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Remarks on an exact Seiberg-Witten map

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We obtain the leading derivative corrections to an expression for the Seiberg-Witten map given by Banerjee and Yang and show how they affect the noncommutative deformation of the Maxwell action, as well as the matter coupling in noncommutative emergent gravity.

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The Seiberg-Witten map relates noncommutative gauge potentials, fields, and transformation parameters to their commutative counterparts [1]. It is defined such that gauge transformations in the commutative theory induce gauge transformations in the corresponding noncommutative theory. This is the Seiberg-Witten consistency condition. Exact solutions [2–7], as well as series expansions in terms of the noncommutative parameters $\theta^{\mu\nu}$ (see for example [8]) have been obtained for the map. They have often been applied for the purpose of finding corrections to standard electromagnetic theory and to general relativity (for example [9–12]). However, as the results are generally rather technically involved, the map has rarely been applied beyond the second order in $\theta^{\mu\nu}$. Since $\theta^{\mu\nu}$ has units of length squared, the corrections are expansions in $\theta^{\mu\nu}/\text{length-squared}$, and so results obtained so far are generally only valid for length scales much larger than $\sqrt{\theta^{\mu\nu}}$.

There exists, on the other hand, an extremely simple expression for the map, known in [3], which was discussed by Banerjee and Yang [13,14] and referred to as exact. It relates commutative field strengths in one coordinate system to their noncommutative counterparts in another coordinate system—without involving derivatives of the fields. The map was applied in [13,14] to give a nonlinear deformation of electrodynamics and an emergent metric for gravity. The usual Maxwell equations result in the former, along with *local* relations between the usual four vector fields \vec{D} , \vec{H} , \vec{E} , and \vec{B} , and describe an effective noncommutative medium. In the context of noncommutative emergent gravity [15–18], Banerjee and Yang recovered the standard minimal coupling of matter fields to gravity. More specifically, for the case of a scalar field ϕ in four dimensional space-time, all interactions with the metric tensor $g_{\mu\nu}$ are contained in the action

$$-\frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi. \quad (1)$$

However, it is well known [1,3–5,19] that the map for the fields involves derivative corrections, and so strictly speaking, the results in [13,14] can only be considered exact for constant field strengths. Just as they have been known to contribute to *D*-brane actions [20–22], the higher derivative corrections will affect the noncommutative deforma-

tion of electrodynamics and noncommutative emergent gravity. So for the former, the exact Lagrangian cannot be written as a local function of the fields, while for the latter, the exact matter coupling should contain infinitely many higher derivative corrections to (1). In this paper we obtain the leading order derivative corrections for these two dynamical systems. The procedure to be used is simply to demand that the Seiberg-Witten consistency condition is satisfied up to this order. It can be extended to arbitrary order, although this procedure most likely will not lead to simple recursion relations.

We begin by showing that the map discussed in [13,14] [which is equivalent to Eqs. (7), (8), and (10) in this article] fails to satisfy the Seiberg-Witten consistency condition starting at third order in $\theta^{\mu\nu}$. We specialize to *U*(1) gauge theory and utilize the Groenewold-Moyal star product [23,24]

$$\star = \exp\left\{\frac{i}{2} \theta^{\mu\nu} \frac{\overleftarrow{\partial}}{\partial \xi^\mu} \frac{\overrightarrow{\partial}}{\partial \xi^\nu}\right\}, \quad (2)$$

where here $\theta^{\mu\nu} = -\theta^{\nu\mu}$ are constants and $\frac{\overleftarrow{\partial}}{\partial \xi^\mu}$ and $\frac{\overrightarrow{\partial}}{\partial \xi^\mu}$ are left and right derivatives, respectively, with respect to some coordinates ξ^μ , $\mu = 0, 1, 2, 3$, which parametrize the space-time manifold. The noncommutative gauge theory is described by potentials \hat{A}_μ and field strengths $\hat{F}_{\mu\nu}$

$$\hat{F}_{\mu\nu}(\xi) = \frac{\partial}{\partial \xi^\mu} \hat{A}_\nu(\xi) - \frac{\partial}{\partial \xi^\nu} \hat{A}_\mu(\xi) - i[\hat{A}_\mu(\xi), \hat{A}_\nu(\xi)]_\star, \quad (3)$$

