \frac{\gamma_1}{\beta}\theta_{h_1} \tag{3.31}$$

By substituting equation (3.9), the consumption choice equation; equation (3.31), the transformed Euler equation for human capital; and equations (3.16) and (3.17) into equation (3.29), the no-arbitrage condition, we obtain an equation for the growth rate of human capital in the home country in terms of the growth rate of human capital in the foreign country:

$$\theta_{h_1} = \frac{(\phi - \rho) + \phi\frac{\gamma_1}{\beta} - (\alpha - 1)\frac{\gamma_1^*}{\beta}\theta_{h_2}^B}{(\alpha - 1)\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha - 1}\right) + \frac{\gamma_1}{\beta}} \tag{3.32}$$

As on the competitive balanced growth path, regional external economies will continue to distort human capital investment in the presence of a domestic planner. This takes place because the domestic planner does not internalize the impact of the domestic human capital stock on investment in the foreign economy. Therefore, the distortions from regional external economies remain.

To solve for the growth rates of human capital in each country, we need the foreign analog to equation (3.32), that is, the growth rate of foreign country human capital in terms of the growth rate of home country human capital. The foreign country remains competitive. Therefore equation (3.23), which was found in the competitive model, will still hold under asymmetric coordination. So then, equations (3.23) and (3.32) can be combined to find the growth rates of human capital in each country. These growth rates are presented in the following proposition:

Proposition 3.3. The growth rates of human capital in the presence of a domestic planner are as follows:

$$\theta_{h_1}^B(P_1) = \frac{\left(\frac{\gamma_2}{\eta} + \frac{a}{\alpha-1}\right)\left[\left(1 + \frac{\gamma_1}{\beta}\right)\phi - \rho\right] - \frac{\gamma_1^*}{\beta}(\chi - \rho)}{(\alpha - 1)\left[\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta} + \frac{a}{\alpha-1}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right] + \frac{\gamma_1}{\beta}\left(\frac{\gamma_2}{\eta} + \frac{a}{\alpha-1}\right)} > \theta_{h_1}^B(CE) \tag{3.33}$$

$$\theta^B_{h_2}(P_1) = \frac{\left\{\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha-1}\right) + \frac{1}{\alpha-1}\frac{\gamma_1}{\beta}\right\}(\chi - \rho) - \frac{\gamma_2^*}{\eta}\left[\left(1 + \frac{\gamma_1}{\beta}\right)\phi - \rho\right]}{(\alpha - 1)\left[\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta} + \frac{a}{\alpha-1}\right) - \frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right] + \frac{\gamma_1}{\beta}\left(\frac{\gamma_2}{\eta} + \frac{a}{\alpha-1}\right)} < \theta^B_{h_2}(CE) \quad (3.34)$$

$$\left(\theta^B_{h_1}(P_1) - \theta^B_{h_1}(CE)\right) = \frac{\frac{\gamma_2}{\eta} + \frac{\alpha}{\alpha-1}}{\frac{\gamma_2^*}{\eta}}\left(\theta^B_{h_2}(CE) - \theta^B_{h_2}(P_1)\right) \quad (3.35)$$

According to equation (3.33), coordination within the home country leads to a higher growth rate of human capital in the home country, relative to the competitive case. The coefficient $\frac{\gamma_1}{\beta}$ augments the positive effect of the efficiency of human capital investment in the home country. The planner considers the full benefit to production from the human capital stock at home, which is felt through the home domestic spillover.

Free-riding on the domestic human capital stock is therefore eliminated in the home country. The planner allocates more time to human capital investment, leading to an increase in the growth of human capital in the home country. Moreover, the higher the level of productivity in the human capital sector in the home country, the greater the gains in human capital accumulation over time.

In contrast, equation (3.34) reveals that coordination within the home country lowers the growth rate of human capital in the foreign country, relative to the competitive case. The decline is directly correlated with the magnitude of $\frac{\gamma_1}{\beta}$. The positive effects, due to coordination, on human capital growth in the home country cause an increase in production in the foreign country through the foreign international spillover. This lowers the incentive for foreign agents to invest in human capital. In response, the growth rate of human capital in the foreign country will fall.

Equation (3.35) provides a relationship between the change in the growth of human capital that each country experiences as a result of coordination in the home country. The following lemma explores this relationship:

Lemma 3.5. Suppose that $\left(\frac{\gamma_2}{\eta} + \frac{\alpha}{\alpha-1}\right) > \frac{\gamma_2^*}{\eta}$. *Relative to the competitive case, the increase in the growth of human capital in the home country from coordination in the home country is greater than the decrease in the growth of human capital in the foreign country. Consequently, domestic coordination will lead to a higher stock of knowledge* *across the world. However, if* $\left(\frac{\gamma_2}{\eta} + \frac{\alpha}{\alpha-1}\right) < \frac{\gamma_2^*}{\eta}$, *the opposite will occur.*

As described above, coordination within the home country ultimately affects the growth rate of human capital in both countries. The primary consequence of the presence of a home national planner is an increase in the growth rate of human capital at home. The secondary effect on the foreign country is felt through the foreign international spillover.

It is interesting to note that the conditions in Lemma 3.5 primarily depend on conditions in the foreign country. Since $\alpha > 1$, a sufficient condition for the first condition in Lemma 3.5 is that domestic spillovers are larger than regional spillovers. This seems to be the most plausible scenario. In that case, the gains from domestic coordination outweigh the losses from human capital accumulation abroad. Consequently, domestic coordination positively contributes to the world stock of knowledge.

However, if the foreign international spillover is extremely large, then the second condition in Lemma 3.5 will be met, and the opposite will be true. In this case, the foreign international spillover plays the most important role in foreign production – larger than the combined role of the foreign domestic spillover and individual human capital. In turn, the increase in the growth rate of human capital in the home country will have a profound impact on foreign production. Foreign agents will shift time towards investment in production, to the extent the decline in human growth in the foreign country will be greater than the original increase in human capital growth in the home country. Therefore, the stock of knowledge in the world economy will suffer.

To continue our analysis, we will next consider the effects of changes in each of the spillovers on the growth rates of human capital in both countries. The following lemma summarizes these changes:

Lemma 3.6. Suppose $\left(\frac{\gamma_2}{\eta} + \frac{a}{\alpha-1}\right)\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)\right] > \frac{\gamma_1^*}{\beta}(\chi-\rho)$. *In addition, let* $\left[\left(\frac{\gamma_1}{\beta} + \frac{\alpha}{\alpha-1}\right) + \frac{1}{\alpha-1}\frac{\gamma_1}{\beta}\right](\chi-\rho) > \frac{\gamma_2^*}{\eta}\left[\left(1+\frac{\gamma_1}{\beta}\right)\phi-\rho\right]$. *Under these conditions,* $\theta_{h_1}^B$ *is positive but it decreasing in* γ_1^* *. However, it is increasing in* γ_1, γ_2, *and* γ_2^*. $\theta_{h_2}^B$ *is positive and increasing* γ_1^* *but it is decreasing in* γ_1, γ_2, *and* γ_2^*.

Lemma 3.6 can best be understood through comparison to Lemma 3.1. Lemma 3.1 states the effect of spillovers on the growth of human capital in the competitive case. Yet, the effect of the home domestic spillover on the growth of human capital in the partially coordinated framework is different than in the competitive equilibrium. While domestic spillovers weaken human capital accumulation at home in the competitive case, they lead to an increase in the stock of human capital in the presence of a domestic planner.

It is not surprising that coordination in the home country alters the effects of the home domestic spillover. As stated previously, the principal feature of the national planner is the internalization of domestic spillovers. Therefore, an increase in this spillover causes the planner to allocate more time to human capital accumulation, resulting in an increase in the growth of human capital. Effectively, coordination eliminates the free-rider problem associated with domestic spillovers in the home country.

Due to coordination in the home country, the home domestic spillover positively affects home human capital growth. The increase in the growth of home human capital benefits foreign production through the foreign international spillover. This, in turn, causes additional free-riding in the foreign country and lowers the growth of human capital in the foreign country. So, an increase in the home domestic spillover causes the growth of human capital in the foreign country to decrease.

For the three remaining spillovers, the effects are the same as in the competitive equilibrium. Specifically, the international spillover into the home country allows home agents to free-ride off of the foreign human capital stock and therefore has a negative effect on the growth of human capital. Similarly, spillovers within and into the foreign country cause free-riding in the foreign country. The loss in human capital growth abroad hurts home production. This ultimately leads to an increase in the growth of human capital at home.

Finally, substitute the human capital growth equations, equations (3.33) and (3.34), into the equations for consumption growth in terms of human capital growth, equations (3.17) and (3.22), in order to obtain the final equations for the growth of consumption in each country:

Proposition 3.4. A unique, balanced growth path exists, and the growth rates of consumption for all goods are positive. The growth rates of consumption in each country are as follows:

$$\begin{aligned}\theta_{c_1}^{B}(P_1) &= \frac{\begin{array}{c}\left[\left(1+\frac{\gamma_1}{\beta}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\left(\frac{\gamma_2^*}{\eta}-\frac{1}{\alpha-1}\right)\right]\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)\right]\\ +\frac{1}{\alpha-1}\frac{\gamma_1^*}{\beta}\left\{(\chi-\phi)\left(1+\frac{\gamma_1}{\beta}\right)-\rho\frac{\gamma_1}{\beta}\right\}\end{array}}{(\alpha-1)\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]+\frac{\gamma_1}{\beta}\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)}\\ &> \theta_{c_1}^{B}(CE)\end{aligned} \tag{3.36}$$

$$\begin{aligned}\theta_{c_2}^{B}(P_1) &= \frac{\begin{array}{c}\left\{\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(1+\frac{\gamma_2}{\eta}\right)-\left(\frac{\gamma_1^*}{\beta}-\frac{1}{\alpha-1}\right)\frac{\gamma_2^*}{\eta}\right]+\frac{1}{\alpha-1}\frac{\gamma_1}{\beta}\left(1+\frac{\gamma_2}{\eta}\right)\right\}(\chi-\rho)\\ +\frac{1}{\alpha-1}\frac{\gamma_2^*}{\eta}\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\chi)\right]\end{array}}{(\alpha-1)\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right]+\frac{\gamma_1}{\beta}\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)}\\ &> \theta_{c_2}^{B}(CE)\end{aligned} \tag{3.37}$$

$$\left[\theta_{c_1}^{B}(P_1)-\theta_{c_1}^{B}(CE)\right]=\frac{\frac{1}{\alpha-1}\frac{\gamma_2^*}{\eta}}{\left(\frac{\gamma_1}{\beta}+1\right)\left(\frac{\gamma_2}{\eta}+\frac{a}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}}\left[\theta_{c_2}^{B}(P_1)-\theta_{c_2}^{B}(CE)\right] \tag{3.38}$$

Equations (3.36) and (3.37) reveal that consumption in both countries increases in the presence of a planner in the home country. This result serves as evidence of the free-rider problem that exists in the competitive equilibrium. The increase in consumption also draws attention to the potential benefits of economic coordination.

As was the case for equations (3.33) and (3.34), we see that the term $\frac{\gamma_1}{\beta}$ plays an important role in equations (3.36) and (3.37). The home country national planner takes into account the home domestic spillover. That is, he or she considers the full external benefit to production of investment in human capital. So then, $\frac{\gamma_1}{\beta}$, the role of the home domestic spillover in production in the home country, has a strong positive effect on consumption in both countries.

The primary purpose of encouraging coordination within a country is to increase that nation's economic growth. However, as shown in Proposition 3.4, coordination within one country increases growth in other countries as well. Therefore, when a nation seeks to increase growth, it should consider not only the level of coordination in its own country, but also the domestic coordination of its trade partners.

The relative benefit to each country can be further understood by an examination of equation (3.38). According to the assumptions of Proposition 3.4, this equation shows that the increase in consumption is greater for the home country than for the foreign country. This result is not surprising, since the benefits to the foreign country from coordination in the home country are indirect through the foreign international spillover.

Next, the effects of changes in each of the spillovers on the growth rates of consumptions will be considered. These effects are presented in the following lemma:

Lemma 3.7. Suppose that $\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)\right] > \frac{\gamma_1^*}{\beta}(\chi-\rho)$*. In addition, let* $\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)+\frac{1}{\alpha-1}\frac{\gamma_1}{\beta}\right](\chi-\rho) > \frac{\gamma_2^*}{\eta}\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)\right]$*. Under these conditions,* $\theta_{c_1}^B$ *is increasing in* γ_1 *and* γ_1^**, but is decreasing in* γ_2 *and* γ_2^**. On the other hand,* $\theta_{c_2}^B$ *is increasing in* γ_1*,* γ_2*, and* γ_2^* *but is decreasing in* γ_1^**.*

First, note that Lemma 3.7 requires the same assumptions as Lemma 3.6. The spillover effects in Lemma 3.7 will be compared to those found in Lemma 3.2. Relative to the competitive balanced growth path, only the effects of the home domestic spillover are altered (in terms of sign) by the presence of a national planner in the home country.

In the competitive case, the home domestic spillover positively affects home production. Free-riding then leads to lower growth of human capital at home. In the home country, the original productivity increase overwhelms the resulting loss in human capital growth. Therefore, the final effect on consumption in the home country is positive. In contrast, foreign production is not aided by the home domestic spillover. The foreign country does, however, feel the loss in home human capital growth through the foreign international spillover. Thus, the effect on consumption in the foreign country is negative.

Through the elimination of free-riding, coordination allows the positive effect of the home domestic spillover to spread to production in both countries. Therefore, and in contrast to the competitive equilibrium, the home domestic spillover positively impacts consumption in both countries.

To complete the analysis of growth rates under partial coordination, a few special cases will be considered briefly. The following lemma will present the outcome if the conditions in Lemma 3.6 and 3.7 are broken:

Lemma 3.8. Let $\left[\left(\frac{\gamma_1}{\beta}+\frac{\alpha}{\alpha-1}\right)+\frac{1}{\alpha-1}\frac{\gamma_1}{\beta}\right](\chi-\rho)<\left[\frac{\gamma_2^*}{\eta}\left(1+\frac{\gamma_1}{\beta}\right)\right](\phi-\rho)$*. As a result,* $\theta_{h_1}^B$ *is positive but* $\theta_{h_2}^B$ *is negative. Alternatively,* $\left[\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)-\frac{\gamma_1^*}{\beta}\right]\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)\right]$ $<\frac{\gamma_1^*}{\beta}\left(\chi-\left(1+\frac{\gamma_1}{\beta}\right)\phi\right)$*. If this condition holds,* $\theta_{h_1}^B$ *is negative and* $\theta_{h_2}^B$ *is positive.*

Interestingly, the first part of Lemma 3.8 provides conditions in which human capital in the foreign country can fall over time in the presence of a domestic planner. As a similar claim was made in Lemma 3.4 about such possibilities along the competitive balanced growth path, we concentrate on making comparisons to Lemma 3.4. First, the left-hand side of the first condition in Lemma 3.8 indicates that the left-hand side is larger than in Lemma 3.4. This suggests it is less likely that the stock of human capital will decline over time in the foreign country in the presence of a domestic planner. However, the right-hand side is also larger, depending on the extent of domestic spillovers. Coordination on human capital investment stimulates human capital accumulation domestically. If the productivity of the education sector in the foreign country is weak and individuals can rely on more productive spillovers from the domestic economy, human capital accumulation may decline over time.

As one would expect, the conditions in which human capital would decline over time at home are much harder to satisfy in the case of a domestic planner. Recall from Lemma 3.3 that the parameter space would need to satisfy $\left(\frac{\gamma_2}{\eta}+\frac{\alpha}{\alpha-1}\right)(\phi-\rho)<\frac{\gamma_1^*}{\beta}(\chi-\rho)$. In other words, along the competitive balanced growth path, all that would be necessary is for the productivity of the foreign education sector to be relatively high and for there to be strong regional external economies into the home country. Moreover, the extent of domestic spillovers (γ_1) would be irrelevant. However, in the case of a domestic planner, the condition is much more involved and depends on γ_1. Interestingly, the higher the value of γ_1, the smaller the parameter space required to satisfy the second condition in Lemma 3.8, indicating that there is less support for negative growth of human capital.

3.4 Symmetric Coordination

We will next consider the balanced growth path for the world economy when a social planner is present in each country. To begin, we concentrate on the allocation imposed by a benevolent planner in the foreign country. In order to solve the problem, we apply Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$\begin{aligned} H &= \frac{(c_2(t))^{1-\alpha}}{1-\alpha} \\ &+\lambda_2(t)\left[B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta+\gamma_2}\left(h_1(t)\right)^{\gamma_2^*}-\delta k_2(t)-c_2(t)\right] \\ &+\mu_2\left(t\right)\left[\chi(1-u_2(t))h_2(t)\right] \end{aligned} \tag{3.39}$$

where $\lambda_2\left(t\right)$ and $\mu_2\left(t\right)$ represent the co-state variables for physical and human capital.

The planner in the foreign country must also decide how to allocate time between the production and learning sectors, which yields the following no-arbitrage condition:

$$\lambda_2(t)\eta B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta-1}\left(h_2(t)\right)^{\eta+\gamma_2}\left(h_1(t)\right)^{\gamma_2^*}=\mu_2(t)\chi h_2(t) \tag{3.40}$$

The Euler equation for human capital represents the capital gains or losses from human capital over time:

$$\begin{aligned} \dot{\mu}_2(t) &= \rho\mu_2(t)-\left(\eta+\gamma_2\right)\lambda_2(t)B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta+\gamma_2-1}\left(h_1(t)\right)^{\gamma_2^*} \\ &-\mu_2(t)\chi(1-u_2(t)) \end{aligned} \tag{3.41}$$

The no-arbitrage condition, equation (3.40), gives a relationship between the value of physical and human capital, and so can be used to rewrite the Euler equation for human capital in terms of only the value of human capital. The Euler equation can be further modified using the evolution of human capital equation for the foreign country, with the resulting growth rate of the human capital co-state variable:

$$\theta_{\mu_2}^B=\rho-\chi\left(1+\frac{\gamma_2}{\eta}\right)+\frac{\gamma_2}{\eta}\theta_{h_2}^B \tag{3.42}$$

By substituting equation (3.9), the consumption choice equation; equation (3.42), the transformed Euler equation for human capital; and the foreign analogs of equations (3.16) and (3.17) into equation (3.40), the no-arbitrage condition, we obtain an equation for the growth rate of human capital in the foreign country in terms of the growth rate of human capital in the home country:

$$\theta_{h_2}^{B} = \frac{\left(1 + \frac{\gamma_2}{\eta}\right)\chi - \rho - (\alpha - 1)\frac{\gamma_2^*}{\eta}\theta_{h_1}^{B}}{\alpha\left(1 + \frac{\gamma_2}{\eta}\right)} \tag{3.43}$$

Equation (3.43) can then be combined with the analogous equation for the case of a planner in the home country, equation (3.18), to solve for the growth rate of human capital in each country. This leads to the following proposition:

Proposition 3.5. The growth rates of human capital across the world are as follows:

$$\theta_{h_1}^{B}(P) = \frac{\alpha\left(1 + \frac{\gamma_2}{\eta}\right)\left[\left(1 + \frac{\gamma_1}{\beta}\right)\phi - \rho\right] - (\alpha - 1)\frac{\gamma_1^*}{\beta}\left[\left(1 + \frac{\gamma_2}{\eta}\right)\chi - \rho\right]}{\alpha\left(1 + \frac{\gamma_1}{\beta}\right)\alpha\left(1 + \frac{\gamma_2}{\eta}\right) - (\alpha - 1)^2\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}} < \theta_{h_1}^{B}(P_1) \tag{3.44}$$

$$\theta_{h_2}^{B}(P) = \frac{\alpha\left(1 + \frac{\gamma_1}{\beta}\right)\left[\left(1 + \frac{\gamma_2}{\eta}\right)\chi - \rho\right] - (\alpha - 1)\frac{\gamma_2^*}{\eta}\left[\left(1 + \frac{\gamma_1}{\beta}\right)\phi - \rho\right]}{\alpha^2\left(1 + \frac{\gamma_1}{\beta}\right)\left(1 + \frac{\gamma_2}{\eta}\right) - (\alpha - 1)^2\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}} > \theta_{h_2}^{B}(P_1) \tag{3.45}$$

$$\theta_{h_1}^{B}(P_1) - \theta_{h_1}^{B}(P) = \frac{(\alpha - 1)\frac{\gamma_1^*}{\beta}}{\alpha\left(1 + \frac{\gamma_1}{\beta}\right)}\left(\theta_{h_2}^{B}(P) - \theta_{h_2}^{B}(P_1)\right) \tag{3.46}$$

Equations (3.44) and (3.45) demonstrate that coordination within both countries will lead to a lower growth rate of human capital in the home country and a higher growth rate of human capital in the foreign country, relative to the case of asymmetric coordination. The term $\frac{\gamma_2}{\eta}$ plays an important role in each of these equations, magnifying the affect of the efficiency of human capital investment in the foreign country. This reflects the fact that the foreign national planner considers fully the external impact of the foreign domestic spillover on production. Furthermore, as $\frac{\gamma_2}{\eta}$ increases, the growth rate of human capital will decline in the home country and rise in the foreign country.

As stated in the previous paragraph, when the case of symmetric coordination is compared to the case of asymmetric coordination, we see that human capital growth is lower in the home country and higher in the foreign country. Equation (3.46) presents a relationship between the change in human capital in each country. This relationship is used to produce the following lemma:

Lemma 3.9. Let $(\alpha-1)\frac{\gamma_1^*}{\beta} < \alpha\left(1+\frac{\gamma_1}{\beta}\right)$. *Relative to the case of asymmetric coordination, the increase in the growth of human capital in the foreign country due to coordination in both countries is greater than the decrease in the growth of human capital in the home country.*

The introduction of a national planner in the foreign country has an effect on human capital growth in both countries. However, the primary effect is an increase in human capital growth in the foreign country. A higher human capital stock in the foreign country benefits production in the home country through the home domestic spillover. This ultimately causes free-riding and a loss in growth of human capital in the home country. This secondary effect on the growth of home country human capital is understandably of a lower magnitude than the original positive effect on the growth of foreign human capital.

In order to gain further understanding of the growth rates of human capital, we will next examine the effects of changes in each of the spillovers on the growth rates of human capital in each country. The following lemma summarizes these results:

Lemma 3.10. Suppose $\alpha\left(1+\frac{\gamma_2}{\eta}\right)\left[\left(1+\frac{\gamma_1}{\beta}\right)\phi-\rho\right] > (\alpha-1)\frac{\gamma_1^*}{\beta}\left[\left(1+\frac{\gamma_2}{\eta}\right)\chi-\rho\right]$. *Also, let* $\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left[\left(1+\frac{\gamma_2}{\eta}\right)(\chi-\rho)\right] > (\alpha-1)\frac{\gamma_2^*}{\eta}\left[\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)\right]$. $\theta_{h_1}^B$ *is positive and decreasing in* γ_1^* *and* γ_2, *but is increasing in* γ_1 *and* γ_2^*. $\theta_{h_2}^B$ *is positive and decreasing in* γ_1 *and* γ_2^*, *but is increasing in* γ_1^* and γ_2.

In order to understand the significance of the results of Lemma 3.10, we will use Lemma 3.6 as a point of comparison. Under the current framework of symmetric coordination, only the effects of the foreign domestic spillover are distinct (in terms of sign), relative to the case of partial coordination. The national planner considers fully the external benefits through the foreign domestic spillover of investment in human capital.

Therefore, the introduction of a national planner in the foreign country transforms the influence that the foreign domestic spillover has on the growth rates of human capital.

To be specific, when the foreign domestic spillover increases, the foreign national planner recognizes the benefit of this spillover for production. Therefore, the planner chooses to invest more heavily in human capital, rather than physical capital. Hence, the growth rate of human capital in the foreign country increases. This decision allows the planner to fully exploit the benefit of the domestic spillover.

The choices of the foreign planner, and the resulting increase in the growth of foreign country human capital, benefit production in the home country through the home international spillover. Effectively, coordination in the foreign country provides home country agents a larger foreign human capital stock from which they can free-ride. Therefore, the growth rate of human capital in the home country declines.

Finally, substitute the human capital growth equations, equations (3.44) and (3.45), into the equations for consumption growth in terms of human capital growth, equations (3.17) and (3.22), in order to obtain the final equations for growth of consumption in each country:

Proposition 3.6. A unique, balanced growth path exists, and the growth rates of consumption for all goods are positive. The growth rates of consumption in each country are as follows:

$$\begin{aligned}\theta_{c_1}^{B}(P) &= \frac{\begin{array}{c}\left\{\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)-(\alpha-1)\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right\}\left[\left(1+\frac{\gamma_1}{\beta}\right)\phi-\rho\right]\\ +\left(1+\frac{\gamma_1}{\beta}\right)\frac{\gamma_1^*}{\beta}\left[\left(1+\frac{\gamma_2}{\eta}\right)\chi-\rho\right]\end{array}}{\alpha^2\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)-(\alpha-1)^2\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}} \\ &> \theta_{c_1}^{B}(P_1) > \theta_{c_1}^{B}(CE) \end{aligned} \tag{3.47}$$

$$\begin{aligned}\theta_{c_2}^{B}(P) &= \frac{\begin{array}{c}\left\{\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)-(\alpha-1)\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}\right\}\left[\left(1+\frac{\gamma_2}{\eta}\right)\chi-\rho\right]\\ +\left(1+\frac{\gamma_2}{\eta}\right)\frac{\gamma_2^*}{\eta}\left[\left(1+\frac{\gamma_1}{\beta}\right)\phi-\rho\right]\end{array}}{\alpha^2\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)-(\alpha-1)^2\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}} \\ &> \theta_{c_2}^{B}(P_1) > \theta_{c_2}^{B}(CE) \end{aligned} \tag{3.48}$$

$$\left[\theta^B_{c_1}(P) - \theta^B_{c_1}(P_1)\right] = \frac{\frac{\gamma_1^*}{\beta}\left(1+\frac{\gamma_1}{\beta}\right)}{\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - (\alpha-1)\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta}} \left[\theta^B_{c_2}(P) - \theta^B_{c_2}(P_1)\right] \quad (3.49)$$

According to equations (3.47) and (3.48), consumption in both countries increases even more in the presence of a planner in both countries, relative to the case of partial coordination. The term $\frac{\gamma_2}{\eta}$ plays an important role in equations (3.47) and (3.48). Recall that this was also the case for equations (3.44) and (3.45). The foreign national planner fully considers external benefits of human capital investment for production. Therefore, the foreign domestic spillover has a strong effect on the growth rates of consumption.

The higher economic growth under symmetric coordination is particularly interesting in the home country. From the case of asymmetric coordination, we learn that a national planner within the home country will increase consumption in the home country. The case of symmetric coordination, however, results in an even higher growth rate of consumption for the home country. To understand the implications of these results, consider a country that is economically coordinated. That nation can experience further growth benefits through interactions with countries that are also economically coordinated.

Equation (3.49) facilitates a comparison between the increase in consumption growth in each country. Given the assumptions of Proposition 3.6, it can be shown that the increase in consumption, due to the introduction of a planner in the foreign country, is greater for the foreign country than for the home country.[4] The foreign national planner directly increases human capital growth, production, and consumption in the foreign country. The effect on the home country, however, is indirect and felt through the home international spillover.

Our analysis of the growth rates of consumption under symmetric coordination will continue with an examination of the effects of changes in each of the spillovers on the growth rates of consumption. The following lemma summarizes these results:

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[4] $\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - (\alpha-1)\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta} > \frac{\gamma_1^*}{\beta}\left(1+\frac{\gamma_1}{\beta}\right)$

if: $\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right) - (\alpha-1)\frac{\gamma_1^*}{\beta}\frac{\gamma_2^*}{\eta} - (\alpha-(\alpha-1))\frac{\gamma_1^*}{\beta}\left(1+\frac{\gamma_1}{\beta}\right) > 0$

$\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right] - (\alpha-1)\frac{\gamma_1^*}{\beta}\left[\frac{\gamma_2^*}{\eta} - \left(1+\frac{\gamma_1}{\beta}\right)\right] > 0$

Lemma 3.11. Assume $\alpha\left(1+\frac{\gamma_2}{\eta}\right)\left[\left(1+\frac{\gamma_1}{\beta}\right)\phi-\rho\right] > (\alpha-1)\frac{\gamma_1^*}{\beta}\left[\left(1+\frac{\gamma_2}{\eta}\right)\chi-\rho\right]$. *In addition,* $\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)(\chi-\rho) > (\alpha-1)\frac{\gamma_2^*}{\eta}\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)$. *Under these conditions,* $\theta^B_{c_1}$ *is positive and is increasing in* γ_1, γ_2, *and* γ_1^* *and decreasing in* γ_2^*. $\theta^B_{c_2}$ *is positive and is increasing in* γ_1, γ_2, *and* γ_2^* *and decreasing in* γ_1^*.