where $[\]_\star$ denotes the star commutator, i.e., $[f, g]_\star \equiv f \star g - g \star f$ for any two functions f and g . So, for example, $[\xi_\mu, \xi_\nu]_\star = i\theta^{\mu\nu}$. The Groenewold-Moyal star implies that only odd powers of $\theta^{\mu\nu}$ appear in the expansion of the star commutator

$$[f, g]_\star = i\theta^{\mu\nu} \frac{\partial f}{\partial \xi^\mu} \frac{\partial g}{\partial \xi^\nu} - \frac{i}{24} \theta^{\mu\nu} \theta^{\rho\sigma} \theta^{\kappa\lambda} \frac{\partial^3 f}{\partial \xi^\mu \partial \xi^\rho \partial \xi^\kappa} \times \frac{\partial^3 g}{\partial \xi^\nu \partial \xi^\sigma \partial \xi^\lambda} + \mathcal{O}(\theta^5). \quad (4)$$

The noncommutative potentials and fields have gauge variations

$$\delta \hat{A}_\mu(\xi) = \frac{\partial}{\partial \xi^\mu} \hat{\Lambda}(\xi) + i[\hat{\Lambda}(\xi), \hat{A}_\mu(\xi)]_\star, \quad (5)$$

$$\delta \hat{F}_{\mu\nu}(\xi) = i[\hat{\Lambda}(\xi), \hat{F}_{\mu\nu}(\xi)]_\star, \quad (6)$$

where $\hat{\Lambda}(\xi)$ are infinitesimal parameters.

The Seiberg-Witten map relates \hat{A}_μ , $\hat{F}_{\mu\nu}$, and $\hat{\Lambda}$ to their commutative counterparts, A_μ , $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, and Λ , respectively. The expression in [13,14] gives the noncommutative fields $\hat{F}_{\mu\nu}$ evaluated for coordinates ξ^μ in terms of the commutative fields $F_{\mu\nu}$ evaluated for coordinates x^μ , where x^μ and ξ^μ are related via the noncommutative potential

$$x^\mu(\xi) = \xi^\mu + \theta^{\mu\nu} \hat{A}_\nu(\xi). \quad (7)$$

Thus

$$\hat{F}_{\mu\nu}(\xi) = \mathcal{G}_{\mu\nu}(x(\xi)), \quad (8)$$

and so

$$\frac{\partial \hat{F}_{\mu\nu}}{\partial \xi^\mu} = \frac{\partial \mathcal{G}_{\mu\nu}}{\partial x^\sigma} \frac{\partial x^\sigma}{\partial \xi^\mu}. \quad (9)$$

According to [13,14], $\mathcal{G}_{\mu\nu}$ depends only on $F_{\mu\nu}$ (and $\theta^{\mu\nu}$), and not its derivatives. Specifically,

$$\mathcal{G}_{\mu\nu}(x) = F_{\mu\rho}(x) \left[\frac{1}{1 + \theta F(x)} \right]^\rho_\nu. \quad (10)$$

This was shown to be valid for constant fields $F_{\mu\nu}$ to all orders in θ [3]. For nonconstant $F_{\mu\nu}$ one can easily compare the commutative and noncommutative fields evaluated in a common coordinate system, upon expanding in θ

$$\begin{aligned} \hat{F}_{\mu\nu} &= F_{\mu\nu} + F_{\mu\nu}^{(1)} + F_{\mu\nu}^{(2)} + \dots, \\ \mathcal{G}_{\mu\nu} &= F_{\mu\nu} - (F\theta F)_{\mu\nu} + (F\theta F\theta F)_{\mu\nu} + \dots, \\ x^\mu &= \xi^\mu + \theta^{\mu\nu} (A_\nu + A_\nu^{(1)} + \dots), \end{aligned} \quad (11)$$

where $A_\mu^{(n)}$ and $F_{\mu\nu}^{(n)}$ denote, respectively, terms in the noncommutative potentials and field strengths of n th order in θ . So, for example,

$$\begin{aligned} F_{\mu\nu}^{(1)} &= -(F\theta F)_{\mu\nu} + \partial_\rho F_{\mu\nu} (\theta A)^\rho, \\ F_{\mu\nu}^{(2)} &= (F\theta F\theta F)_{\mu\nu} - \partial_\rho (F\theta F)_{\mu\nu} (\theta A)^\rho + \partial_\rho F_{\mu\nu} (\theta A^{(1)})^\rho \\ &\quad + \frac{1}{2} \partial_\rho \partial_\sigma F_{\mu\nu} (\theta A)^\rho (\theta A)^\sigma. \end{aligned} \quad (12)$$