Lemma 3.11 contains the same assumptions as Lemma 3.10. In order to understand the most important elements, we will compare Lemma 3.11 to Lemma 3.7 and Lemma 3.2. There is one significant difference in the effects of the spillovers on consumption growth. Under symmetric coordination, both home domestic spillovers and foreign domestic spillovers have a positive effect on consumption growth in both countries. This result serves as evidence of the elimination of the free-rider problem associated with domestic spillovers, brought about through the efforts of the national planners.

The discussion of the effects of symmetric national coordination will conclude by briefly mentioning a few special cases:

Lemma 3.12. Let $\alpha\left(1+\frac{\gamma_2}{\eta}\right)\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho) < (\alpha-1)\frac{\gamma_1^*}{\beta}\left(1+\frac{\gamma_2}{\eta}\right)(\chi-\rho)$. *Then,* $\theta^B_{h_1}$ *is negative and* $\theta^B_{h_2}$ *is positive. In contrast, assume* $\alpha\left(1+\frac{\gamma_1}{\beta}\right)\left(1+\frac{\gamma_2}{\eta}\right)(\chi-\rho) < (\alpha-1)\frac{\gamma_2^*}{\eta}\left(1+\frac{\gamma_1}{\beta}\right)(\phi-\rho)$. *Then,* $\theta^B_{h_1}$ *is positive and* $\theta^B_{h_2}$ *is negative.*

Lemma 3.12 states the conditions that will bring about negative growth in human capital in each country. Recall similar conditions that were provided in Lemma 3.3 and Lemma 3.4 for the competitive case. Under symmetric coordination, we see that in order for these conditions to hold, extreme asymmetry must exist across countrie–not only in the efficiency of human capital investment, but also in the domestic spillovers. Domestic coordination causes the domestic spillovers to take on a much larger role in the determination of the growth rates of human capital.

3.5 Conclusion

The objective of this essay is to study the role of uncompensated spillovers of knowledge in the development process in the presence of regional variation and asymmetries in the degree of economic coordination across countries. The model accounts for regional variation in the rate of economic growth due to differences in the productivity of human capital investment in each economy.

In particular, if the growth rate of human capital is stronger abroad than domestically, the difference acts as a drag on human capital investment in the home country. However, the magnitude of the problem depends on the extent of regional external economies across countries. In fact, such differences might actually cause the stock of human capital to decline over time. Given the potential for inefficiencies to occur, economic coordination on human capital investment can be important. Yet, it could also further exacerbate inefficiencies across countries. Only symmetric coordination will lead to greater human capital investment in the presence of heterogeneous regional external economies.

CHAPTER 4

KNOWLEDGE SPILLOVERS AND ECONOMIC GROWTH IN OPEN ECONOMIES

4.1 Introduction

The standard Lucas model views economies as entirely isolated entities with homogeneous final output. Yet, a wide array of evidence highlights that economies are connected, heterogeneous clusters of activity – therefore, inferences and policy prescriptions obtained from closed economy models of the development process may be inappropriate. For example, Coe and Helpman (1995) find that total factor productivity in OECD countries is higher in the presence of higher levels of foreign research and development expenditures. Jaffe, Trajtenberg, and Henderson (1996) document that knowledge spillovers flow across nearby economies.

Moreover, information flow across borders depends on the degree to which countries seek to access international markets. In particular, Coe, Helpman, and Hoffmaister (1997) observe that developing economies obtain higher rates of productivity growth by trading with countries with more research and development activity. Miller and Upadhyay (2000) stress that countries with greater export orientation (as measured by a higher ratio of exports to gross domestic product) are also more productive.

Consequently, the objective of this paper is to study the role of uncompensated knowledge spillovers on human capital accumulation and economic growth in an open economy setting. In contrast to Lucas, we consider an economy with two different countries that each produce heterogeneous consumption goods. In order to study the impact of information flows on trading patterns, we follow Lucas (1988) by allowing for external economies from human capital accumulation. Yet, we recognize that there may be asymmetries in the transmission of information in the world economy. That is, we study a setting where the extent of spillovers *within* and *across* borders varies between the home and foreign economies.[1]

In our model, there are two countries: home and foreign. There are also two different consumption goods, but countries vary in their capacity to produce them. In particular, each country specializes in the production of one good.[2] However, agents derive utility from consumption of both goods. We allow for both local and cross-border external economies in the production of final output in each country.

Related Literature

As previously mentioned, the Lucas model studies human capital accumulation and growth in a closed economy. By comparison, models of innovation and R&D activity study the *production* of human capital and economic growth.[3] Occasionally, R&D based models are used to show the impact of opening economies to trade on economic growth. The argument advanced is that each country has a stock of knowledge, and opening markets combines the stocks of knowledge in each country to promote growth. However, R&D-based models typically do not include physical capital accumulation. Moreover, such models are often criticized because of their reliance on "scale effects" – larger economies, measured in terms of population size, are predicted to grow faster than smaller economies. By comparison, in our model and in the original Lucas framework, uncompensated spillovers arise from the *average* stock of knowledge.

Holod and Reed (2004) extend the Lucas (1988) model to allow for both local and regional external economies, but they impose that the extent of spillovers are symmetric across countries. Their principal focus is to study the impact of economic integration on growth rates across the world. Holod and Reed (2009) introduce asymmetry into the framework by allowing for differences in knowledge spillovers both within and across

[1]Asymmetries in external economies may arise due to a variety of factors. Matutes, Regibeau, and Rockett (1996) show that patent policies both affect the diffusion of knowledge and protect innovative effort. Lee and Mansfied (1996) emphasize that differences in intellectual property rights across countries affect the amount of foreign direct investment. In addition, the diffusion of knowledge is affected by the extent of labor mobility – see Franco and Filson (2006). Gould (1994) documents that trade flows are linked to immigration flows, since immigrants share their information about foreign markets.

[2]This represents the benefits of specialization. As we are interested in the impact of spillovers on trade flows, the terms of trade, and human capital investment in an *open economy setting,* we do not study growth paths under autarky.

[3]See Grossman and Helpman (1993).

economies. In contrast to our work, neither paper allows for the flow of goods across countries – only information flows are considered. In this manner, neither Holod and Reed (2004) nor Holod and Reed (2009) studies the relationships between trade volumes, relative prices, human capital investment, and the diffusion of knowledge. Bond, Trask, and Wang (2005) study trade flows, endogenous relative prices, and human capital accumulation in which physical capital is traded between countries over time. In contrast to our framework, they do not allow for externalities from human capital.

The remainder of the paper is as follows. The second section describes the benchmark model. The third section presents a model of partial economic coordination. In the fourth section, the impact of economic coordination simultaneously within each country will be studied.

4.2 Benchmark Model

There are two countries, home and foreign. In this model, home country variables will be specified with the subscript "1" and foreign country variables with the subscript "2." In each location, individuals are endowed with some positive amount of country-specific knowledge. Each agent in each country has the opportunity to either work or study. The time spent working is represented by $u_i(t)$, where $i = 1, 2$. If the agent chooses to study, then he or she will accumulate human capital, as represented by the evolution of human capital equations:

$$\dot{h}_1(t) = \phi(1 - u_1(t))h_1(t) \tag{4.1}$$

$$\dot{h}_2(t) = \chi(1 - u_2(t))h_2(t) \tag{4.2}$$

where h_i is the human capital of each individual in country i. $\phi > 0$ represents the growth rate of human capital in the home country if agents were to devote all of their time to investment in human capital. In contrast, $\chi > 0$ represents the growth rate of human capital in the foreign country if agents were to devote all of their time to investment in human capital.

Human capital spillovers are present within each country and also across countries.

γ_i represents the level of domestic human capital spillovers. In contrast, γ_i^* represents the level of international human capital spillovers that country i experiences. Assume $\gamma_i, \gamma_i^* \geq 0$. These spillovers add to productivity, as can be seen in the production functions for each country:

$$\begin{aligned} y_1(t) &= f_1(k_1(t), u_1(t), h_1(t), H_1(t), H_2(t) \\ &= A\,(k_1(t))^{1-\beta}\,(u_1(t)h_1(t))^{\beta}\,(H_1(t))^{\gamma_1}\,(H_2(t))^{\gamma_1^*} \end{aligned} \tag{4.3}$$

$$\begin{aligned} y_2(t) &= f_2\,(k_2(t), u_2(t), h_2(t), H_1(t), H_2(t)) \\ &= B\,(k_2(t))^{1-\eta}\,(u_2(t)h_2(t))^{\eta}\,(H_2(t))^{\gamma_2}\,(H_1(t))^{\gamma_2^*} \end{aligned} \tag{4.4}$$

where the knowledge level of each economy as a whole is denoted by $H_i(t)$. In addition, $0 < \beta, \eta < 1$.

Production is completely specialized, but agents in each country have a preference for consumption of both goods. The home country produces (and exports) good A and the foreign country produces (and exports) good B. Consumption by the home country of good A and good B is represented by $c_1^A(t)$ and $c_1^B(t)$ respectively. Consumption by the foreign country of good A and good B is represented by $c_2^A(t)$ and $c_2^B(t)$ respectively.

From this point on, the decision making of the home country will be the primary focus of the analysis. Foreign country decision making is completely analogous to home country decision making.

As mentioned above, agents have a preference for consumption of both goods. This is reflected in the lifetime utility maximization problem for agents in the home country:

$$\underset{c_1^A(t);c_1^B(t)}{Max} \int_0^{\infty} e^{-\rho t}\left[\ln\left(c_1^A(t)\right) + \ln\left(c_1^B(t)\right)\right] dt \tag{4.5}$$

where $\frac{1}{\alpha} > 0$ is the intertemporal elasticity of substitution between the two goods and $\rho > 0$ is the discount rate.

The home country produces good A according to the production function given in equation (4.3). Part of the production is depreciated $(\delta k_1(t))$ and part is reinvested $\left(\dot{k}_1(t)\right)$. What remains of good A production is consumed, both in the home country $\left(c_1^A(t)\right)$ and in the foreign country $\left(c_2^A(t)\right)$. The resulting market-clearing condition for the home country is as follows:

$$A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)h_1(t)\right)^{\beta}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} = c_1^A(t) + \left(\dot{k}_1(t) + \delta k_1(t)\right) + c_2^A(t) \quad (4.6)$$

Since the countries have specialized production, they must trade with each other in order to consume both goods. The home country will trade some of its good A in exchange for some of the foreign country's good B. The terms of trade will be determined by $p(t)$, the relative price of good B measured in units of good A. So then, we have the following budget constraint:

$$c_2^A(t) = p(t)c_1^B(t) \quad (4.7)$$

Note that this constraint implies that the amount spent on imported goods in the home country must equal the amount spent on imported goods in the foreign country. That is, there are no trade deficits or surpluses in the model.

The budget constraint can be combined with the market-clearing condition to construct the evolution of physical capital, using all home country variables:

$$\dot{k}_1(t) = A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)h_1(t)\right)^{\beta}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} - \delta k_1(t) - c_1^A(t) - p(t)c_1^B(t) \quad (4.8)$$

In order to solve the problem, we apply Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$\begin{aligned} H \;=\; & \left(\ln\left(c_1^A(t)\right) + \ln\left(c_1^B(t)\right)\right) \quad (4.9) \\ & +\lambda_1(t)[A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)h_1(t)\right)^{\beta}\left(H_1(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} - \delta k_1(t) - c_1^A(t) - p(t)c_1^B(t)] \\ & +\mu_1(t)[\phi(1-u_1(t))h_1(t)] \end{aligned}$$

where λ_1 and μ_1 represent the co-state variables for physical and human capital.

First, agents need to decide how much of each good to consume each period:

$$(c_1^A(t))^{-1} = \lambda_1(t) \tag{4.10}$$

$$(c_1^B(t))^{-1} = p(t)\lambda_1(t) \tag{4.11}$$

Since λ_1 represents the value of an additional unit of physical capital, these equations represent the standard trade-off between current consumption and capital accumulation. These two equations yield a relationship between consumption of the two goods:

$$c_1^A(t) = p(t)c_1^B(t) \tag{4.12}$$

Equation (4.12) demonstrates that expenditures on each good are the same.

Individuals must also decide how to allocate their time between the production and learning sectors, which yields the following no-arbitrage condition:

$$\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta-1}\left(h_1(t)\right)^{\beta}\left(H_2(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} = \mu_1(t)\phi h_1(t) \tag{4.13}$$

Equation (4.13) reflects that the choice of working time is actually an investment decision. The representative agent chooses to invest his or her time in either physical capital accumulation or human capital accumulation. Each side of the equation represents the value of an additional unit of capital ($\lambda_1(t)$ or $\mu_1(t)$) times the additional units of capital that would be created by devoting an additional unit of time to that sector. At the margin, the return to investment must be equal for each type of capital.

It is significant that the left side displays diminishing marginal return to time devoted to production. However, the right side reveals that the marginal return of time devoted to human capital accumulation is independent of the amount of time already spent accumulating human capital. This is due to the linearity of the human capital accumulation formula, equation (4.1).

Some of the main results of this paper are driven by equation (4.13). For example, if spillovers increase at home, this will increase productivity. Productivity in the learning sector is constant. Therefore, agents will shift time to the production sector until the marginal return from production is equal to the marginal return from learning. Stated differently, we see that agents have more incentive to produce since the increased spillover allows for increased free-riding off of others' knowledge.

The Euler equations represent the capital gains or losses from physical and human capital over time:

$$\dot{\lambda}_1(t) = \lambda_1(t)(\rho - f_{k_1}(t) + \delta) \tag{4.14}$$

$$\begin{aligned} \dot{\mu}_1(t) \quad &= \quad \rho\mu_1(t) - \lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta-1}\left(H_2(t)\right)^{\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} \\ &\quad -\mu_1(t)\phi(1-u_1(t)) \end{aligned} \tag{4.15}$$

Interestingly, since physical capital is only used in the production sector, the Euler equation for physical capital is only dependent on the value of physical capital. Human capital, however, is used to accumulate more human capital – but is also used in the production sector. Therefore, the Euler equation for human capital is dependent on the value of both physical and human capital. The no-arbitrage condition, equation (4.13), gives a relationship between the value of physical and human capital, and so can be used to rewrite the Euler equation for human capital in terms of only the value of human capital:

$$\dot{\mu}_1(t) = \mu_1(t)\left(\rho - \phi\right) \tag{4.16}$$

The goal of this paper is to analyze the effects of human capital spillovers on the growth of consumption across time. To achieve this, we will convert all equations to growth equations, and then solve for the growth rates of each variable along the balanced growth path. Along the BGP, all growth rates must be constant. The growth rate of variable x along the BGP is represented by θ_x^B.

First, we write a new equation for the growth rate of consumption of the exportable good by combining the consumption choice condition with the Euler equation for physical capital:

$$\theta^B_{c^A_1} = \frac{f_{k_1} - (\rho + \delta)}{\alpha} \tag{4.17}$$

Therefore, in order for $\theta^B_{c^A_1}$ to be constant, f_{k_1}must be constant. We impose the condition that in equilibrium the average stock of knowledge within each country will be equal to the representative agent's human capital stock, explicitly, $H_i = h_i$[4].This yields:

$$\theta^B_{k_1} = \left(1 + \frac{\gamma_1}{\beta}\right)\theta^B_{h_1} + \frac{\gamma^*_1}{\beta}\theta^B_{h_2} \tag{4.18}$$

By combining the evolution of physical capital, equation (4.8), with the equation derived from consumption choice, equation (4.12), we find that physical capital can only be constant if the following condition holds:

$$\theta^B_{k_1} = \theta^B_{c^A_1} \tag{4.19}$$

Together, equations (4.18) and (4.19) form a new expression for the growth of consumption of the exportable, now in terms of the growth rates of human capital of each country:

$$\theta^B_{c^A_1} = \left(1 + \frac{\gamma_1}{\beta}\right)\theta^B_{h_1} + \frac{\gamma^*_1}{\beta}\theta^B_{h_2} \tag{4.20}$$

In the absence of human capital spillovers, the growth rate of consumption of a good would simply be equal to the growth rate of human capital in the country where that good is produced, bringing attention to the role of human capital as the "engine of growth."

Equation (4.20) also shows the direct impact of spillovers on the growth of consumption of the exportable. The coefficients $\frac{\gamma_1}{\beta}$ and $\frac{\gamma^*_1}{\beta}$ can be understood by considering the production function of the home country. These coefficients represent the relative importance to production of each of the spillovers as compared to the extent productivity is enhanced by the human capital of the individual agent. Human capital is the main driver of economic growth, but that growth is augmented by the knowledge spillovers,

[4]It is of course important that this condition was imposed after individual agents made their decisions regarding human capital accumulation.

both from within the home country and from the foreign country.

An analogous analysis can be performed for the foreign country, with the resulting equations:

$$\theta^B_{u_2} = \rho - \chi \tag{4.21}$$

$$\theta^B_{k_2} = \theta^B_{c_2^B} \tag{4.22}$$

$$\theta^B_{c_2^B} = \theta^B_{h_2} + \frac{\gamma_2}{\eta}\theta^B_{h_2} + \frac{\gamma_2^*}{\eta}\theta^B_{h_1} \tag{4.23}$$

Consumption choice in the foreign country yields the following relationships:

$$(c_2^B(t))^{-1} = \lambda_2(t) \tag{4.24}$$

$$(c_2^A(t))^{-1} = \frac{\lambda_2(t)}{p(t)} \tag{4.25}$$

$$c_2^A(t) = p(t)c_2^B(t) \tag{4.26}$$

As was the case in the home country, expenditures are the same for each good in the foreign country. Equation (4.26) can be combined with the analogous consumption choice relationship found for the home country, equation (4.12), and the budget constraint, equation (4.7), to find that consumption of each good is the same across the two countries:

$$c_1^A(t) = c_2^A(t) \tag{4.27}$$

$$c_2^B(t) = c_1^B(t) \tag{4.28}$$

By substituting equation (4.10), the consumption choice equation; equation (4.16), the transformed Euler equation for human capital; and equations (4.19) and (4.20) into equation (4.13), the no-arbitrage condition; we obtain an equation for the growth rate of human capital in the home country. Human capital growth in the foreign country can be found in a similar manner, yielding the following proposition:

Proposition 4.1. The growth rates of human capital are as follows:

$$\theta^B_{h_1} = \phi - \rho \tag{4.29}$$

$$\theta^B_{h_2} = \chi - \rho \tag{4.30}$$

According to equations (4.29) and (4.30), the growth of human capital in each country is primarily determined by the efficiency of human capital investment in that country. Therefore, whichever country is more efficient in human capital accumulation will experience higher human capital growth over time. Growth of human capital is also affected by the discount rate. By investing in human capital, agents decrease production of goods in the present and increase production and consumption of goods in the future. Therefore, when agents discount future consumption more, the growth of human capital will be lower.