These results agree with known solutions for the Seiberg-Witten map up to second order in θ . (See, for example, [25].) Below we show that (10) cannot be relied upon for nonconstant $F_{\mu\nu}$ beyond the second order, or more generally, that any $\mathcal{G}_{\mu\nu}$ expressed in terms of only $F_{\mu\nu}$ does not satisfy the Seiberg-Witten consistency conditions beyond the second order.

Since $\mathcal{G}_{\mu\nu}$ is a function only of the commutative field strengths it is gauge invariant. On the other hand, from (5) and (7), the coordinates x^μ of the domain of $\mathcal{G}_{\mu\nu}$ have nonvanishing gauge variations,

$$\delta x^\mu(\xi) = i[\hat{\Lambda}(\xi), x^\mu(\xi)]_\star \quad (13)$$

Infinitesimal gauge variations of the right-hand side of (8) are then

$$\begin{aligned} \delta \mathcal{G}_{\mu\nu}(x) &= i \frac{\partial \mathcal{G}_{\mu\nu}}{\partial x^\eta} [\hat{\Lambda}, x^\eta]_\star \\ &= \frac{\partial \mathcal{G}_{\mu\nu}}{\partial x^\eta} \left(-\theta^{\alpha\beta} \frac{\partial \hat{\Lambda}}{\partial \xi^\alpha} \frac{\partial x^\eta}{\partial \xi^\beta} + \frac{1}{24} \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \right. \\ &\quad \left. \times \frac{\partial^3 \hat{\Lambda}}{\partial \xi^\alpha \partial \xi^\rho \partial \xi^\kappa} \frac{\partial^3 x^\eta}{\partial \xi^\beta \partial \xi^\sigma \partial \xi^\lambda} + \mathcal{O}(\theta^5) \right). \end{aligned} \quad (14)$$

We wish to compare this to infinitesimal gauge variations of the left-hand side of (8) given by (6),

$$\begin{aligned} \delta \hat{F}_{\mu\nu}(\xi) &= -\theta^{\alpha\beta} \frac{\partial \hat{\Lambda}}{\partial \xi^\alpha} \frac{\partial \hat{F}_{\mu\nu}}{\partial \xi^\beta} + \frac{1}{24} \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \\ &\quad \times \frac{\partial^3 \hat{\Lambda}}{\partial \xi^\alpha \partial \xi^\rho \partial \xi^\kappa} \frac{\partial^3 \hat{F}_{\mu\nu}}{\partial \xi^\beta \partial \xi^\sigma \partial \xi^\lambda} + \mathcal{O}(\theta^5). \end{aligned} \quad (15)$$

Using (9), the terms proportional to $\frac{\partial \hat{\Lambda}}{\partial \xi^\alpha}$ in (14) and (15) agree, but the terms proportional to $\frac{\partial^3 \hat{\Lambda}}{\partial \xi^\alpha \partial \xi^\rho \partial \xi^\kappa}$ do not. In (15) the latter contribute at third order in θ , while they contribute at fourth order in (14) (since $\frac{\partial^3 x^\eta}{\partial \xi^\beta \partial \xi^\sigma \partial \xi^\lambda}$ goes like θ to leading order). Therefore (8) can only be trusted up to second order in θ , and moreover, the proof did not require knowledge of the explicit expression for $\mathcal{G}_{\mu\nu}$ in terms of $F_{\mu\nu}$ given in (10).