Equations (4.29) and (4.30) can then be substituted into equations (4.20) and (4.23) to find the growth rates of consumption in each country. This leads to the following proposition:

Proposition 4.2. Assume $\phi > \rho$ and $\chi > \rho$. A unique, balanced growth path exists, and the growth rates of consumption for all goods are positive. The growth rates of consumption in each country are as follows:

$$\theta^B_{c_1^A} = \theta^B_{c_2^A} = \left(1 + \frac{\gamma_1}{\beta}\right)(\phi - \rho) + \frac{\gamma_1^*}{\beta}(\chi - \rho) \tag{4.31}$$

$$\theta^B_{c_2^B} = \theta^B_{c_1^B} = \left(1 + \frac{\gamma_2}{\eta}\right)(\chi - \rho) + \frac{\gamma_2^*}{\eta}(\phi - \rho) \tag{4.32}$$

Each country in the model experiences different degrees of efficiency from human capital investment. One advantage of this feature is that it allows more detailed consideration of how human capital from each country is driving growth. As can be seen in equations (4.31) and (4.32), the growth of consumption of each good depends on the efficiency of human capital investment in both countries.

The growth rate of consumption of the home exportable across the world depends heavily on the degree of human capital spillovers that affect home production. However, the role of spillovers to the home country also depends on the efficiency of human capital investment in both the home and foreign country. If international spillovers into each country are the same, and if investment in human capital is relatively more efficient in the foreign country, then international spillovers will have a stronger effect on consumption growth of the home country exportable than on consumption growth of the foreign country exportable.

When considering trade partnerships to enhance economic growth, nations should seek out countries with high productivity in human capital accumulation. This will then benefit the domestic production sector. In addition, nations should consider the efficiency in human capital investment in their own country, relative to other nations. If a country is relatively efficient in human capital investment, then encouraging domestic spillovers will increase growth. If a country is relatively inefficient in human capital investment, then encouraging international spillovers will increase growth.

The final step is to combine the consumption growth equations with the budget constraint to obtain the growth of the relative price, which leads to the following proposition:

Proposition 4.3. The growth rate of the relative price of the foreign produced good (B) in terms of the domestically produced good (A) is as follows:

$$\theta_p^B = \left[\left(1+\frac{\gamma_1}{\beta}\right)-\frac{\gamma_2^*}{\eta}\right](\phi-\rho)-\left[\left(1+\frac{\gamma_2}{\eta}\right)-\frac{\gamma_1^*}{\beta}\right](\chi-\rho) \tag{4.33}$$

Equation (4.33) highlights the effect of both spillovers and the relative productivity in human capital accumulation on the growth of the terms of trade along the balanced growth path. The growth rate of relative prices depends on the relative efficiency of human capital accumulation across both countries. Assuming that the production technology is the same across countries, then relative prices are increasing over time if human capital accumulation is more efficient in the home country than in the foreign country.

This is due to the fact that higher productivity in human capital in the home country will lead to a relative increase in the supply of the home produced good. This then creates upward pressure on the price of the foreign good.

Correspondingly, assume that the efficiency of human capital accumulation is the same in both countries. Then, relative prices are increasing if the sum of spillovers that affect production in the home country are higher than the sum of spillovers that affect production in the foreign country. Again, this is due to the relative increase in the supply of the home produced good.

4.3 Asymmetric National Coordination

In the benchmark model, decision making is decentralized. In particular, a representative agent in each country allocates time between the learning and production sectors in a way that maximizes his or her own lifetime utility, ignoring the effect of this decision on the average human capital stock in his or her country. In contrast to the benchmark model, we will next look at a setting with partial economic coordination. That is, we seek to understand the global ramifications from economic coordination in one country. This framework introduces a planner in the home country, but the competitive equilibrium will still hold in the foreign country. By setting the average nationwide stock of human capital equal to the individual stock of human capital (i.e. $H_1 = h_1$), the home national planner internalizes the positive domestic external effects of increases in the average human capital stock.

Our analysis will focus on the changes in the decision making in the home country caused by the presence of a planner. In order to solve the problem, we apply Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$\begin{aligned} H \;=\;& \left(\ln\left(c_1^A(t)\right) + \ln\left(c_1^B(t)\right)\right) \qquad (4.34)\\ &+\lambda_1(t)[A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta+\gamma_1}\left(H_2(t)\right)^{\gamma_1^*} - \delta k_1(t) - c_1^A(t) - p(t)c_1^B(t)]\\ &+\mu_1(t)[\phi(1-u_1(t))h_1(t)] \end{aligned}$$

The planner must determine how to allocate time between the production and learning sectors, which yields the following no-arbitrage condition for capital investment in each sector:

$$\lambda_1(t)\beta A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta-1}\left(h_1(t)\right)^{\beta+\gamma_1}\left(H_2(t)\right)^{\gamma_1^*}=\mu_1(t)\phi h_1(t) \tag{4.35}$$

The Euler equation for human capital is as follows:

$$\begin{aligned}\dot{\mu}_1(t) &= \rho\mu_1(t)-\lambda_1(t)\left(\beta+\gamma_1\right)A\left(k_1(t)\right)^{1-\beta}\left(u_1(t)\right)^{\beta}\left(h_1(t)\right)^{\beta+\gamma_1-1}\left(H_2(t)\right)^{\gamma_1^*} \quad (4.36)\\ &\quad -\mu_1(t)\phi(1-u_1(t))\end{aligned}$$

As in the competitive case, the new no-arbitrage condition, equation (4.35), can be used to rewrite the Euler equation for human capital in terms of only the value of human capital. The resulting equation can then be further transformed using the evolution of human capital, equation (4.1), leading to the following equation for the growth of the human capital co-state variable:

$$\theta_{\mu_1}^{B}\left(P_1\right)=\rho-\left(1+\frac{\gamma_1}{\beta}\right)\phi+\frac{\gamma_1}{\beta}\theta_{h_1} \tag{4.37}$$

Important intuition can be gleaned from equation (4.37). Recall that μ_1 represents the value of an additional unit of human capital. The growth rate of μ_1 is decreasing in ϕ, the productivity of human capital accumulation in the home country. This is due to the fact that any increase in ϕ will lead to a greater accumulation of human capital. This in turn causes a lower marginal value of human capital. The negative relationship between the productivity of human capital accumulation and the growth of the human capital co-state variable was also seen in the competitive case. However, in the presence of a national planner, the negative effect is magnified by the spillover (hence the $1+\frac{\gamma_1}{\beta}$ coefficient on ϕ).

Recall that $\frac{\gamma_1}{\beta}$ represents the relative importance to production of the home domestic spillover. The home domestic spillover increases the benefit of human capital accumulation on production. In the competitive equilibrium, individual agents did not consider this effect and so the spillover did not add to the marginal value of human capital. However, for the planner, the third term of equation (4.37) shows the positive effect that the home domestic spillover has on the growth of the co-state variable. This indicates that the planner is including the external benefits of the domestic spillover in decision making.

By substituting equation (4.10), the consumption choice equation; equation (4.37), the transformed Euler equation for human capital; and equations (4.19) and (4.20) into equation (4.35), the no-arbitrage condition, we obtain an equation for the growth rate of human capital in each country. This is stated in the following proposition:

Proposition 4.4. The growth rates of human capital are as follows:

$$\theta^B_{h_1}(P_1) = \phi - \frac{\rho}{1 + \frac{\gamma_1}{\beta}} > \theta^B_{h_1}(CE) \tag{4.38}$$

$$\theta^B_{h_2}(P_1) = \theta^B_{h_2}(CE) \tag{4.39}$$

$$\theta^B_{h_1}(P_1) - \theta^B_{h_1}(CE) = \frac{\gamma_1}{\beta + \gamma_1}\rho \tag{4.40}$$

According to equations (4.38) and (4.40), the planner's involvement in the home country alleviates the cost of subjective discounting on the growth rate of home country human capital. Under the competitive equilibrium, the growth rate of human capital is $\phi - \rho$. In the case of coordination in the home country, it is higher depending on the value of $\frac{\gamma_1}{\beta}$. The planner internalizes the benefit to production due to increases in the average human capital stock. This leads to a reduction in free-riding and an increase in human capital growth in the home country, relative to the competitive case.

The increase in home human capital growth does not have any impact on human capital growth in the foreign country, as seen in equation (4.39). This is in contrast to Cai and Reed (2015), which found that human capital growth in the foreign country actually declined as a result of coordination in the home country. However, the inclusion of relative prices in our model may adjust incentives so that the desire to free-ride on domestic human capital stock is mitigated.

The growth rates of human capital, equations (4.38) and (4.30), can be combined with equations (4.20) and (4.23) to find the growth rates of consumption in each country, which are given in the following proposition:

Proposition 4.5. Assume $\phi > \rho$ and $\chi > \rho$. A unique, balanced growth path exists, and the growth rates of consumption for all goods are positive. The growth rates of consumption in each country are as follows:

$$\begin{aligned} \theta^B_{c_1^A}(P_1) &= \theta^B_{c_2^A}(P_1) = \left(1+\frac{\gamma_1}{\beta}\right)\left(\phi - \frac{\beta}{\beta+\gamma_1}\rho\right) + \frac{\gamma_1^*}{\beta}(\chi-\rho) \\ &> \theta^B_{c_1^A}(CE) = \theta^B_{c_2^A}(CE) \end{aligned} \tag{4.41}$$

$$\begin{aligned} \theta^B_{c_2^B}(P_1) &= \theta^B_{c_1^B}(P_1) = \left(1+\frac{\gamma_2}{\eta}\right)(\chi-\rho) + \frac{\gamma_2^*}{\eta}\left(\phi - \frac{\beta}{\beta+\gamma_1}\rho\right) \\ &> \theta^B_{c_2^B}(CE) = \theta^B_{c_1^B}(CE) \end{aligned} \tag{4.42}$$

$$\theta^B_{c_1^A}(P_1) - \theta^B_{c_1^A}(CE) = \left(1+\frac{\gamma_1}{\beta}\right)\frac{\gamma_1}{\beta+\gamma_1}\rho \tag{4.43}$$

$$\theta^B_{c_2^B}(P_1) - \theta^B_{c_2^B}(CE) = \frac{\gamma_2^*}{\eta}\frac{\gamma_1}{\beta+\gamma_1}\rho \tag{4.44}$$

$$\theta^B_{c_1^A}(P_1) - \theta^B_{c_1^A}(CE) = \frac{1+\frac{\gamma_1}{\beta}}{\frac{\gamma_2^*}{\eta}}\left[\theta^B_{c_2^B}(P_1) - \theta^B_{c_2^B}(CE)\right] \tag{4.45}$$

As explained following Proposition 4.4, the presence of a national planner in the home country leads to higher human capital growth in the home country. Equations (4.41) and (4.42) show that the increase in the growth of human capital at home leads to an increase in consumption of both goods in both countries. This result highlights the free-riding problem that exists in the competitive equilibrium and the potential benefits of economic coordination. Efforts to encourage human capital accumulation within a country would generally be for the purpose of increasing economic growth in that country. However, the benefits of coordination are also felt by the trading partners of that nation. Therefore, nations considering ways to increase economic growth should both encourage human capital accumulation in their own country and seek trade partnerships with nations that also provide policies to encourage human capital investment.

Equations (4.43) and (4.44) specifically state the amount that consumption growth under partial coordination exceeds consumption growth in the competitive equilibrium. In addition, the relationship between the increase in consumption growth of each good is given in equation (4.45). The following lemma is derived from these three equations:

Lemma 4.1. Assume $\left(1+\frac{\gamma_1}{\beta}\right) > \frac{\gamma_2^*}{\eta}$. *Relative to the competitive case, the increase in the growth of consumption of the home exportable due to coordination in the home country is greater than the increase in the growth of consumption of the foreign exportable.*

Due to the existence of international spillovers, the increase in human capital growth in the home country brought about through coordination positively affects production in both countries. However, the increase in the growth of consumption of the home produced good is greater than the increase in the growth of consumption of the foreign produced good.

In the home country, the human capital stock at home affects home production both directly and indirectly through the home domestic spillover. In contrast, production in the foreign country is affected by home human capital stock only indirectly through the foreign international spillover.

Combining the growth rates of consumption with the budget constraint will yield the growth rate of the relative price, as stated in the following proposition:

Proposition 4.6. Assume $\left(1+\frac{\gamma_1}{\beta}\right) > \frac{\gamma_2^*}{\eta}$. *The growth rates of the relative price of the foreign produced good (B) in terms of the domestically produced good (A) are as follows:*

$$\theta_p^B\left(P_1\right) = \left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right]\left(\phi - \frac{\beta}{\beta+\gamma_1}\rho\right) - \left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]\left(\chi-\rho\right) \tag{4.46}$$

$$\theta_p^B\left(P_1\right) - \theta_p^B\left(CE\right) = \left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right]\frac{\gamma_1}{\beta+\gamma_1}\rho > 0 \tag{4.47}$$

Interestingly, the model predicts that improved coordination at home leads to improved terms of trade in the foreign country. This can be seen in equations (4.46) and (4.47). Coordination within the home country benefits production in both countries.

However, the growth of consumption of the home produced good increases by a greater amount than the increase in the growth of consumption of the foreign produced good. This shifts the terms of trade. Coordination in the home country leads to a relative increase in productivity at home. Since agents in both countries have a preference for consuming both goods, the increase in the relative supply of the home exportable places upward pressure on the price of the foreign produced good.

4.4 Symmetric National Coordination

To complete our analysis, a setting with more extensive economic coordination will be considered. Specifically, the effects of economic coordination within each country will be compared to the setting of partial economic coordination and the competitive equilibrium. That is, in this framework, a national planner exists within each country. Each national planner considers the full effect of human capital investment decisions on production within his or her own country, including the effect of the domestic spillovers. However, they national planners do not consider the effect their decision making has on the other country. This is modeled by setting the average nationwide stock of human capital equal to the individual human capital stock within each country.

In this model, decision making within the home country will be the same as in the model of asymmetric national coordination. Therefore, our analysis will focus on the changes in decision making in the foreign country, relative to the previous models. We begin by applying Pontryagin's maximum principle, which yields the following current-valued Hamiltonian:

$$\begin{aligned} H &= \left(\ln\left(c_2^A(t)\right) + \ln\left(c_2^B(t)\right)\right) \\ &\quad + \lambda_2(t)\left[B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta+\gamma_2}\left(H_1(t)\right)^{\gamma_2^*} - \delta k_2(t) - c_2^B(t) - \frac{c_2^A(t)}{p(t)}\right] \\ &\quad + \mu_2(t)\left[\chi(1-u_2(t))h_2(t)\right] \end{aligned} \tag{4.48}$$

The foreign national planner must decide between investment in physical capital and investment in human capital, which yields the following no-arbitrage condition:

$$\lambda_2(t)\eta B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta-1}\left(h_2(t)\right)^{\eta+\gamma_2}\left(H_1(t)\right)^{\gamma_2^*}=\mu_2(t)\chi h_2(t) \tag{4.49}$$

The Euler equation for human capital is as follows:

$$\begin{aligned}\dot{\mu}_2(t) &= \rho\mu_2(t)-\left(\eta+\gamma_2\right)\lambda_2(t)B\left(k_2(t)\right)^{1-\eta}\left(u_2(t)\right)^{\eta}\left(h_2(t)\right)^{\eta+\gamma_2-1}\left(H_1(t)\right)^{\gamma_2^*} \quad (4.50)\\ &\quad -\mu_2(t)\chi(1-u_2(t))\end{aligned}$$

The no-arbitrage condition, equation (4.49), which provides a relationship between the value of physical and human capital, can be used to rewrite the human capital Euler equation in terms of only the value of human capital. The Euler equation can be further modified using the evolution of human capital equation for the foreign country, with the resulting growth rate of the human capital co-state variable:

$$\theta_{\mu_2}^B=\rho-\chi\left(1+\frac{\gamma_2}{\eta}\right)+\frac{\gamma_2}{\eta}\theta_{h_2}^B \tag{4.51}$$

As the foreign analog of equation (4.37), equation (4.51) provides key insight into the decision making of the foreign national planner. This equation demonstrates the way that the planner fully considers the foreign domestic knowledge externality. The term $\frac{\gamma_2}{\eta}$ represents the relative importance to foreign production of the foreign domestic spillover. This term magnifies the negative effect that the efficiency in human capital accumulation has on the growth of the value of an additional unit of human capital in the foreign country. Furthermore, the third term of equation (4.51), which was not present in the competitive case, reveals that the growth of the average stock of human capital adds to the marginal value of human capital to the extent that the foreign domestic spillover affects foreign production.

Next, substitute equation (4.51), the transformed Euler equation for human capital; equation (4.24), the consumption choice equation for the foreign country; and equations (4.22) and (4.23) into equation (4.49), the no-arbitrage condition. This yields an equation for the growth rate of human capital in the foreign country, which is stated in the following proposition:

Proposition 4.7. Assume $\phi > \rho$ and $\chi > \rho$. The growth rates of human capital are as follows:

$$\theta^B_{h_2}(P) = \chi - \frac{\rho}{1+\frac{\gamma_2}{\eta}} > \theta^B_{h_2}(P_1) = \theta^B_{h_2}(CE) \tag{4.52}$$

$$\theta^B_{h_1}(P) = \theta^B_{h_1}(P_1) > \theta^B_{h_1}(CE) \tag{4.53}$$

$$\theta^B_{h_2}(P) - \theta^B_{h_2}(P_1) = \frac{\gamma_2}{\eta+\gamma_2}\rho \tag{4.54}$$

Equations (4.52) and (4.54) show that coordination in the foreign country leads to an increase in the growth of human capital in the foreign country. This is indicative of the free-rider problem in the competitive equilibrium. The cost of subjective discounting is reduced to the extent that the foreign domestic spillover is important to foreign production. According to equation (4.53), coordination in the foreign country does not affect human capital growth in the home country. When a national planner is present in each country, human capital growth in the home country is the same as when a national planner was present only in the home country.