The exact result for $\mathcal{G}_{\mu\nu}$ in (8) must contain infinitely many derivatives of the field strengths. Here we can easily obtain the leading order derivative corrections to (10) and to the noncommutative deformation of the Maxwell action. In order for (14) and (15) to agree at third order in θ , we have to add a term to (10) which at leading order has the gauge variation

$$\frac{1}{24} \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \partial_\alpha \partial_\rho \partial_\kappa \Lambda \partial_\beta \partial_\sigma \partial_\lambda F_{\mu\nu}. \quad (16)$$

So if we now replace (10) by

$$\begin{aligned} \mathcal{G}_{\mu\nu} &= F_{\mu\nu} - (F\theta F)_{\mu\nu} + (F\theta F\theta F)_{\mu\nu} - (F\theta F\theta F\theta F)_{\mu\nu} \\ &\quad + \frac{1}{24} \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \partial_\alpha \partial_\rho A_\kappa \partial_\beta \partial_\sigma \partial_\lambda F_{\mu\nu} + \mathcal{O}(\theta^4), \end{aligned} \quad (17)$$

we get a corrected solution to the Seiberg-Witten equations which is valid up to third order in θ and which now involves derivatives of the field strength. From (8), the action for noncommutative $U(1)$ gauge theory in four-dimensional space-time

$$\mathcal{S}_F = -\frac{1}{4} \int d^4\xi \hat{F}_{\mu\nu}(\xi) \star \hat{F}^{\mu\nu}(\xi) \quad (18)$$

can be reexpressed as an action for commutative fields

$$\mathcal{S}_F = -\frac{1}{4} \int d^4x \left| \frac{\partial \xi}{\partial x} \right| \mathcal{G}_{\mu\nu}(x) \mathcal{G}^{\mu\nu}(x), \quad (19)$$

since for the Groenewold-Moyal star $\int d^4\xi \alpha(\xi) \star \beta(\xi) = \int d^4\xi \alpha(\xi) \beta(\xi)$, for well-behaved functions $\alpha(\xi)$ and $\beta(\xi)$. Then (17) leads to the presence of terms in the action which involve derivatives of the commutative field strength. The leading third order contribution of such terms can be expressed as

$$\frac{1}{192} \int d^4x \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \partial_\alpha \partial_\rho F_{\kappa\lambda} \partial_\beta F_{\mu\nu} \partial_\sigma F^{\mu\nu}. \quad (20)$$

For this we did not need to consider the Jacobian factor $|\frac{\partial \xi}{\partial x}|$. Higher derivative terms from the Jacobian factor can also contribute, but these occur at higher order in θ . This is more easily seen in two dimensions where the inverse Jacobian is simply

$$\left| \frac{\partial x}{\partial \xi} \right| = 1 + \theta^{01} \left[\frac{\partial}{\partial \xi_0} \hat{A}_1(\xi) - \frac{\partial}{\partial \xi_1} \hat{A}_0(\xi) - i[\hat{A}_0(\xi), \hat{A}_1(\xi)]_1 \right], \quad (21)$$

where $[\]_1$ denotes the first order term of the star commutator. Up to third order in θ , one can approximate the right-hand side by $1 + \theta^{01} \hat{F}_{01}$, and upon substituting in (17), one gets that the leading higher derivative term contributes at fourth order in θ .

In [15] the Seiberg-Witten map was used to transform noncommutative $U(1)$ gauge theory into an effective theory of gravity, or noncommutative emergent gravity. For this one can couple the gauge theory to a scalar field and obtain the Seiberg-Witten map from the commutative to the noncommutative covariant derivative of the field, which we denote, respectively, by $\partial_\mu \phi$ and $\hat{D}_\mu \hat{\phi}$. Here the noncommutative scalar field $\hat{\phi}$ is assumed to be in the adjoint representation of the gauge group with

$$\hat{D}_\mu \hat{\phi}(\xi) = \frac{\partial}{\partial \xi^\mu} \hat{\phi}(\xi) + i[\hat{\phi}(\xi), \hat{A}_\mu(\xi)]_\star, \quad (22)$$

while the commutative field ϕ is gauge invariant. A simple expression for the Seiberg-Witten map was given in [13], which is of the form

$$\hat{D}_\mu \hat{\phi}(\xi) = \mathcal{D}_\mu \phi(x(\xi)), \quad (23)$$

where $\mathcal{D}_\mu \phi$ is assumed to depend only on $\partial\phi$ and F . Specifically,

$$\begin{aligned} \mathcal{D}_\mu \phi &= \left[\frac{1}{1 + F\theta} \right]_\mu^\nu \partial_\nu \phi \\ &= (1 - F\theta + F\theta F\theta + \dots)_\mu^\nu \partial_\nu \phi. \end{aligned} \quad (24)$$