Insert equations (4.52) and (4.53) into equations (4.20) and (4.23) to find the growth rates of consumption in each country to produce the following proposition:

Proposition 4.8. Assume $\phi > \rho$ and $\chi > \rho$. A unique, balanced growth path exists, and the growth rates of consumption for all goods are positive. The growth rates of consumption in each country are as follows:

$$\begin{aligned} \theta^B_{c_1^A}(P) &= \theta^B_{c_2^A}(P) = \left(1+\frac{\gamma_1}{\beta}\right)\left(\phi - \frac{\rho}{1+\frac{\gamma_1}{\beta}}\right) + \frac{\gamma_1^*}{\beta}\left(\chi - \frac{\rho}{1+\frac{\gamma_2}{\eta}}\right) \\ &> \theta^B_{c_1^A}(P_1) = \theta^B_{c_2^A}(P_1) \end{aligned} \tag{4.55}$$

$$\begin{aligned} \theta^B_{c_1^B}(P) &= \theta^B_{c_2^B}(P) = \left(1+\frac{\gamma_2}{\eta}\right)\left(\chi - \frac{\rho}{1+\frac{\gamma_2}{\eta}}\right) + \frac{\gamma_2^*}{\eta}\left(\phi - \frac{\rho}{1+\frac{\gamma_1}{\beta}}\right) \\ &> \theta^B_{c_1^B}(P_1) = \theta^B_{c_2^B}(P_1) \end{aligned} \tag{4.56}$$

$$\theta^B_{c_1^A}(P) - \theta^B_{c_1^A}(P_1) = \frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2}\rho \tag{4.57}$$

$$\theta^B_{c_2^B}(P) - \theta^B_{c_2^B}(P_1) = \left(1+\frac{\gamma_2}{\eta}\right)\frac{\gamma_2}{\eta+\gamma_2}\rho \tag{4.58}$$

$$\theta^B_{c_1^A}(P) - \theta^B_{c_1^A}(P_1) = \frac{\frac{\gamma_1^*}{\beta}}{1+\frac{\gamma_2}{\eta}}\left[\theta^B_{c_2^B}(P) - \theta^B_{c_2^B}(P_1)\right] \tag{4.59}$$

$$\theta^B_{c_1^A}(P) - \theta^B_{c_1^A}(CE) = \left[\frac{\gamma_1}{\beta} + \frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2}\right]\rho \tag{4.60}$$

$$\theta^B_{c_2^B}(P) - \theta^B_{c_2^B}(CE) = \left[\frac{\gamma_2}{\eta} + \frac{\gamma_2^*}{\eta}\frac{\gamma_1}{\beta+\gamma_1}\right]\rho \tag{4.61}$$

From Proposition 4.5, we learned that consumption of both goods in both countries increased as a result of economic coordination in only the home country. Now, from equations (4.55) and (4.56), we see that with economic coordination in both countries, consumption of both goods in both countries increases even more. As seen in proposition 4.7, coordination in the foreign country directly benefits human capital growth in the foreign county. This benefit spreads to production in both countries through the spillovers.

When the case of symmetric coordination is compared to the competitive equilibrium, it is apparent that coordination promotes growth in both countries. However, it is not clear whether each country is benefitting solely due to its own coordination, or if coordination in the other country is also growth-enhancing. Comparison of the case of asymmetric coordination to both the competitive equilibrium and the case of symmetric coordination sheds light on this issue.

Relative to the competitive equilibrium, in the case of asymmetric coordination, the foreign country benefits from the coordination within the home country even though the foreign country remains competitive. Relative to asymmetric coordination, in the case of symmetric coordination, the effect on the home country is most interesting. In this case, the home country benefits from coordination within the foreign country. Economic growth in the home country is higher when both countries are coordinated, compared to the case when only the home country is coordinated.

Together, the analysis of the differing degrees of coordination demonstrates that a country will benefit from pursuing economic coordination. However, the country will see even higher economic growth if at the same time they trade with nations that are also economically coordinated.

Equations (4.57) and (4.58) present the amount that consumption growth under symmetric coordination exceeds consumption growth under asymmetric coordination. Furthermore, Equation (4.59) provides a relationship between the consumption growth increase of the two goods. These equations lead to the following lemma:

Lemma 4.2. Assume $\left(1+\frac{\gamma_2}{\eta}\right) > \frac{\gamma_1^*}{\beta}$. *Relative to the case of asymmetric coordination, the increase in the growth of consumption of the foreign exportable due to coordination in both countries is greater than the increase in the growth of consumption of the home exportable.*

The increased human capital growth in the foreign country created through coordination will both directly and indirectly (through the spillover) affect production in the foreign country. Production in the home country will only experience an indirect effect through the spillover.

Equations (4.60) and (4.61) represent the amount that consumption growth under symmetric coordination exceeds consumption growth in the competitive equilibrium. The following lemma is derived from these equations:

Lemma 4.3. Assume $\frac{\gamma_1}{\beta}+\frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2} > \frac{\gamma_2}{\eta}+\frac{\gamma_2^*}{\eta}\frac{\gamma_1}{\beta+\gamma_1}$. *Relative to the competitive equilibrium, the increase in the growth of consumption of the home exportable due to coordination in both countries is greater than the increase in the growth of consumption of the foreign exportable.*

In contrast, assume $\frac{\gamma_1}{\beta}+\frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2} < \frac{\gamma_2}{\eta}+\frac{\gamma_2^*}{\eta}\frac{\gamma_1}{\beta+\gamma_1}$. *Relative to the competitive equilibrium, the increase in the growth of consumption of the foreign exportable due to coordination in both countries is greater than the increase in the growth of consumption of the home exportable.*

It is clear that relative to the competitive equilibrium, coordination in both countries leads to higher consumption of both goods. However, it is the degree of the spillovers in each country that will determine which good will experience greater consumption growth.

The left-hand side of the assumptions in Lemma 4.3 can be understood as the positive effect of coordination on the growth of consumption of the home country exportable. The first term $\left(\frac{\gamma_1}{\beta}\right)$ represents the benefit on home production from coordination within the home country, since the planner takes into account the full effect of the home domestic spillover. The second term $\left(\frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2}\right)$ represents the benefit to home production from coordination within the foreign country, which is felt through the home international spillover. If domestic spillovers and the productivity of labor are assumed to be the same in both countries, then whichever country experiences higher international spillovers will also experience a greater increase in production growth due to coordination.

Finally, combine the growth rates of consumption with the budget constraint to obtain the growth of the relative price, yielding the following proposition:

Proposition 4.9. Assume $\phi > \rho$ and $\chi > \rho$. Also assume that $\left(1+\frac{\gamma_2}{\eta}\right) > \frac{\gamma_1^*}{\beta}$. The growth rate of the price of the foreign produced good (B) in terms of the domestically produced good (A) is as follows:

$$\theta_p^B(P) = \left[\left(1+\frac{\gamma_1}{\beta}\right) - \frac{\gamma_2^*}{\eta}\right]\left(\phi - \frac{\rho}{1+\frac{\gamma_1}{\beta}}\right) - \left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]\left(\chi - \frac{\rho}{1+\frac{\gamma_2}{\eta}}\right) \quad (4.62)$$

$$\theta_p^B(P) - \theta_p^B(P_1) = -\left[\left(1+\frac{\gamma_2}{\eta}\right) - \frac{\gamma_1^*}{\beta}\right]\frac{\gamma_2}{\eta+\gamma_2}\rho \quad (4.63)$$

$$\theta_p^B(P) - \theta_p^B(CE) = \left[\frac{\gamma_1}{\beta} + \frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2} - \frac{\gamma_2}{\eta} - \frac{\gamma_2^*}{\eta}\frac{\gamma_1}{\beta+\gamma_1}\right]\rho \quad (4.64)$$

Equations (4.62) and (4.63) reveal that relative to the case of asymmetric coordination, improved coordination in the foreign country leads to a deterioration in the terms of trade in the foreign country. This is due to the fact that economic coordination in the foreign country has a greater impact on production in the foreign country, relative to the impact on production in the home country. The relative increase in the supply of the foreign exportable causes the growth of the relative price of the foreign exportable to fall.

As demonstrated by equation (4.64), the relationship between the growth of the terms of trade under symmetric coordination and the growth of the terms of trade in the competitive equilibrium varies based on the relative strength of the spillovers that affect the production sectors in each country. The growth of relative price will change based on changes in the relative supply of the two goods. If production in the home country expands more than production in the foreign country, then the growth of the relative price of the foreign exportable will rise.

According to Lemma 4.3, when $\frac{\gamma_1}{\beta} + \frac{\gamma_1^*}{\beta}\frac{\gamma_2}{\eta+\gamma_2} > \frac{\gamma_2}{\eta} + \frac{\gamma_2^*}{\eta}\frac{\gamma_1}{\beta+\gamma_1}$, the increase in the growth of consumption of the home exportable due to coordination will be greater than the increase in the growth of consumption of the foreign exportable due to coordination. Note that the magnitudes of the domestic spillovers within each country are most important in this comparison. This is because it is domestic spillovers that the national planners incorporate into their decision making. Human capital in each country increases depending on the relative importance of the domestic spillover for production. Production in each country also benefits through the international spillover from coordination-induced human capital growth in the other country.

4.5 Conclusion

The inclusion of trade allows both ideas and goods to travel across borders, making our model significantly more complex than the original closed economy Lucas (1988) framework. In addition, we allow for asymmetry in the efficiency of human capital investment across countries, giving added insight. We find that knowledge spillovers are growth enhancing to the extent of the efficiency of human capital investment. The spillovers that are the most beneficial to growth are the spillovers that originate in areas of rapid knowledge accumulation, whether at home or abroad. Our analysis of economic coordination draws attention to a free-rider problem and the benefit of pursuing policies to encourage investment in human capital, such as educational grants.

CHAPTER 5
CONCLUSION

The groundbreaking work of Lucas (1988) highlights the importance of uncompensated knowledge spillovers in economic development. However, the simplicity of the model allows for the study of only one type of spillover. This dissertation has examined various extensions of the original Lucas model. The effects on growth of several distinct types of knowledge spillovers have been studied at length. In particular, the potential for positive growth effects of external economies from human capital have been demonstrated, but the results also point to a free-rider problem.

In particular, our work within a two-sector environment is more supportive of the intra-industry spillover theories of Marshall, Arrow, and Romer, as opposed to the inter-industry spillover theory of Jacobs. Our open economy analysis reveals that human capital growth may decline over time in the presence of strong international spillovers and severe asymmetry in the efficiency of human capital investment across countries. When trade is introduced, we find that stronger home spillovers, relative to foreign spillovers, improve the terms of trade in the foreign country due to supply effects.

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