This was obtained using a compactification of (10). We can use it to compare $\hat{D}_\mu \hat{\phi}$ and $\partial\phi$ evaluated in a common coordinate system. Expanding up to second order in θ ,

$$\hat{D}_\mu \hat{\phi} = \partial_\mu \phi + [D_\mu \phi]^{(1)} + [D_\mu \phi]^{(2)} + \dots \quad (25)$$

Then

$$\begin{aligned} [D_\mu \phi]^{(1)} &= -(F\theta)_\mu^\nu \partial_\nu \phi + (\theta A)^\nu \partial_\mu \partial_\nu \phi, \\ [D_\mu \phi]^{(2)} &= (F\theta F\theta)_\mu^\nu \partial_\nu \phi - \partial_\rho (F\theta)_\mu^\nu (\theta A)^\rho \partial_\nu \phi \\ &\quad - (F\theta)_\mu^\nu (\theta A)^\rho \partial_\rho \partial_\nu \phi + (\theta A^{(1)})^\rho \partial_\mu \partial_\rho \phi \\ &\quad + \frac{1}{2} (\theta A)^\rho (\theta A)^\sigma \partial_\mu \partial_\rho \partial_\sigma \phi, \end{aligned} \quad (26)$$

which can be verified to satisfy the Seiberg-Witten consistency condition up to this order. Unfortunately, (24) cannot be trusted beyond second order in θ . Following the previous arguments, we get a mismatch of the gauge variations of the left-hand side of (23), i.e.,

$$\delta \hat{D}_\mu \hat{\phi} = i[\hat{\Lambda}, \hat{D}_\mu \hat{\phi}]_\star, \quad (27)$$

with those of the right-hand side

$$\delta \mathcal{D}_\mu \phi = i \frac{\partial \mathcal{D}_\mu \phi}{\partial x^\eta} [\hat{\Lambda}, x^\eta]_\star \quad (28)$$

As before, the exact result for the Seiberg-Witten map must contain infinitely many derivatives of the fields. In order to obtain the correct result up to third order we need to add a term to $\mathcal{D}_\mu \phi$ in (24) which has the gauge variation

$$\frac{1}{24} \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \partial_\alpha \partial_\rho \partial_\kappa \Lambda \partial_\beta \partial_\sigma \partial_\lambda \partial_\mu \phi, \quad (29)$$

and so up to third order $\mathcal{D}_\mu \phi$ can take the form

$$\begin{aligned} \mathcal{D}_\mu \phi &= (1 - F\theta + F\theta F\theta - F\theta F\theta F\theta + \dots)_\mu^\nu \partial_\nu \phi \\ &\quad + \frac{1}{24} \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \partial_\alpha \partial_\rho A_\kappa \partial_\beta \partial_\sigma \partial_\lambda \partial_\mu \phi + \dots \end{aligned} \quad (30)$$

Using the map, the action for noncommutative scalar field in four-dimensional space-time

$$\mathcal{S}_\phi = -\frac{1}{2} \int d^4\xi \hat{D}_\mu \hat{\phi}(\xi) \star \hat{D}^\mu \hat{\phi}(\xi) \quad (31)$$

can be reexpressed as an action for the commutative fields ϕ

$$\mathcal{S}_\phi = -\frac{1}{2} \int d^4x \left| \frac{\partial \xi}{\partial x} \right| \mathcal{D}_\mu \phi(x) \mathcal{D}^\mu \phi(x). \quad (32)$$

Then (30) implies the presence of terms in the action which involve higher derivatives of the scalar field, which to leading order can be written

$$\frac{1}{192} \int d^4x \theta^{\alpha\beta} \theta^{\rho\sigma} \theta^{\kappa\lambda} \partial_\alpha \partial_\rho F_{\kappa\lambda} \partial_\beta \partial_\mu \phi \partial_\sigma \partial^\mu \phi, \quad (33)$$

where again we did not need to consider the Jacobian factor at this order. Clearly, terms such as these cannot be absorbed in the standard coupling of the scalar field to gravity (1). Thus, as before, higher derivative terms appear beginning at third order in θ . They represent additional and

novel contributions to the dynamics for noncommutative emergent gravity theories [15–18].

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